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Correlators of hadron currents: The model and the ALEPH data on τ-decay

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Abstract

The model with the meson spectrum, consisting of zero-width equidistant resonances, is considered with connection to current correlators in coordinate space. The comparison of the explicit expressions for the correlators, obtained in this model, with the experimental data of ALEPH Collaboration on τ-decay is made and good agreement is found.

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1. Introduction

The main objects of our studies are correlators of charged hadron currents with the value of isospin $I = 1$

$$P_{\mu\nu}^{(c)}(x) = \langle j_\mu^{(c)}(x) j_\nu^{(c)}(0)^\dagger \rangle \qquad (1)$$

for vector and axial channels, i.e., for

$$j_\mu^{(v)}(x) = \bar{u}\gamma_\mu d, \qquad j_\mu^{(a)}(x) = \bar{u}\gamma_\mu\gamma_5 d. \qquad (2)$$

These objects play important role in QCD, since they contain information of two kinds: one is the spectral features of hadrons—the set of the mass poles m_n^2 and the corresponding quark coupling constants c_n, which define behavior of the correlators for large x, on the other hand, at small distances, where perturbation theory works, and OPE is assumed to be valid, the coefficients of the correlation functions expansion give us matrix elements of local field operators—the vacuum condensates like $\langle G_{\mu\nu}^2 \rangle$.

Current correlators were studied long ago in the framework of perturbation theory, which works at small x because of asymptotic freedom of QCD [1]. Power corrections to this behavior, expressed in terms of gluon and quark condensates, were found with the help of OPE, which is consistent also only for small enough distances [2]. There are many papers, where calculations of (1) are given on the base of different approaches. So, the random instanton liquid model, where it is supposed, that the nonperturbative QCD-vacuum is dominated by strong classical field configuration—instantons, whose position and color orientations are distributed randomly,—gives good result for the difference of the correlation functions in all region of distances ($0 < x < 1.5$ fm) [3]. Correlation functions and their moments in momentum space are considered in some papers with the purpose of extraction from them the values of the lowest OPE condensates [4–6]. There is another model—the so-called minimal hadronic ansatz (MHA), in which the spectral density consists of the pion state, a vector state and an axial state plus continuum. In this approximation one can compute corresponding condensates [7] or compare the moments of the polarization operators with the experimental data [8]. The method of calculation of correlators, more general, than MHA, was proposed in the paper [9]. This method, as well as MHA, assumes the limit of large number of colors N_c, when the hadronic spectrum is represented by the set of zero-width

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levels. One can calculate the spectrum quasiclassically [10,11], restore the correlator in momentum space by this spectrum and by residues and then go to coordinate space. In the same paper was made the comparison of obtained expressions for the correlators with the lattice data [12,13] and good agreement of the results was found. However, the results of lattice computations have a large errors, moreover, the data from the cited papers do not agree with each other even in the limits of this errors, therefore comparison of the model calculations with the experimental data is of particular interest both for the model and lattice results.

This Letter continues the line of research, started in [9]. We will be interested in explicit expressions for current correlators (1) in the model with equidistant spectrum, and the main purpose is to compare these expressions with the real experimental data of ALEPH Collaboration [14] on hadronic τ-decay. Note also, that the model of zero-width equidistant resonances has been considered with respect to correlation function in [15–17], where the main attention was spared to the momentum-space representation and the corresponding OPE coefficients were estimated, also the behavior of Regge trajectories has been studied in [18]. We will be interested in the current correlators, calculated in the Euclidean coordinate space and compare them directly with the experimental curves.

