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Wess–Zumino actions and Dirichlet boundary conditions for super p-branes with exotic fractions of supersymmetry

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Abstract

The general solutions in the models of closed and open superstring and super p-branes with exotic fractions of the $N=1$ supersymmetry are considered and the spontaneously broken character of the $OSp(1,2M)$ symmetry of the models is established. It is shown that extending these models by Wess–Zumino terms generates the Dirichlet boundary conditions for superstring and super p-branes. Using the generalized Wess–Zumino terms new $OSp(1,2M)$-invariant super p-brane and Dp-brane-like actions preserving $(M-1)/M$ fraction of supersymmetry are proposed. For $M=32$ these models suggest new superbrane vacua of M-theory preserving 31 from 32 global supersymmetries.

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1. Introduction

Recently new progress in the tracing of M-theory symmetries [1,2] based on the development of the generalized holonomy conception [3] has been achieved.[1] The generalized holonomy conception classifies vacuum states permitted by the centrally extended supersymmetry algebra [6,7] and introduces new hidden space–time symmetries. It was shown in [2] that the holonomy extension in M-theory to the $SL(32,R)$ local symmetry is necessary to include the fermionic degrees of freedom and to permit exotic vacuum states preserving 31 from 32 supersymmetries [8,9].

The string/brane description of the vacuum state with the so high supersymmetry was given by the model [10] of tensionless superstring and super p-brane. A connection of this model with the description [9] (see also [11]) of the BPS states in M-theory was discussed in [12]. The model [10] develops the

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[1] Let us note that this conception permits an extension by the lengthening of the spinor components of the connection Ω_M. An example of the extension has been studied in [4] for $N=1,2$ supersymmetric electrodynamics, where the covariant derivative D_M lengthening $D_M \to \nabla_M = D_M + i\mu\tilde{W}_M$ with $\tilde{W}_M = \frac{i}{4}(0, -\sigma_{\mu\alpha\dot{\alpha}}F^{\mu\dot{\alpha}}, \tilde{\sigma}^{\mu\dot{\alpha}\alpha}F_{\mu\alpha})$ for the $N=1$ spinor derivatives, and with $\tilde{W}_M = -\frac{i}{4}(0, D^i_\alpha W, \bar{D}^{\dot{\alpha}i}\bar{W})$ for the $N=2$ spinor derivatives, were considered. The spinor components of the connection $\tilde{W}_M$ take into account the anomalous magnetic moment (AMM) μ of charged and neutral particles with spin $1/2$ and generate the Pauli term. Taking into account of the AMM of $N=2$ massive superparticles is necessary to restore κ-symmetry in its interactions with $N=2$ extended Maxwell supermultiplet [5].

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approach [13–15] to the description of string/brane dynamics in superspaces extended by the addition of tensor central charge (TCC) coordinates. The centrally extended superspaces are characterized by the orthosymplectic symmetries and are closely connected with gauge theory of massless fields with higher spins [16,17] which already appear in the quantized superparticle models with exotic supersymmetries [18,19].

It was observed in [20] that $OSp(1,64)$ symmetry is spontaneously broken in $D=11$ supergravity which is the low energy phase of M/string-theory containing massive higher spin states. This observation gave a reason to suppose that the superbrane microscopic structure also may be described in terms of the spontaneously broken orthosymplectic symmetries [17]. Taking into account the connection of tensionless strings and branes with higher spin field theory [21,22] it is important to understand whether the $OSp(1,2M)$ symmetry of the model [10] is spontaneously broken.

Here we study this question for the case of closed and open tensionless superstring and super p-brane and find that the $OSp(1,2M)$ symmetry of the model is spontaneously broken by the general static solutions of the brane equations of motion. This effect is similar to the partial supersymmetry breaking by the super four-brane [23] and the generalized coordinates of the model [10] are the Goldstone fields of the $OSp(1,2M)$ symmetry. These Goldstone fields may be associated with effective long wave description of the vacua in microscopic higher spin theories. Also we construct new topological Wess–Zumino like superstring and super p-brane actions generating the Dirichlet boundary conditions and spontaneously breaking supersymmetry and $OSp(1,2M)$ symmetry. In addition we propose a new set of the $OSp(1,2M)$ invariant super p-brane and Dp-brane-like actions preserving $\frac{M-1}{M}$ fraction of the $N=1$ supersymmetry.

