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Quark–hadron phase transition in a magnetic field

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ABSTRACT

Quark–hadron phase transition in QCD in the presence of magnetic field is studied. It is shown that both the temperature of a phase transition and latent heat decrease compared to the case of zero magnetic field. The phase diagram in the plane temperature-magnetic field is presented. Critical point, $T_* = 104$ MeV, $\sqrt{eH_*} = 600$ MeV, for which the latent heat goes to zero, is found.

1. Introduction

QCD is essentially nonperturbative at low temperatures $T < T_c$ (T_c is a quark–hadron phase transition temperature) and is characterized by confinement and spontaneous chiral symmetry breaking. In the hadron phase, at low temperature, the dominating contribution to the partition function of the system is given by lightest particles in the physical spectrum. In the case of QCD these particles are π-mesons, which in the limit of massless quarks are Goldstone excitations in the chiral condensate. It is a common method in the low temperature physics of QCD to use effective chiral theory [1–3], often called chiral perturbation theory (ChPT).

One of the important questions is the phase structure of vacuum in the external magnetic field H. In the recent paper [4] it is argued that high magnetic fields $eH \sim 10^2$–10^4 MeV^2 are created in heavy ion collisions. Such magnetic fields should lead to effects that can be experimentally observed at RHIC. Also high magnetic fields $eH \sim \Lambda^2_{\mathrm{QCD}}$ could exist in the early universe at the scale of strong interactions. Such high field strength can lead to new interesting phenomena at the stage of quark–hadron phase transition. At the same time it is interesting to study the influence of the external magnetic field on the dynamics of strong interactions from purely theoretical point of view. Different nonperturbative phenomena in Abelian magnetic fields were previously studied in [5–20].

In this Letter, we study the quark–hadron phase transition in QCD in Abelian magnetic field. The physics of the considered phenomenon is the following. The plasma of hot quarks and gluons at $T > T_c$ in the magnetic field is a thermodynamic system in a paramagnetic phase. On the other hand, at low temperature, $T < T_c$, hadron matter, which mainly consists of scalar π-mesons, is in diamagnetic phase. The paramagnetic phase is preferable thermodynamically, because it minimizes the free energy (maximizes pressure). Therefore the temperature of the transition from hadron phase to the quark-gluon phase decreases as compared to the case of zero magnetic field $H = 0$. Thus, there is the analogy with the physics of condensed matter: confinement phase corresponds to the diamagnetic gas of scalar π-mesons (we neglect the contribution of heavier hadrons), and deconfinement phase corresponds to the paramagnetic phase of quarks and gluons.

2. Free energy of the QCD vacuum at $T \neq 0$ and $H \neq 0$

The partition function of QCD in Euclidean formulation in the presence of external Abelian field A_μ can be written in a form (here $T = 1/\beta$ is temperature)

$$Z = \exp\left\{-\frac{1}{4}\int_0^\beta dx_4 \int_V d^3x\, F^2_{\mu\nu}\right\} \times \int [DB][D\bar{q}][Dq] \exp\left\{-\int_0^\beta dx_4 \int_V d^3x\, \mathcal{L}\right\}, \quad (1)$$

where QCD Lagrangian in the background field is

$$\mathcal{L} = \frac{1}{4g_0^2}\left(G^a_{\mu\nu}\right)^2 + \sum_{q=u,d} \bar{q}\left[\gamma_\mu\left(\partial_\mu - iQ_q e A_\mu - i\frac{\lambda^a}{2}B^a_\mu\right) + m_q\right]q, \quad (2)$$

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here Q_q is the charge matrix of quarks with flavor $q=(u,d)$, and we omitted ghost and gauge fixing terms for simplicity. Free energy density is given by the expression $\beta V F(T,H,m_q)=-\ln Z$.