2. Current correlator: the model and experiment

We work with QCD in the limit of large number of colors N_c. In the framework of this limit the hadronic spectrum is a set of zero-width resonances. Masses m_n and residues c_n of current correlators are stable at large N_c limit and define the imaginary part of the polarization operators (see Appendix A):

$$\frac{1}{\pi}\operatorname{Im}\Pi(s)=\sum_{n=0}^{\infty}c_n\delta\left(s-m_n^2\right). \tag{3}$$

On the other hand, one can obtain the equidistant mass spectrum and the set of residues in analytic form for all n quasiclassically, solving the corresponding Hamiltonian problem for two quarks [9,10]:

$$\left(m_n^{(c)}\right)^2=m^2n+m_c^2,\qquad c_n=\frac{N_c}{12\pi^2}m^2, \tag{4}$$

where m_c^2 are the masses of low lying resonances in each channel, and the quantity $m^2=4\pi\sigma$ is defined universally for all channels, where σ is physical string tension.

In that way one can easily find with the help of (3) the correlation function in the momentum space and then go to the coordinate one (see Appendix A).

It is convenient to consider the normalized correlation functions and to define the ratio $R^{(c)}(x)$ as

$$R^{(c)}(x)=\frac{P^{(c)}(x)}{P_{\text{free}}(x)}\equiv\frac{g^{\mu\nu}P_{\mu\nu}^{(c)}(x)}{P_{\text{free}}(x)}, \tag{5}$$

where

$$P_{\text{free}}(x)=\frac{6}{\pi^4x^6}. \tag{6}$$

Experimental data provide us with information about the spectral functions $v_1(q^2)$ and $a_1(q^2)$ in the interval $0<s<s_0=3.5\ \text{GeV}^2$:

$$\operatorname{Im}\Pi^{(v)}(s)=\frac{1}{2\pi}v_1(s),\qquad \operatorname{Im}\Pi_1(s)=\frac{1}{2\pi}a_1(s), \tag{7}$$

i.e., about the imaginary parts of polarization operators $\Pi^{(v)}(q^2)$ and $\Pi_1(q^2)$, defining the correlation functions in momentum space (A.1) and (A.2).

Using the dispersion relation

$$\Pi\left(q^2\right)=\int_0^{\infty}dt\,\frac{1}{t-q^2}\frac{1}{\pi}\operatorname{Im}\Pi(t) \tag{8}$$

and the known representation for the McDonald function $K_1(z)$ for $z=\sqrt{-sx^2}$, it is easy to obtain the spectral representation for correlation functions in coordinate space:

$$P^{(c)}(x_E)=\int_0^{\infty}s\rho^{(c)}(s)D(\sqrt{s},x_E)\,ds, \tag{9}$$

where $x_E=\sqrt{-x^2}$ is a distance in Euclidean space, $\rho^{(v\backslash a)}(s)=\frac{3}{2\pi^2}v_1(s)\setminus a_1(s)$ are corresponding spectral functions, and

$$D(\sqrt{s},x_E)=\frac{\sqrt{s}}{4\pi^2x_E}K_1(\sqrt{s}x_E). \tag{10}$$

The integrals in considered combinations

$$P_{v\pm a}(x_E)=\int_0^{\infty}s\rho_{v\pm a}(s)D(\sqrt{s},x_E)\,ds, \tag{11}$$

where $\rho_{v\pm a}(s)=\rho^{(v)}(s)\pm\rho^{(a)}(s)$, $P_{v\pm a}=P^{(v)}\pm P^{(a)}$, were taken numerically by Simpson method, using the experimental data for difference and sum of the spectral functions in the region $0<s<s_0$. The spectral functions $\rho_{v\pm a}(s)$ were replaced by the parton model prediction for $s>s_0$, i.e., $v_1(s)+a_1(s)=1$, $v_1(s)-a_1(s)=0$ for $s>s_0$. Under this replacement we do not take into account perturbation theory corrections, which change little behavior of the sum of the current correlators for small x. However, this perturbative corrections, corresponding to the hybrid states in the spectral sum in terms of our model, are not taken into account also, so the disregarding of them in the experimental curves is coordinated with the model calculations.

The experimental curves, normalized to the free answer (6), and the corresponding model predictions are shown on Figs. 1 and 2 for two values of the cut-off scale $s_0=2.3\ \text{GeV}^2$ and $s_0=3.1\ \text{GeV}^2$. Notice, that the choice of the parameter s_0 changes noticeably behavior of $P_{v+a}(x_E)$ only in the intermediate region of x, pushing away the theoretical curve from the experimental one. In addition, the error bars increase with increasing s_0 in this region because of large uncertainties of the experimental data for $s_0>2.8\ \text{GeV}^2$, so agreement of the experimental curves with the model predictions remains acceptable even on the quantitative level.