2. A simple super *p*-brane model with extra supersymmetry: the general solution and symmetries

The exactly solvable supersymmetric model of closed tensionless super p-brane ($p=1,2,3,\ldots$) with extra κ-symmetry

$$S_p = \frac{1}{2}\int d\tau\, d^p\sigma\, \rho^\mu \big(U_a W_\mu^{ab} U_b\big) \tag{1}$$

has been studied in [10]. This model includes the Cartan differential one-form W_{ab}

$$W_{ab} = dY_{ab} - 2i(d\theta_a\,\theta_b + d\theta_b\,\theta_a) \tag{2}$$

invariant under the $N=1$ global supersymmetry transformations

$$\delta_\varepsilon\theta_a = \varepsilon_a, \qquad \delta_\varepsilon Y_{ab} = 2i(\theta_a\varepsilon_b + \theta_b\varepsilon_a),$$
$$\delta_\varepsilon U_a = 0 \tag{3}$$

of the generalized superspace composed by the spin-tensor Y_{ab}, the Grassmannian Majorana spinor θ_a and an auxiliary commuting Majorana spinor U_a [24] parametrizing the light-like density of the brane momentum. The worldvolume density $\rho^\mu = (\rho^\tau, \vec{\rho})$ [25], invariant under the $N=1$ supersymmetry (3), provides reparametrization invariance of S_p (1). The real symmetric spin-tensor Y_{ab}

$$Y_{ab} \equiv x_{ab} + z_{ab} \tag{4}$$

unifies the space–time coordinates x_m and the TCC coordinates $z_{kl\ldots m}$

$$x_{ab} = x_m\big(\gamma^m C^{-1}\big)_{ab},$$
$$z_{ab} = i z_{mn}\big(\gamma^{mn} C^{-1}\big)_{ab} + z_{mnl}\big(\gamma^{mnl} C^{-1}\big)_{ab} + \cdots \tag{5}$$

of the D-dimensional Minkowski space–time with $D = 2,3,4 \bmod(8)$. The spin-tensor Y_{ab} is a realization of the symmetric matrix of generalized symplectic coordinates previously considered in [16,17]. The action S_p is invariant under the transformations of the enhanced κ-symmetry

$$\delta_\kappa\theta_a = \kappa_a, \qquad \delta_\kappa Y_{ab} = -2i(\theta_a\kappa_b + \theta_b\kappa_a),$$
$$\delta_\kappa U_a = 0, \qquad \delta_\kappa\rho^\mu = 0, \tag{6}$$

with the parameter κ restricted by one real condition

$$U^a\kappa_a = 0 \tag{7}$$

and the super p-brane model (1) preserves $\frac{M-1}{M}$ fraction of the $N=1$ supersymmetry, where M is the dimension of the Majorana spinors θ_a and U_a.

The action (1) is presented in the equivalent form [10]

$$S_p = \frac{i}{2}\int d\tau\, d^p\sigma\, \rho^\mu \big\{\big[\big(U^a\partial_\mu \tilde{Y}_a\big) - \big(\partial_\mu U^a \tilde{Y}_a\big)\big] - \tilde{\eta}\partial_\mu\tilde{\eta}\big\}, \tag{8}$$

where the Majorana spinor $\tilde{Y}_a$ is defined by the relation

$$i\tilde{Y}_a = Y_{ab}U^b - \tilde{\eta}\theta_a \tag{9}$$

and is a new effective variable substituted for Y_{ab} and $\tilde{\eta}$

$$\tilde{\eta} = -2i\left(U^a\theta_a\right) \tag{10}$$

is the Lorentz-invariant Grassmannian field describing the Goldstone fermion of the model. The action (8) is the component representation of the $OSp(1,2M)$-invariant action

$$S_p = \frac{1}{2}\int d\tau\, d^p\sigma\, \rho^\mu \partial_\mu Y^\Lambda G_{\Lambda\Xi} Y^\Xi, \tag{11}$$

where $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ is a real $OSp(1,2M)$ supertwistor and $G_{\Lambda\Xi} = (-1)^{\Lambda\Xi+1}G_{\Xi\Lambda}$ is the invariant supersymplectic metric previously considered in superparticle dynamics [18]. The equations of motion following from S_p (11)

