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Determination of elastic properties in a plate by Lamb wave analysis and particle swarm optimisation

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Abstract

Traditionally dispersion characteristics for any plate structure are evaluated using the Rayleigh-Lamb solution and an assumption that the material properties are known. In cases where the material properties may be difficult to obtain the opposite is proposed. Specifically, the dispersive characteristics of Lamb waves may be used to extract the mechanical properties of a given test coupon. This paper presents a particle swarm optimisation (PSO) methodology to achieve this. The results from a series of tests on an aluminium coupon were used to validate the approach. The PSO was found to successfully match dispersion characteristics identified in a 2D FFT.

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Nomenclature

v	velocity
x	position
xb	Particle's best position
sb	Swarm's best position
f	frequency
E	Young's modulus
G	Shear modulus

Greek symbols

ω	*inertial constant*

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φ	*attraction constant*
v	*Poisson's ratio*
ρ	*Density*
Subscripts	
numerical	COMSOL numerically obtained results
disperse	DISPERSE global matrix method obtained results

1. Introduction

Lamb waves are considered to be inherently complex due to their dispersive characteristics and multi-mode nature. However, having been extensively studied for isotropic materials such as steel and aluminium. Lamb wave propagation in these materials is well understood and accurate models can be established. In relation to metallic structures it is possible to find numerous papers covering numerical modelling with respect to the prediction of displacement[1, 2], attenuation[3], scatter[4-6], modal content and transduction[7]. The approach presented in this paper takes the inverse approach where experimental measurements of the dispersion spectrum are used to determine the elastic properties of a material. This technique is currently being extended to other material types including fibre reinforced plastics.

This paper uses a numerical model and particle swarm optimisation (PSO) which simplifies the task and provides versatility. The PSO is a stochastic optimisation technique based on the movement of swarms. It is considered to be robust and stable. It does not require calculation of a gradient function which makes it far easier to use. This paper begins with a brief explanation of the PSO method, followed by a study of the effects of numerical resolution on the modelling of Lamb waves. The resolution study shows that an inaccurate simulation is possible if care is not taken. A verification of the PSO algorithm was done on an aluminium specimen with known material properties.

2. Particle Swarm Optimisation Method

The standard PSO method [8] was employed in this study due to its robustness and ease of use. This makes it suitable for this paper's objective of optimising material properties to achieve a given target modal content. In the present case the difference between known dispersion characteristics and those obtained with estimated material properties is used to provide feedback to the optimisation algorithm. The PSO algorithm has been summarised in the flow chart shown in **Fig 1**. Using this method, a number of particles are employed which are treated as points moving in an n-dimensional space. Each particle adjusts its direction and velocity according to its previous experience and the experience of its neighbouring particles, mimicking the behaviour of a swarm. Its key objective is to minimise the fitness function in a user defined search space. The PSO's appeal comes from the simplicity of its algorithm.

The calculation of each particle's velocity occurs iteratively and is central to the performance of the PSO and is given by

$$v=\omega v+\varphi_p r_p(xb-x)+\varphi_g r_g(sb-x) \qquad \textit{Equation (1)}$$

where ω, φ_p and φ_g are user selected constants. r_p and r_g are randomly generated numbers between 0 and 1.

The inertial constant ω localises a search when decreased. φ_p and φ_g govern the influence from the particle's best position and the swarm's best position respectively. These constants are often adjusted depending on the behaviour of the fitness function e.g. where multiple positions may present local minima in the fitness function the inertial constant ω may be increased to globalise the search. The fitness function $f(x)$ has the biggest impact on the PSO's function and may widely vary according the application of the PSO. This will be discussed further for each individual optimisation.

3. Experimental results and discussions

The validity of the PSO method was tested on an aluminium plate. Due to its isotropic nature, only 3 material properties play a role in the characteristics of propagating Lamb waves. Young's modulus (E), shear modulus (G) and density (ρ) can be used to completely define its Lamb wave dispersion characteristics. In this test of the PSO, the fitness function was programmed to measure the fit between a direct solution generated by DISPERSE and the numerical solution generated by COMSOL.

The PSO algorithm was written in MATLAB which is able to interface with COMSOL. This allows the PSO to perform necessary changes to optimise the numerical model's material properties. A fitness function has been written for the PSO by calculating the difference between numerical and DISPERSE generated wave numbers. This is summed for all modes and frequencies present to create the final fitness function

$$fitness = \sum n_f \sum n_{modes}\, k_{numerical}(f,mode) - k_{disperse}(f,mode)$$

Where n_f is the number of frequencies and n_{modes} is the number of modes.