On the other hand, using meson masses and decay constants as fitting parameters, one can place the curves, obtained in the

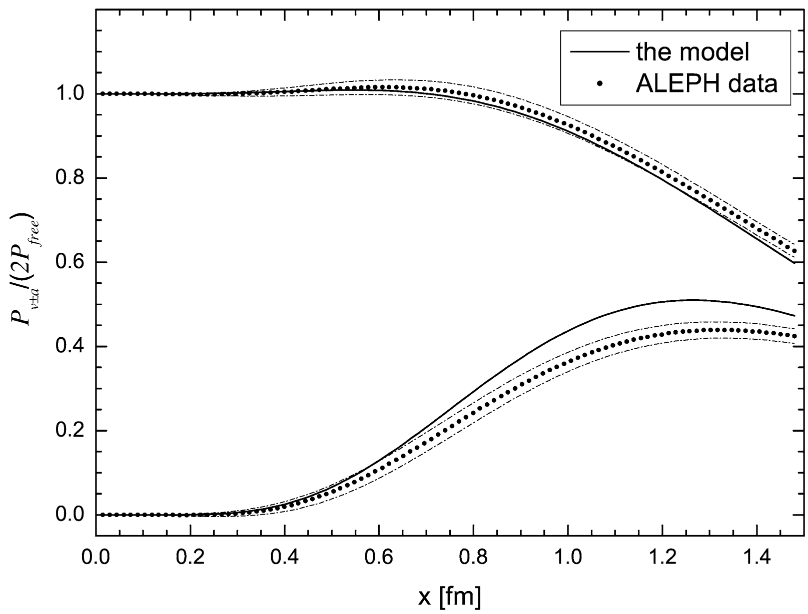

Fig. 1. Normalized Euclidean coordinate space correlation functions $P_{v\pm a}(x)$ for $s_0 = 2.3\ \mathrm{GeV}^2$.

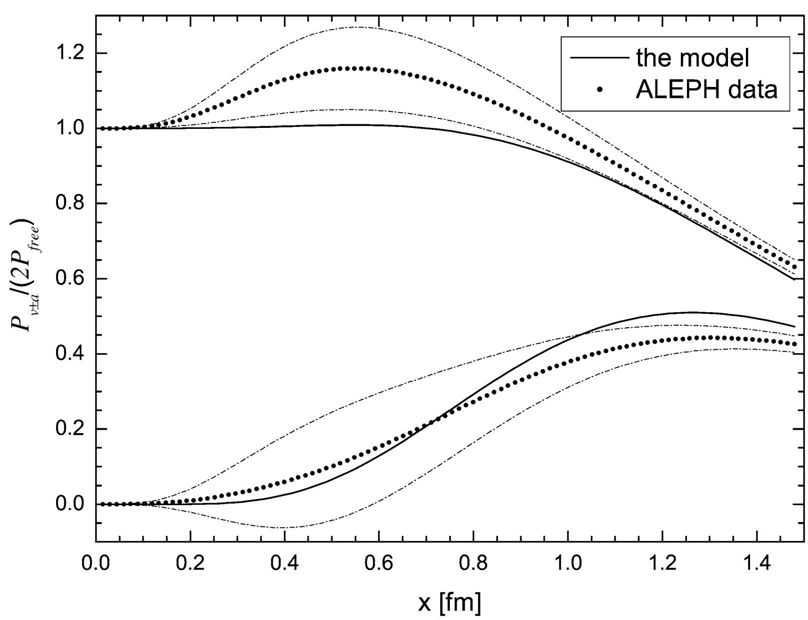

Fig. 2. Normalized Euclidean coordinate space correlation functions $P_{v\pm a}(x)$ for $s_0 = 3.1\ \mathrm{GeV}^2$.

framework of the model at large N_c, inside the error corridors completely (see Fig. 3). The corresponding values

$$m_\rho^2 = 0.616\ \mathrm{GeV}^2, \qquad \lambda_\rho^2 = 0.0475\ \mathrm{GeV}^2, \tag{12}$$

$$m_{a_1} = 1.181\ \mathrm{GeV}, \qquad \sigma = 0.176\ \mathrm{GeV}^2 \tag{13}$$

reproduce the table values (A.6)–(A.7) with 2–4% accuracy.