$$\begin{aligned} &2\rho^\mu\partial_\mu Y^\Lambda + \partial_\mu\rho^\mu Y^\Lambda = 0,\\ &\partial_\tau Y^\Lambda G_{\Lambda\Xi} Y^\Xi = 0, \qquad \partial_{\vec{\sigma}} Y^\Lambda G_{\Lambda\Xi} Y^\Xi = 0 \end{aligned} \tag{12}$$

are invariant under the linearly realized $OSp(1,2M)$ symmetry, worldvolume reparametrizations and the Weyl gauge symmetry [26]

$$\rho'^\mu = e^{-2\lambda(\tau,\vec{\sigma})}\rho^\mu, \qquad Y'^\Sigma = e^{\lambda(\tau,\vec{\sigma})}Y^\Sigma. \tag{13}$$

In the partially fixed reparametrization gauge [10]

$$\rho^i(\tau,\vec{\sigma}) = 0 \quad (i = 1,2,\ldots,p), \tag{14}$$

removing p of $(p+1)$ components of the worldvolume density $\rho^\mu(\tau,\vec{\sigma})$ without breaking of the Weyl and $OSp(1,2M)$ symmetries, the general solution of Eq. (12) is given by

$$\begin{aligned} &Y^\Lambda(\tau,\vec{\sigma}) = \frac{1}{\sqrt{\rho^\tau(\tau,\vec{\sigma})}}\mathcal{Y}^\Lambda(\vec{\sigma}),\\ &\rho^i(\tau,\vec{\sigma}) = 0 \quad (i = 1,2,\ldots,p). \end{aligned} \tag{15}$$

The static fields $\mathcal{Y}^\Lambda(\vec{\sigma})$ in (15) are restricted by the p initial data constraints

$$\partial_{\vec{\sigma}}\mathcal{Y}^\Lambda(\vec{\sigma})G_{\Lambda\Xi}\mathcal{Y}^\Xi(\vec{\sigma}) = 0, \tag{16}$$

which are the invariants of the Weyl and $OSp(1,2M)$ symmetries. In the case of closed super p-brane the components of $\mathcal{Y}^\Lambda(\vec{\sigma})$ and $\rho^\tau(\tau,\vec{\sigma})$ are periodic functions of σ^i

$$\begin{aligned} &\mathcal{Y}^\Lambda(\sigma^i + 2\pi) = \mathcal{Y}^\Lambda(\sigma^i),\\ &\rho^\tau(\tau,\sigma^i + 2\pi) = \rho^\tau(\tau,\sigma^i). \end{aligned} \tag{17}$$

The components of the arbitrary supertwistor $\mathcal{Y}^\Lambda(\vec{\sigma})$ in the general solution (15) are the invariants of the Weyl gauge symmetry (13) due to the presence of the $\rho^\tau(\tau,\vec{\sigma})$ factor. However, they form the linear representation of the $OSp(1,2M)$ group, because ρ^τ is the invariant of this group. The $\rho^\tau(\tau,\vec{\sigma})$-factor in (15) concentrates all dependence of the general solution on the evolution parameter τ and it may be removed by the additional to (14) gauge fixing

$$\partial_\tau\rho^\tau(\tau,\vec{\sigma}) = 0. \tag{18}$$

The gauge condition (18) breaks the Weyl symmetry, but preserves the $OSp(1,2M)$ symmetry and simplifies the general solution (15) to the pure static form

$$\begin{aligned} &Y^\Lambda(\tau,\sigma^i) = Y_0^\Lambda(\sigma^i),\\ &\partial_\tau\rho^\tau(\tau,\vec{\sigma}) = 0,\\ &\rho^i(\tau,\vec{\sigma}) = 0 \quad (i = 1,2,\ldots,p), \end{aligned} \tag{19}$$

where $\rho^\tau(\tau,\vec{\sigma}) = \rho_0^\tau(\sigma)$ was moved in $Y_0^\Lambda(\sigma^i)$. One remarks that the solutions (15) and (19) are equivalent on the classical level, because of a correlation between the Weyl and space–time conformal symmetries on the quantum level of the tensionless string treatment [27].