A 2 dimensional PSO was run to optimise E and ν. For isotropic materials X can be related to G by

$$G = \frac{E}{2(1+\upsilon)}$$

To test the PSO, density was prescribed the known correct value to reflect the fact that in most practical circumstances the density of a material is easily measurable, while E and ν were given incorrect values as shown in Table 1. Table 2 summarises the PSO constants which are used throughout this paper.

4. Optimisation of material properties

Each model was excited with a 400 kHz and 800 kHz Hanning windowed sinusoid with 10 cycles. S0 and A0 modes were excited in separate models by applying the appropriate forcing to obtain the desired respective displacement profile. A total of 200 numerical simulations were performed to optimise the material properties. 2D FFTs were performed on the top surface using out of plane displacement data.

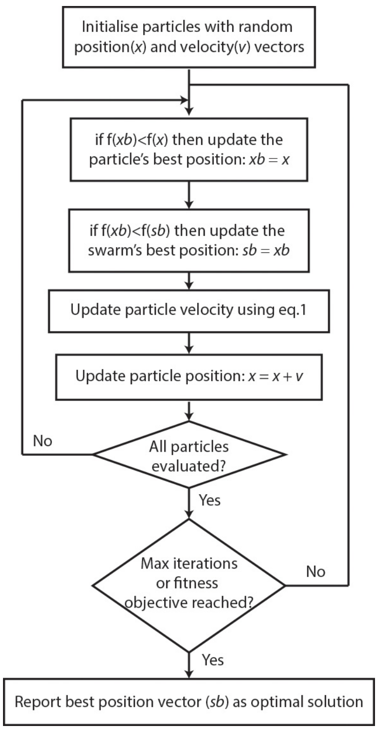

Fig 1 Flow chart describing PSO methodology

E	62 Gpa
ν	0.3
ρ	2700 kg/m^3
thickness	2.08 mm

Table 1: Initial conditions

Swarm size	10
Max Iterations	10
w	0.5
φ_p	1.8
φ_g	1.8

Table 2: PSO constants

After 10 iterations the PSO settled at a solution 70.34 GPa Young's modulus and 0.301 Poisson's ratio. Visually, the results shown in Figure 2 shows a good fit between the recorded Lamb modes and the DISPERSE reference. Figure 3 suggests a reasonable fit can be achieved for a wide range of Poisson's ratio values, with the correct value falling within this range. The fact the PSO converged to an incorrect value suggests a deficiency in the fitness function formulation. Table 3 below summarises the outcomes:

Table 3 Summary of PSO outcome

	E (GPa)	υ
Disperse	70.76	0.338
2 frequency excitation	70.34	0.301

5. Optimisation using multiple excitations

The number of frequencies employed in the optimisation was increased from two to ten spanning a range from 100 to 1000 kHz at intervals of 100kHz. These waveforms were summed into a single excitation resulting in the signal and spectrum shown in Figure 4. As done previously A0 and S0 modes were considered separately. This is demonstrated in the spectrums presented in Figure 5.

After 10 iterations the PSO settled on a Young's modulus of 70.50 GPa and Poisson's ratio of 0.342. As expected, the added information from multiple excitations yields improved accuracy. Table 4 summarises the results from both PSO attempts. In this context the PSO searches for the appropriate solutions over a wide band of excited Lamb waves. Intuitively with 2 excitation frequencies it is possible to achieve a good fit using a variety of material properties. Conversely involving more frequencies helps alleviate this ambiguity which leads the PSO to a single solution much closer to the correct material properties. This is also reflected in the smoother and more consistent optimisation map shown in Figure 6.