The errors was computed with the help of experimental covariance matrices

$$\rho_{v\pm a}(s,s') = \langle \Delta\rho_{v\pm a}(s)\Delta\rho_{v\pm a}(s')\rangle \tag{14}$$

by the standard expression

$$\begin{aligned} &\Delta P_{v\pm a}(x_E) \\ &\quad = \left[\langle P_{v\pm a}^2\rangle - \langle P_{v\pm a}\rangle^2\right]^{1/2} \\ &\quad = \left[\int_0^{s_0} ds \int_0^{s_0} ds'\, \rho_{v\pm a}(s,s') s D(\sqrt{s}, x_E) s' D(\sqrt{s'}, x_E)\right]^{1/2}. \end{aligned} \tag{15}$$

3. Conclusion

We have considered the model, based on QCD in the limit of large number of colors N_c, in which the meson spectrum is taken as the series of zero-width equidistant levels. These spec-

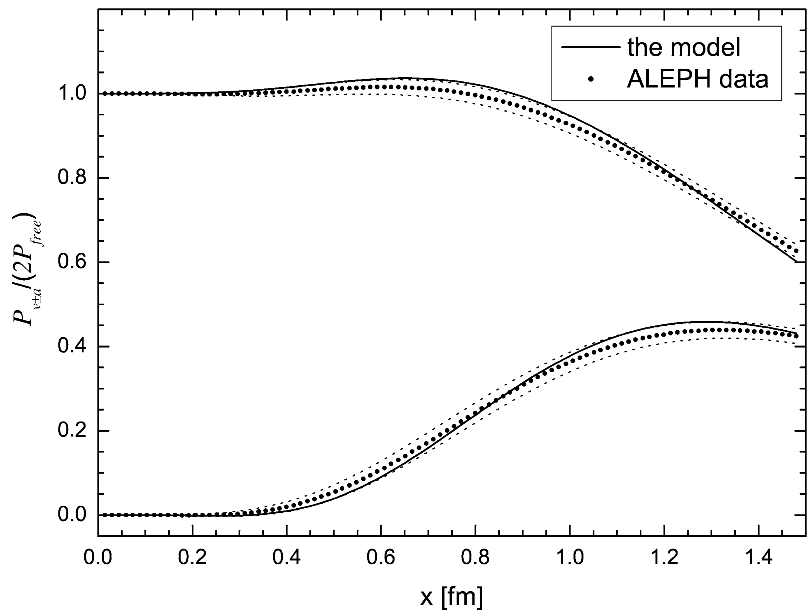

Fig. 3. Normalized Euclidean coordinate space correlation functions $P_{v\pm a}(x)$ for the set of parameters (12), (13) and $s_0 = 2.3\ \text{GeV}^2$.