The components of the static field $Y_0^\Lambda(\sigma^i)$ describe the shape of the super p-brane. The superbrane has a freedom to choose any shape restricted by the initial data constraints (16) and this shape will remains frozen during the evolution. Any other shape obtained from the initially randomly chosen by any transformation belonging to the $OSp(1,2M)$ group will have the same rights. However, a fixing of the brane shape by any fixed initial data for $Y_0^\Lambda(\sigma^i)$ will break the $OSp(1,2M)$ symmetry. From the point of view of the general theory of system with broken global symmetry [28] fixing of the form of $Y_0^\Lambda(\sigma^i)$ may be interpreted as a choice of the vacuum state of the underlying field system with the spontaneously broken global $OSp(1,2M)$ symmetry. As a result, the static fields $Y_0^\Lambda(\sigma^i)$ are interpreted similarly to [23] as the Goldstone fields associated with the spontaneously broken $OSp(1,2M)$ symmetry and the action (11) is an effective long wave action for the Goldstone fields associated with the super p-brane. It proves the spontaneously broken character of the $OSp(1,2M)$ symmetry as the symmetry of the brane action (11).

Using the supersymmetry laws (3) and the definitions (9), (10) of the components of the supertwistor $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ we find the transformation properties of the Goldstone fields under the supersymmetry transformations from $OSp(1,2M)$

$$\delta_\varepsilon \tilde{Y}_a = 2i\tilde{\eta}\varepsilon_a, \qquad \delta_\varepsilon \tilde{\eta} = -2iU^a\varepsilon_a,$$
$$\delta_\varepsilon U_a = 0, \qquad \delta_\varepsilon \rho^\mu = 0. \tag{20}$$

The $N=1$ supersymmetry transformations (20) are nonlinear, as it have to be for the spontaneously broken symmetries [28], because the original Goldstone fields θ_a are presented in (20) by only one their projection $(U^a\theta_a)$. The absence of other $M-1$ projections of θ_a on $M-1$ basis spinors means the disappearance of $M-1$ Goldstone fermions corresponding to the unbroken fractions of the $N=1$ supersymmetry, because of the presence of the enhanced κ-symmetry (6) restricted by the condition (7)

$$U^a\kappa_a = 0.$$

The non-zero projection $(U^a\varepsilon_a)$ of the supersymmetry parameter ε_a

$$U^a\varepsilon_a \neq 0 \tag{21}$$

defines the direction of the spontaneously broken $\frac{1}{M}$ fraction of the $N=1$ supersymmetry which cannot be compensated any of the $M-1$ κ-symmetry transformations. So, the condition (21) is antipodal to the condition (7) in the correspondence with the aforesaid and the Goldstone fermion $\tilde{\eta}$ has a non-zero shift (21).

It is easy to check that the action S_p (8), and respectively (11), are invariant under the $N=1$ global supersymmetry transformations (20), because of the cancellation between the contributions given by $\tilde{Y}_a$ and the fermionic Goldstone field $\tilde{\eta}$

$$\delta_\varepsilon S_p = -\int d\tau\, d^p\sigma\, \rho^\mu \big\{ \big[U^a\partial_\mu\tilde{\eta} - \partial_\mu U^a\tilde{\eta}\big]\varepsilon_a - \big[U^a\partial_\mu\tilde{\eta} - \tilde{\eta}\partial_\mu U^a\big]\varepsilon_a \big\} = 0. \tag{22}$$

An interesting and open question is to clarify the effect of the boundary terms for the dynamics of the open super p-branes and we turn to this question below.

3. Boundary conditions for the open super p-brane

Here we study the case of open super p-brane (11). The contribution of the boundary terms in the variation of S_p (11) is given by

$$\delta S_p\big|_\Gamma = \oint ds_\mu\, \rho^\mu Y^\Lambda G_{\Lambda\Xi}\delta Y^\Xi, \tag{23}$$

where $ds^\nu = \frac{1}{p!}\varepsilon^{\nu\mu_1\mu_2\ldots\mu_p}\, dS_{\mu_1\mu_2\ldots\mu_p}$. Here, we consider the variational problem with the fix initial ($\tau = \tau_i$) and final ($\tau=\tau_f$) data, so the integral along the super p-brane profile for $\tau = (\tau_i, \tau_f)$ does not contribute to $\delta S_p|_\Gamma$ (23)

$$\int_{s_\tau} ds_\tau\, \rho^\tau Y^\Lambda G_{\Lambda\Xi}\delta Y^\Xi \bigg|_{\tau_i}^{\tau_f} = 0. \tag{24}$$