Let us consider the hadron phase. At low temperature, $T<T_c$, (T_c is the temperature of chiral phase transition) and at weak external field, $H<\mu^2_{\text{hadr}}\sim(4\pi F_\pi)^2$, the characteristic momentum in the vacuum loops is small, and the theory is described by the effective low-energy Lagrangian L_{eff} [2,3].

$$Z_{\text{eff}}[T,H]=e^{-\beta V F_{\text{eff}}[T,H]} = Z_0[H]\int[DU]\exp\left\{-\int_0^\beta dx_4\int_V d^3x\, L_{\text{eff}}[U,A]\right\}. \tag{3}$$

L_{eff} can be represented as a decomposition in a series of powers of momentum (derivatives) and masses

$$L_{\text{eff}}=L^{(2)}+L^{(4)}+L^{(6)}+\cdots. \tag{4}$$

The leading term in (4) is a Lagrangian of nonlinear σ-model in external field V_μ

$$L^{(2)}=\frac{F_\pi^2}{4}\text{Tr}\left(\nabla_\mu U^+\nabla_\mu U\right)+\Sigma\, Re\,\text{Tr}\left(\hat{M}U^+\right),$$
$$\nabla_\mu U=\partial_\mu U-i[U,V_\mu]. \tag{5}$$

Here U is unitary $SU(2)$ matrix, $F_\pi=93$ MeV is a pion decay constant, and the parameter Σ has the meaning of quark condensate, $\Sigma=|\langle\bar{u}u\rangle|=|\langle\bar{d}d\rangle|$. External Abelian magnetic field H, directed along z axis, corresponds to $V_\mu(x)=(\tau^3/2)eA_\mu(x)$, and vector-potential A_μ is taken in the form $A_\mu(x)=\delta_{\mu2}Hx_1$. We will further neglect the breaking of isospin symmetry of strong interactions and consider masses of light u- and d-quarks equal, $m_u=m_d=m_q$, thus the mass matrix diagonal, $\hat{M}=m_q\hat{I}$.

In one-loop approximation it is sufficient to use the decomposition of L_{eff} up to quadratic over pion field terms. In exponential parametrization of the matrix $U(x)=\exp\{i\tau^a\pi^a(x)/F_\pi\}$ we find that

$$L^{(2)}=\frac{1}{2}\left(\partial_\mu\pi^0\right)+\frac{1}{2}M_\pi^2\left(\pi^0\right)^2 +\left(\partial_\mu\pi^++ieA_\mu\pi^+\right)\left(\partial_\mu\pi^--ieA_\mu\pi^-\right)+M_\pi^2\pi^+\pi^-, \tag{6}$$

where we introduced the fields of charged $\pi^\pm$ and neutral π^0 mesons

$$\pi^\pm=\left(\pi^1\pm i\pi^2\right)/\sqrt{2},\quad \pi^0=\pi^3. \tag{7}$$

The QCD partition function (1) in one-loop approximation of the effective chiral theory takes the form[1]

$$Z^R_{\text{eff}}[T,H]=Z^{-1}_{PT}Z_0[H]\int\left[D\pi^0\right]\left[D\pi^+\right]\left[D\pi^-\right] \times\exp\left\{-\int_0^\beta dx_4\int_V d^3x\, L^{(2)}[\pi,A]\right\}. \tag{8}$$

Here the partition function is normalized to the case of perturbation theory at $T=0$, $H=0$

$$Z_{PT}=\left[\det\left(-\partial_\mu^2+M_\pi^2\right)\right]^{-3/2}. \tag{9}$$

Integrating (8) over π-fields one gets

$$Z^R_{\text{eff}}=Z^{-1}_{PT}Z_0[H]\left[\det{}_T\left(-\partial_\mu^2+M_\pi^2\right)\right]^{-1/2} \times\left[\det{}_T\left(-|D_\mu|^2+M_\pi^2\right)\right]^{-1}, \tag{10}$$

[1] The partition function Z^R_{eff} describes charged $\pi^\pm$ and neutral π^0 Bose gases in magnetic field.

where $D_\mu=\partial_\mu-ieA_\mu$ is a covariant derivative, and the subscript T means that the determinant is evaluated at finite temperature T using standard Matsubara rules. Using (9) and regrouping multipliers in (10) we arrive at the following expression for Z^R_{eff}

$$Z^R_{\text{eff}}[T,H]=Z_0[H]\left[\frac{\det_T\left(-\partial_\mu^2+M_\pi^2\right)}{\det\left(-\partial_\mu^2+M_\pi^2\right)}\right]^{-1/2}\left[\frac{\det\left(-|D_\mu|^2+M_\pi^2\right)}{\det\left(-\partial_\mu^2+M_\pi^2\right)}\right]^{-1} \times\left[\frac{\det_T\left(-|D_\mu|^2+M_\pi^2\right)}{\det\left(-|D_\mu|^2+M_\pi^2\right)}\right]^{-1}. \tag{11}$$