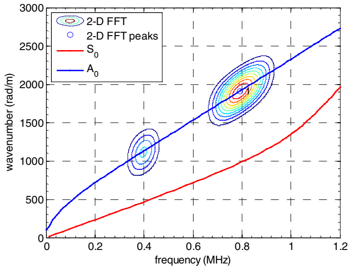

Fig 2(a): 2DFFT with DISPERSE overlay with A0 excitation

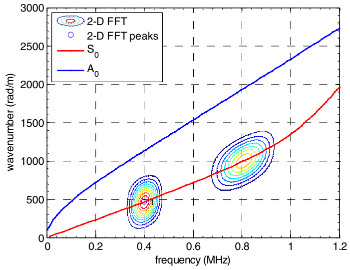

Fig 2(b): 2DFFT with DISPERSE overlay with S0 excitation

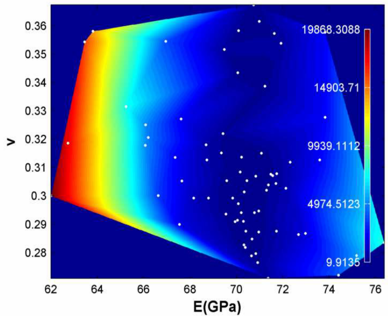

Fig 3: Interpolated fitness map in the 2 variable optimisation space. White dots indicate particles where fitness functions are evaluated

Table 4 Summary of PSO results

	E (GPa)	υ
Disperse	70.76	0.3375
2 frequency excitation	70.3358	0.3014
Multipoint excitation	70.4963	0.3418

6. Conclusions

An optimisation approach has been presented as an efficient method of obtaining material properties by analysing Lamb waves. The PSO algorithm was chosen due to its robust nature and simple implementation. This combination proved adequate in the two variable optimisations performed in this paper. A series of studies were performed to provide confidence in the results by ensuring convergence and accuracy of the numerical simulations. Future work will focus on composite materials an expanded range of variables as well as greater numbers of variables being simultaneously optimised. However the efficacy of such an approach will depend on the supply of sufficient information and on an appropriate specification of a fitness function. The next stage will most likely require further modifications to the correlation method or sophisticated image processing methods to interpret the modes present in a 2D FFT.

Acknowledgements

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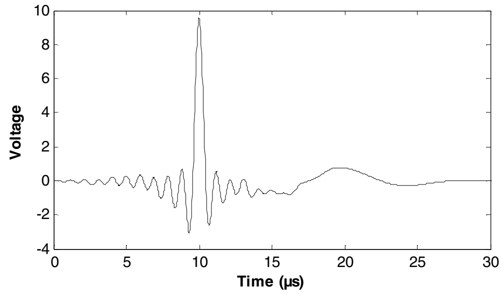

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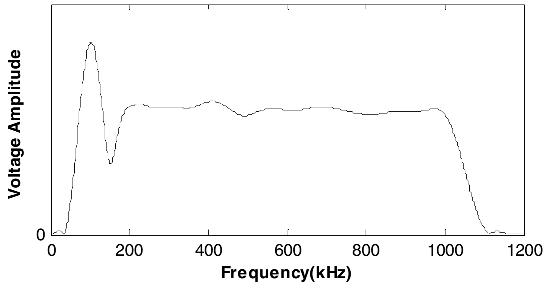

b)

Figure 4 a) Excitation Signal. b) Excitation spectrum

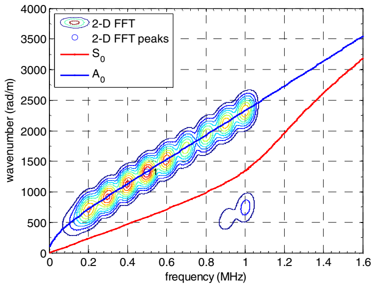

Fig 5(a): 2DFFT with DISPERSE overlay with A0 excitation

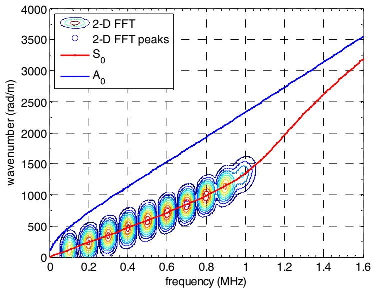

Fig 5(b): 2DFFT with DISPERSE overlay with S0 excitation

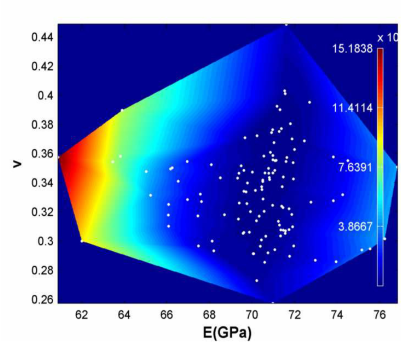

Fig 6: 1 Interpolated fitness map in the 2 variable optimisation space. White dots indicate particles where fitness functions are evaluated.

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