As a result, the variation $\delta S_p|_\Gamma$ (23) is filled out by the integrals along the p-dimensional boundaries of the brane worldvolume containing the τ-direction

$$\delta S_p\big|_\Gamma = \sum_{i=1}^{i=p}\int_{s_i} ds_i\, \rho^i Y^\Lambda G_{\Lambda\Xi}\delta Y^\Xi \bigg|_{\sigma^i=0}^{\sigma^i=\pi}. \tag{25}$$

In the case of variational problem with free ends, i.e., when the field variations on the p-brane boundaries are arbitrary, the vanishing of these hypersurface terms in $\delta S_p|_\Gamma$ (25) gives the open super p-brane boundary conditions

$$\rho^i Y^\Lambda\big|_{\sigma^i=0,\pi} = 0 \quad (i = 1,2,\ldots,p). \tag{26}$$

One of the solutions of (26) is

$$\rho^i(\tau,\vec{\sigma})\big|_{\sigma^i=0,\pi} = 0 \quad (i=1,2,\ldots,p). \tag{27}$$

The second possibility to satisfy the boundary conditions (26) implies the zero boundary conditions for the supertwistor $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ values on the boundaries

$$Y^\Lambda(\tau,\sigma)\big|_{\sigma^i=0,\pi} = 0 \quad (i=1,2,\ldots,p), \tag{28}$$

or, equivalently, in terms of the supertwistor components

$$U^a\big|_{\sigma^i=0,\pi} = 0, \qquad \tilde{Y}_a\big|_{\sigma^i=0,\pi} = 0,$$
$$\tilde{\eta}\big|_{\sigma^i=0,\pi} = 0 \quad (i=1,2,\ldots,p). \tag{29}$$

The boundary conditions (27) for ρ^i and (28) for Y^Λ are invariant under the Weyl symmetry (13), $N=1$ global supersymmetry (20) and other homogeneous transformations of $OSp(1,2M)$. The boundary conditions (27) will be automatically satisfied in the invariant gauge (14)

$$\rho^i(\tau,\vec{\sigma})=0 \quad (i=1,2,\ldots,p).$$

As a result, the general solution (15) for the closed super p-brane in this gauge gives also the general solution of the boundary problem (12), (27) for the open super p-brane.

Concerning the boundary conditions (28), one can note that the zero boundary values $U_a|_{\sigma^i=0,\pi}=0$ (29) result in some problem in the geometric interpretation of the auxiliary spinor field U_a as a basic constituent of the local spinor repere attached to the super p-brane worldvolume. For example, in the case of the 4-dimensional Minkowski space, where U^a is treated [24] as one of the components of the Newman–Penrose dyads [29], these boundary conditions result in the condition

$$\big(U^a(\tau,\vec{\sigma})V_a(\tau,\vec{\sigma})\big)\big|_{\sigma^i=0,\pi}=0 \quad (i=1,2,\ldots,p), \tag{30}$$

which breaks the basis relation $U^aV_a=1$ defining the dyads U_a, V_a [29]. To preserve this condition the spinor field V_a should be singular on the brane/string boundaries and it signals on some instabilities on the brane boundaries. Therefore, the solution (27) have to be choosen for the considered simple model (11) and in this case the open and closed super p-brane are described by the same general solution (15) for the static Goldstone fields. This result is based on use of the gauge condition (14) for the auxiliary field ρ^μ.

To overcome the problem of the singular character of the boundary conditions (28) we need to extend the simple action (11) and to this end we may generalize the topological actions studied in [11,27]. An example of that generalization will be done in the next section, where we will present of a topological action which yields the Dirichlet boundary conditions for open superstring, resulting to the spontaneous breakdown of the $OSp(1,2M)$ symmetry and $N=1$ supersymmetry.