Free energy then takes the form [7]

$$F^R_{\text{eff}}(T,H)=-\frac{1}{\beta V}\ln Z^R_{\text{eff}} =\frac{H^2}{2}+F_{\pi^0}(T)+F_{\pi^\pm}(H)+F_{\text{dia}}(T,H). \tag{12}$$

Here F_{π^0} is the free energy of massive scalar boson

$$F_{\pi^0}(T)=T\int\frac{d^3p}{(2\pi)^3}\ln\left(1-\exp\left(-\sqrt{\mathbf{p}^2+M_\pi^2}/T\right)\right), \tag{13}$$

$F_{\pi^\pm}$ is Schwinger's result for the vacuum energy density of charged scalar particles in magnetic field

$$F_{\pi^\pm}(H)=-\frac{1}{16\pi^2}\int_0^\infty\frac{ds}{s^3}e^{-M_\pi^2 s}\left[\frac{eHs}{\sinh(eHs)}-1\right]. \tag{14}$$

Next, it is technically not hard to generalize the case of the vacuum $H=0$, $T=0$ for charged $\pi^\pm$-mesons

$$F=\text{Tr}\ln\left(p_4^2+\omega_0^2(\mathbf{p})\right) \tag{15}$$

to the case of $H\neq0$, $T\neq0$. Omitting the details, we will note that it corresponds to the following substitutions

$$p_4\to\omega_k=2\pi kT\quad(k=0,\pm1,\ldots),$$
$$\omega_0=\sqrt{\mathbf{p}^2+M_\pi^2}\to\omega_n=\sqrt{p_z^2+M_\pi^2+eH(2n+1)} \tag{16}$$

and

$$\text{Tr}\to\frac{eHT}{2\pi}\sum_{n=0}^\infty\sum_{k=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi},$$

where the degeneracy multiplicity of Landau levels $eH/2\pi$ is taken into account. Summing over Matsubara frequencies, one obtains the following result for the diamagnetic part of free energy of charged Bose gas

$$F_{\text{dia}}(T,H)=\frac{eHT}{\pi^2}\sum_{n=0}^\infty\int_0^\infty dk\ln\left(1-\exp(-\omega_n/T)\right),$$
$$\omega_n=\sqrt{k^2+M_\pi^2+eH(2n+1)}, \tag{17}$$

here ω_n are Landau levels of $\pi^\pm$-mesons in a constant magnetic field H.

Expanding $\ln(\cdots)$ in the integrand of (13), (17) in a series, one gets the following expressions:

$$F_{\pi^0}=-\frac{M_\pi^2T^2}{2\pi^2}\sum_{n=1}^\infty\frac{1}{n^2}K_2\left(n\frac{M_\pi}{T}\right) \tag{18}$$

and

$$F_{\text{dia}}=-\frac{eHT}{\pi^2}\sum_{n=0}^\infty\sqrt{M_\pi^2+eH(2n+1)} \times\sum_{k=1}^\infty\frac{1}{k}K_1\left(\frac{k}{T}\sqrt{M_\pi^2+eH(2n+1)}\right), \tag{19}$$

where K_n is the Macdonald function.

3. Quark–hadron phase transition in magnetic field

In order to determine the temperature of phase transition we will write down the pressure in two phases. At zero chemical potential pressure is equal to minus free energy.

In the confinement phase pressure of π-mesons in magnetic field takes the form (see (12))

$$P_1(T,H)=P_{\pi^0}(T)+P_{\pi^\pm}(H)+P_{\rm dia}(T,H), \tag{20}$$

where the pressure of neutral gas of π^0-mesons is $P_{\pi^0}(T)=-F_{\pi^0}(T)$. Renormalized contribution of vacuum polarization of charged $\pi^\pm$-mesons to the pressure, which does not depend on temperature (Schwinger polarization) is

$$P_{\pi^\pm}(H)=\frac{1}{16\pi^2}\int_0^\infty\frac{ds}{s^3}e^{-M_\pi^2/s}\left[\frac{eHs}{\sinh(eHs)}-1+\frac{1}{6}(eHs)^2\right]. \tag{21}$$

Diamagnetic term in the pressure, which comes from charged $\pi^\pm$ mesons is $P_{\rm dia}(T,H)=-F_{\rm dia}(T,H)$.