tral functions as a set of delta-functional peaks are surely far from the real physical world—experimental curves have much richer structure. In practice we deal with nonzero width of resonances, however, precise solutions and systematic procedures, allowing to account for this subleading effects, are absent. One may try to introduce finite width on a phenomenological level, as it was done in the papers [19,20]. The shape of the spectral functions is reproduced by model curves in momentum space rather well now, however, it depends strongly on a new fitting parameter. Another way is the calculation not of the spectral functions—imaginary parts of the correlators in momentum space, but of their moments or another integral characteristics, for instance, correlators in coordinate space. In this case, as can be seen from the results of comparison of the calculations, performed in the framework of this model, with the high precision experimental data of ALEPH Collaboration for the correlators of hadron currents, the answer turns out to be not strongly sensitive to detailed spectral features. There is no need to introduce resonance width in some way, but it is enough to know several global spectral features to reproduce correctly on qualitative and even on quantitative level experimental curves. These global features are values, which define asymptotic behavior of the correlators for small and large values of the Euclidean coordinate, namely—the data, related to low lying resonances (their masses and decay constants), which define behavior for large x, and correct slope of the Regge trajectories $m^2 = 4\pi\sigma$, which is fixed by quark–hadron duality requirement and which defines behavior of the correlators for small x. Details of the asymptotics sewing (or details of the spectral functions shape) turn out to be not so essential, since the regions of validity for both asymptotic solutions are wide enough: even primitive prediction of parton model (one for the sum of the correlation functions and zero for the difference) works well for distances up to 0.5–0.9 fm as it is seen on Fig. 1, on the other hand, because of the steep slope of the Regge trajectories the ρ-meson mass turns out to be small in comparison with the masses of higher lying resonances, and this is the cause of ground state dominance for enough wide region of coordinate variation. Besides, it is worth noting, that all the parameters of the model (meson masses, decay constants, string tension) are fixed by their physical values, and calculated correlation function have in principle a form of a fixed prediction without any fitting parameters, however, having shifted quite a little the values of these input parameters ($\sim$ 2–3%), one can reach a very good (nearly complete) agreement with the experimental data (see Fig. 3).

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Appendix A

For conserved vector current one can write the following expression in momentum space:

$$P^{(v)}_{\mu\nu}(q) = i\int d^4x\, P^{(v)}_{\mu\nu}(x)\exp(iqx) = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(v)}\left(q^2\right). \quad \text{(A.1)}$$

There are two Lorentz structure in the axial channel:

$$P^{(a)}_{\mu\nu}(q) = i\int d^4x\, P^{(a)}_{\mu\nu}(x)\exp(iqx) = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi_1\left(q^2\right) + q_\mu q_\nu \Pi_2\left(q^2\right). \quad \text{(A.2)}$$

The polarization operator $\Pi_2(q^2)$ is dominated by the pion pole contribution, and we will omit this term, since the experimental data are concerned to the $\Pi^{(v)}(q^2)$ and $\Pi_1(q^2)$ operators.

The normalized correlation functions in the Euclidean coordinate space are given by following expressions in accordance with result of [9]:

$$R^{(v)}(z_v) = \xi z_v^5 K_1(z_v) + \frac{b_v z_v^6}{2^7}\bar{p}(b_v, z_v), \tag{A.3}$$

$$R^{(a)}(z_a) = \frac{b_a z_a^6}{2^7}\bar{p}(b_a, z_a), \tag{A.4}$$

which are depend on the dimensionless coordinates $z_c = \sqrt{-x^2}m_c$, where m_c are the masses of the resonances in vector (ρ-meson) and axial (a_1-meson) channels. The contribution of the ρ-meson state is taken into account separately and is described by the first member in $R^{(v)}(x)$. The parameters

$$b_c = \frac{4\pi\sigma}{m_c^2}, \qquad \xi = \frac{\pi^2}{8}\left(\frac{\lambda_\rho^2}{m_v^2} - \frac{b_v}{4\pi^2}\right) \tag{A.5}$$

are fixed by the physical values of meson masses, decay constants and string tension constant:

$$m_\rho^2 = 0.6\ \text{GeV}^2, \qquad \lambda_\rho^2 = 0.047\ \text{GeV}^2, \tag{A.6}$$

$$m_{a_1} = 1.23\ \text{GeV}, \qquad \sigma = 0.17\ \text{GeV}^2. \tag{A.7}$$

Remaining terms, containing the function

$$\bar{p}(b,z) = \int_0^\infty \frac{du}{u^2}\exp\left(-\frac{z^2}{4u} - u\right)\frac{1 + \exp(-bu)(b-1)}{(1-\exp(-bu))^2}, \tag{A.8}$$

are determined by contribution of a set of the equidistant levels with the masses (4).

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