4. A topological action generating the Dirichlet boundary conditions for the superstring

The superstring action with enhanced supersymmetry given by

$$S_{\mathrm{WZ}}=\frac{\beta}{2}\int_{\tau_i}^{\tau_f}\int_0^\pi G_{\Lambda\Xi}\,dY^\Lambda\wedge dY^\Xi \tag{31}$$

contributes only on the superstring ends and yields the Dirichlet boundary conditions similar to those for the Nambu strings [30]. The integrand in the integral S_{WZ} (31) is a total derivative and is presented in the form of the integral along the one-dimensional boundary of the superstring worldsheet

$$S_{\mathrm{WZ}}=-\frac{\beta}{2}\oint dY^\Lambda\,G_{\Lambda\Xi}\,Y^\Xi. \tag{32}$$

The integral (31) is similar to the curvature integral for the open string

$$S_R=-\frac{C}{4\pi}\int_{\tau_i}^{\tau_f}\int_0^\pi R\sqrt{-g}\,d\tau\,d\sigma, \tag{33}$$

where $R/2$ is the Gauss curvature of the string worldsheet. It was shown in [31] that taking into account of the nonlinear boundary conditions generated by S_R reveals a topological structure of the string action extrema. To find the effect resulted in by S_{WZ} (31) one notes that the integrand of S_{WZ} (32) coincides with the differential form $(U_aW_\mu^{ab}U_b)$ in (1) and therefore S_{WZ} is invariant of the original symmetries of the action (11) besides of the Weyl gauge symmetry (13). The latter restriction follows from the absence of the ρ^μ density in the integral (32) which results in it change

$$S'_{\mathrm{WZ}}=-\frac{\beta}{2}\oint e^{2\lambda}\,dY^\Lambda\,G_{\Lambda\Xi}Y^\Xi \tag{34}$$

under the Weyl transformation (11). It means that the Weyl symmetry is explicitly broken by the boundary terms, already on the classical level unlike the Green–Schwarz superstring, where the breakdown appears only on the quantum level.

The variation of the Wess–Zumino term (32) gives

$$\delta S_{\mathrm{WZ}} = -\beta \oint dY^{\Lambda} G_{\Lambda\Xi}\delta Y^{\Xi} - \frac{\beta}{2}\oint d\big(\delta Y^{\Lambda} G_{\Lambda\Xi} Y^{\Xi}\big) = \beta \int_{\tau_i}^{\tau_f} \partial_\tau Y^{\Lambda} G_{\Lambda\Xi}\delta Y^{\Xi}\Big|_{\sigma=0}^{\sigma=\pi}, \tag{35}$$

where the initial and final variational conditions $\delta Y^{\Xi}(\tau_i,\sigma) = 0$, $\delta Y^{\Xi}(\tau_f,\sigma) = 0$ have been used. Next, taking into account the freedom in the variations $\delta Y^{\Lambda}(\tau_i,\sigma)|_{\sigma=0,\pi}$ on the string ends we obtain the following boundary conditions

$$\partial_\tau Y^{\Lambda}(\tau,\sigma)\big|_{\sigma=0,\pi} = 0, \tag{36}$$

presented in the component form as

$$\partial_\tau U^{a}\big|_{\sigma=0,\pi} = 0, \qquad \partial_\tau \tilde{Y}_a\big|_{\sigma=0,\pi} = 0, \qquad \partial_\tau \tilde{\eta}\big|_{\sigma=0,\pi} = 0. \tag{37}$$

The boundary conditions (36) and (37) are the equations of motion of the string ends and they are invariant under the $OSp(1,2M)$ symmetry and supersymmetry transformations, because of their global character. However, the general solution of these equations

$$Y^{\Lambda}(\tau,\sigma)\big|_{\sigma=0} = \mathcal{A}^{\Lambda}, \qquad Y^{\Lambda}(\tau,\sigma)\big|_{\sigma=\pi} = \mathcal{B}^{\Lambda}, \tag{38}$$

which contains the integration constants $\mathcal{A}^{\Lambda}$ and $\mathcal{B}^{\Lambda}$, defined by the initial data

$$\mathcal{A}^{\Lambda} \equiv \big(iU^{a}_{\mathcal{A}}, \tilde{Y}_{\mathcal{A}a}, \tilde{\eta}_{\mathcal{A}}\big), \qquad \mathcal{B}^{\Lambda} \equiv \big(iU^{a}_{\mathcal{B}}, \tilde{Y}_{\mathcal{B}a}, \tilde{\eta}_{\mathcal{B}}\big), \tag{39}$$