Fermion (quark) determinant at finite temperature in the magnetic field can be considered in a similar way. Then the pressure in the quark–gluon plasma phase has the form

$$P_{\rm pl}(T,H)=2\left(N_c^2-1\right)\frac{\pi^2}{90}T^4+P_q(H)+P_{\rm para}(T,H). \tag{22}$$

The term proportional to $\propto T^4$ comes from the gas of hot gluons, $P_q(H)$ is the contribution from vacuum polarization of quarks to the pressure

$$P_q(H)=-\frac{1}{8\pi^2}\sum_{q=u,d}\int_0^\infty\frac{ds}{s^3}e^{-m_q^2s}\times\left[|e_q|Hs\,\mathrm{cth}\left(|e_q|Hs\right)-1-\frac{1}{3}(e_qHs)^2\right] \tag{23}$$

and $P_{\rm para}$ is a paramagnetic term

$$P_{\rm para}=2N_c\sum_{q=u,d}\frac{|e_q|H}{2\pi}T\sum_{n=0}^\infty\sum_{\sigma=\pm\frac12}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}\ln\left(1+e^{-\omega_q/T}\right),$$
$$\omega_q=p_z^2+m_q^2+|e_q|H(2n+1+2\sigma). \tag{24}$$

An important physical property of the phase transition is the rearrangement of the nonperturbative QCD vacuum. Due to the scale anomaly in the trace of the energy-momentum tensor the new dimensional quantity, gluon condensate $\langle G^2\rangle\equiv\langle(gG^a_{\mu\nu})^2\rangle$, emerges in QCD. Nonperturbative energy density of the vacuum is related to $\langle G^2\rangle$:

$$\varepsilon_{\rm vac}=-\frac{b}{128\pi^2}\langle G^2\rangle, \tag{25}$$

where $b=(3N_c-2N_f)/3$, $N_c=3$ is the number of colors, and $b=29/3$ for $N_f=2$.

Energy density of vacuum is the negative quantity, and the state with $\langle G^2\rangle\neq0$ turns out to be thermodynamically advantageous. Theoretical studies [21,22] and numerical computations in the lattice approximation of QCD [23] show that at the point of phase transition T_c one part of the condensate (chromoelectric part) turns to zero, while the chromomagnetic condensate remains almost unchanged as compared to the case $T=0$. In the vacuum at $T=0$ $\langle(E_i^a)^2\rangle=\langle(H_i^a)^2\rangle$, and therefore vacuum energy density above phase transition appears to be less in magnitude than below phase transition, and the difference is approximately

$$\Delta\varepsilon_v=\frac12|\varepsilon_{\rm vac}|=\frac{b}{256\pi^2}\langle G^2\rangle. \tag{26}$$

Taking this into account, the quantity $-\Delta\varepsilon_v$ should be added to the equation of state in the plasma phase. Thus, the pressure in the quark-gluon plasma state is given by the expression

$$P_2(T)=P_{\rm pl}(T,H)-\Delta\varepsilon_v. \tag{27}$$

Eq. (27) is similar to the equation for the phase transition in MIT bag model, where bag constant B plays the role of $\Delta\varepsilon_v$.