defining the position of string ends in the symplectic superspace. The choice of different values for the constant supertwistors $\mathcal{A}^{\Lambda}$ and $\mathcal{B}^{\Lambda}$ means the choice of different vacuum states breaking the $OSp(1,2M)$ symmetry. Note that $\mathcal{A}^{\Lambda}$ and $\mathcal{B}^{\Lambda}$ have dimension L^1 and their choice define a length scale in the model fixing the scale of β in (31). Let us note the particular solution of Eq. (38) fixed by the zero values of the Goldstone fermion on the string ends

$$\tilde{\eta}_{\mathcal{A}} = 0, \qquad \tilde{\eta}_{\mathcal{B}} = 0. \tag{40}$$

The solution (40) will partially preserve the supersymmetry if the conditions

$$U^{a}_{\mathcal{A}}\varepsilon_a = 0, \qquad U^{a}_{\mathcal{B}}\varepsilon_a = 0 \tag{41}$$

for the projection $(U^{a}\varepsilon_a)$ on the superstring ends are satisfied, as it follows from the transformation rules (20). The conditions (41) impose two real conditions for the supersymmetry parameters ε_a resulting to the breaking of $\frac{2}{M}$ fraction of $N = 1$ supersymmetry or, in the special case

$$U^{a}_{\mathcal{A}} = U^{a}_{\mathcal{B}}, \tag{42}$$

to the breaking only $\frac{1}{M}$ fraction of $N = 1$ supersymmetry.

5. The superstring model with the Wess–Zumino term

Here we show that the addition of the Wess–Zumino term (31) in the original action removes the problem of the singular character of the second solution (28) of the boundary conditions (26). The extended action

$$S = S_1 + S_{\mathrm{WZ}} = \frac{1}{2}\int_{\tau_i}^{\tau_f}\int_0^{\pi} d\tau\, d\sigma\, \rho^{\mu}\partial_\mu Y^{\Lambda} G_{\Lambda\Xi} Y^{\Xi} + \frac{\beta}{2}\int_{\tau_i}^{\tau_f}\int_0^{\pi} G_{\Lambda\Xi}\, dY^{\Lambda}\wedge dY^{\Xi} \tag{43}$$

modifies the boundary conditions (26) to the conditions

$$\big[\rho^{\sigma} Y^{\Lambda} + \beta\partial_\tau Y^{\Lambda}(\tau,\sigma)\big]\big|_{\sigma=0,\pi} = 0. \tag{44}$$

Conditions (44) are invariant under the $OSp(1,2M)$ symmetry similarly to (27) and (28) and their general solution

$$Y^{\Lambda}(\tau,\sigma)\big|_{\sigma=0} = \exp\left\{-\int_{\tau_i}^{\tau}\frac{\rho^{\sigma}(\tau,0)}{\beta}\right\}\mathcal{A}^{\Lambda}, \qquad Y^{\Lambda}(\tau,\sigma)\big|_{\sigma=\pi} = \exp\left\{-\int_{\tau_i}^{\tau}\frac{\rho^{\sigma}(\tau,\pi)}{\beta}\right\}\mathcal{B}^{\Lambda}, \tag{45}$$

includes the arbitrary integration constants $\mathcal{A}^{\Lambda}$, $\mathcal{B}^{\Lambda}$ similar to (39). So, one can see that the boundary conditions (45) are not singular when $\rho^{\sigma}|_{0,\pi} \neq 0$. A fixing of the constant $\mathcal{A}^{\Lambda}$ and $\mathcal{B}^{\Lambda}$ means a vacuum state choice and shows the spontaneously broken character of the $OSp(1,2M)$ symmetry of the action (43).