Phase transition temperature can be found from the condition of equality of pressures in both phases

$$P_1(T_c,H)=P_2(T_c,H). \tag{28}$$

Let us now consider "weak" magnetic field, $eH\ll T_c^2$. Then the Schwinger contribution to the pressure can be neglected. In the weak field one can use Euler–Maclaurin formula for $P_{\rm dia}$

$$\frac12F(a)+\sum_{n=1}^\infty F(a+n)\approx\int_0^\infty dx\,F(x)-\frac{1}{12}F'(a) \tag{29}$$

and (17) can be rewritten in the form

$$P_{\rm dia}=2P_{\pi^0}(T)-\frac{(eH)^2}{12\pi^2}h_1\left(\frac{M_\pi}{T}\right),$$
$$h_1(z)=\int_0^\infty\frac{dx}{\sqrt{x^2+z^2}\,(e^{\sqrt{x^2+z^2}}-1)}. \tag{30}$$

For the paramagnetic term $P_{\rm para}$ we find in the weak field

$$P_{\rm para}=P_{0q}(T,H=0)+\frac{N_cQ^2(eH)^2}{6\pi^2}f_1\left(\frac{m}{T}\right),$$
$$f_1(z)=\int_0^\infty\frac{dx}{\sqrt{x^2+z^2}\,(e^{\sqrt{x^2+z^2}}+1)}, \tag{31}$$

where $m_u=m_d=m$ and $Q^2=(e_u^2+e_d^2)/e^2=(\frac49+\frac19)=\frac59$, and pressure $P_{0q}(T,H=0)$ is given by[2]

$$P_{0q}(T,H=0)=\frac{2N_c}{\pi^2}T^4\int_0^\infty x^2\,dx\ln\left(1+e^{-\omega_q/T}\right),$$
$$\omega_q=\sqrt{x^2+m^2/T^2}. \tag{32}$$

In the absence of magnetic field and in the chiral limit one finds [21]

$$T_c=\left(\frac{\Delta\varepsilon_v}{(\gamma-3)(\pi^2/90)}\right)^{1/4}. \tag{33}$$

Here $\gamma=2\cdot(N_c^2-1)+(7/8)\cdot2\cdot2\cdot N_c\cdot N_f$ is the number of independent degrees of freedom of quarks and gluons, and $\gamma=37$ for $N_c=3$, $N_f=2$. Lattice calculations give the value $\langle G^2\rangle=0.87\ \mathrm{GeV}^4$, and one finds from (33) phase transition temperature $T_c\simeq177$ MeV at $H=0$.

The influence of magnetic field can be taken into account in the first approximation by redefining vacuum energy density

$$\Delta\varepsilon_v^H=\Delta\varepsilon_v-(eH)^2V, \tag{34}$$

where

$$V=\frac{1}{12\pi^2}\left[h_1\left(\frac{M_\pi}{T_c}\right)+2N_cQ^2f_1\left(\frac{m}{T_c}\right)\right]. \tag{35}$$

$V=6.1\cdot10^{-2}$ for $M_\pi=140$ MeV and $m\approx5$ MeV.

[2] Using $\int_0^\infty x^2\,dx\ln(1+e^{-x})=\frac{7\pi^4}{360}$, one finds in the limit $m=0$ that $P_{0q}(T,H=0)=4N_c\frac78\frac{\pi^2}{90}T^4$.

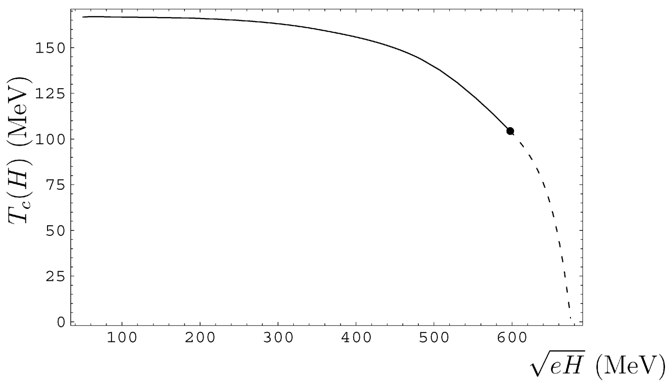

Fig. 1. Quark–hadron phase transition temperature vs. external magnetic field. Critical point, $T_* = 104$ MeV, $\sqrt{eH_*} = 600$ MeV, where latent heat turns to zero, is marked with the dot. Solid line corresponds to the first order phase transition, and dashed line corresponds to the crossover.

Considering the term $(eH)^2V$ as a perturbation, one finds from (33) and (34) that the relative shift of the deconfinement phase transition temperature in the presence of magnetic field is

$$T_c^H = T_c\left(1 - \frac{V}{4\Delta\varepsilon_v}(eH)^2\right) \tag{36}$$

and $V/4\Delta\varepsilon_v \simeq 9.2\ \text{GeV}^{-4}$.

Thus, we see that the presence of a magnetic field leads to a decrease of the quark–hadron phase transition temperature, and $\Delta T/T_c \approx 10^{-2}(eH)^2/\Delta\varepsilon_v$.

4. Results of numerical simulations

Eqs. (20) and (27) allow to evaluate the pressure in both phases, and to numerically find the dependence of phase transition temperature on the magnetic field from (28). Results of numerical calculations for $N_c = 3$, $N_f = 2$ are presented in Fig. 1. Phase transition temperature, as discussed above, decreases with increasing external magnetic field.

Thermodynamics in each phase is defined by the pressure, and we can evaluate energy density and latent heat in both phases. Energy density is given by

$$\varepsilon_1 = T\frac{dP_1}{dT} - P_1, \qquad \varepsilon_2 = T\frac{dP_2}{dT} - P_2. \tag{37}$$

Latent heat equals to the difference of energy densities of two phases in the point of phase transition:

$$\Delta\varepsilon(H) = (\varepsilon_2 - \varepsilon_1)|_{T_c(H)}. \tag{38}$$

The dependence of $\Delta\varepsilon(H)$ is plotted in Fig. 2. The value of the magnetic field $\sqrt{eH_*} = 600$ MeV where latent heat turns to zero corresponds to the critical point, at which first order phase transition changes to the crossover.

5. Conclusion

We have studied the quark–hadron phase transition in QCD in the presence of external magnetic field, and have shown, that the temperature of the phase transition decreases in comparison to the case of zero magnetic field. Eq. (28) was solved numerically, the phase diagram in the plane temperature-magnetic field and critical point were found.

As was shown above, there are two phases in the presence of magnetic field: diamagnetic phase below T_c and paramagnetic above T_c. Correspondingly magnetic susceptibility, $\chi = -\partial^2 P/\partial H^2$, changes it's sign at the critical temperature. Thus, magnetic susceptibility may be considered as the order parameter of the model of thermal QCD in the presence of magnetic field.

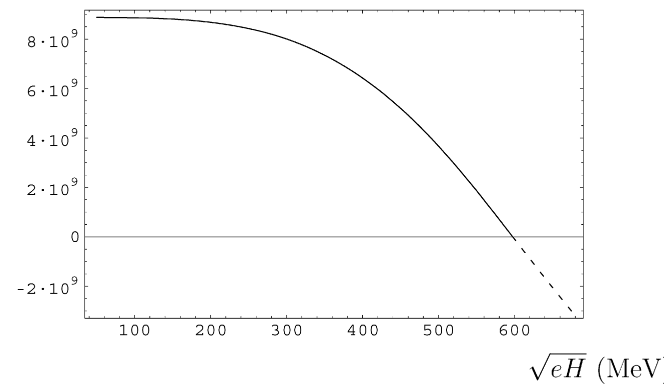

Fig. 2. Latent heat $\Delta\varepsilon(H)$ vs external magnetic field $\sqrt{eH}$.

It is known from lattice calculations that there is a crossover for finite temperature QCD with physical quark masses. In the presence of magnetic field there are additional magnetic terms in the pressure, which give different contribution to the energy density in two phases. Thus we expect that a crossover is replaced by a first order phase transition. Analogous phenomenon was found in [24], where it was shown that chiral transition changes from crossover to the weak first order transition in the linear sigma model in a magnetic field.

The following remark should be made. From [7] it is known that the chiral phase transition temperature grows with the magnetic field. As it follows from lattice calculations, deconfinement and chiral phase transitions take place at the same temperature T_c in case of zero magnetic field $H = 0$. On the other hand, as was shown above, quark–hadron phase transition temperature at nonzero magnetic field is lower than in case of $H = 0$. Thus, chiral and quark–hadron phase transition temperatures are separated in the presence of magnetic field. Correspondingly, there appears a temperature interval, where the quark–hadron phase transition is already passed, but the chiral symmetry is still broken. This phenomenon may be important for the consideration of quark–hadron phase transition in heavy ion collisions and in the early universe.

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