The action (43) differs from the Wess–Zumino like action (31) by the presence of the equations of motion (12) having the general solution (15)

$$Y^{\Lambda}(\tau,\sigma)=\frac{1}{\sqrt{\rho^{\tau}(\tau,\sigma)}}Y_0^{\Lambda}(\sigma),\quad \rho^{\sigma}(\tau,\sigma)=0 \tag{46}$$

if the gauge (14) (for $p=1$) is choosen. The substitution of (46) in the boundary conditions (44) with $\rho^{\sigma}=0$ results in the boundary conditions

$$\partial_{\tau}\rho^{\tau}(\tau,\sigma)\big|_{\sigma=0,\pi}=0 \tag{47}$$

which are satisfied by the additional gauge fixing (18)

$$\partial_{\tau}\rho^{\tau}(\tau,\sigma)=0.$$

In this gauge the general solution (46) coincides with the static solution (19) describing the above studied closed and opened superstrings

$$Y^{\Lambda}(\tau,\sigma)=Y_0^{\Lambda}(\sigma), \tag{48}$$

but has the Dirichlet boundary conditions (38). One notes that the initial data $Y_0^{\Lambda}(\sigma)$ (48) are restricted by the constraint (16)

$$Y_0'^{\Lambda}(\sigma)G_{\Lambda\Xi}Y_0^{\Xi}(\sigma)=0. \tag{49}$$

The matching (45) and (48) confirms that the integration constants $\mathcal{A}^{\Lambda},\mathcal{B}^{\Lambda}$ coincide with the $Y_0^{\Lambda}(\sigma)$ values taken on the string ends $\sigma=0,\pi$

$$\mathcal{A}^{\Lambda}\equiv Y_0^{\Lambda}(0),\qquad \mathcal{B}^{\Lambda}\equiv Y_0^{\Lambda}(\pi). \tag{50}$$

We conclude that the superstring action (43) with the Dirichlet boundary conditions (45) describes a static BPS state with the spontaneously broken $OSp(1,2M)$ symmetry.

6. Wess–Zumino actions of higher orders

Using the $OSp(1,2M)$ invariant character of the differential one-form $Y^{\Lambda}G_{\Lambda\Xi}\,dY^{\Xi}$ and two-form $dY^{\Lambda}G_{\Lambda\Xi}dY^{\Xi}$ one can construct more general $OSp(1,2M)$ invariant super p-brane actions with enhanced supersymmetry. At first, we note that the closed $2n$-differential form $\Omega_{2n}=(G_{\Lambda\Xi}\,dY^{\Lambda}\wedge dY^{\Xi})^n$

$$\begin{aligned}\Omega_{2n}&=d\wedge\Omega_{(2n-1)}\\&\equiv G_{\Lambda_1\Xi_1}\,dY^{\Lambda_1}\wedge dY^{\Xi_1}\wedge\cdots\\&\quad\wedge G_{\Lambda_n\Xi_n}\,dY^{\Lambda_n}\wedge dY^{\Xi_n},\end{aligned} \tag{51}$$

which is not equal to zero, because of the symplectic character of the supertwistor metric $G_{\Lambda\Xi}$, can be used to generate the Dirichlet boundary terms for the open super p-brane $(p=2n-1)$ described by the generalized action (43)

$$S=S_{2n-1}+\beta_{(2n-1)}\int_{M_{2n}}\Omega_{2n}. \tag{52}$$

Similarly to the open superstring case (32), the Wess–Zumino integral in (52) is transformed to the integral along the $(2n-1)$-dimensional boundary M_{2n-1} of the super $(2n-1)$-brane worldvolume

$$\int_{M_{2n}}\Omega_{2n}=\oint_{M_{2n-1}}G_{\Lambda_1\Xi_1}Y^{\Lambda_1}\wedge dY^{\Xi_1}\wedge\cdots\wedge G_{\Lambda_n\Xi_n}\,dY^{\Lambda_n}\wedge dY^{\Xi_n}. \tag{53}$$

The sufficient conditions for the vanishing of the variations of the integral (53) with the fix initial and final data are the conditions

$$\partial_{\tau}Y^{\Lambda}(\tau,\sigma)\big|_{\sigma^i=0,\pi}=0\quad(i=1,2,\ldots,2n-1) \tag{54}$$

generalizing the Dirichlet boundary condition (36). Therefore, in the gauge (14) and (18) this open super p-brane is described by the pure static solution

$$Y^{\Lambda}(\tau,\sigma)=Y_0^{\Lambda}(\sigma^i)\quad(i=1,2,\ldots,2n-1) \tag{55}$$

generalizing the superstring static solution (48). On the other hand, the integrals (53)

$$\begin{aligned}&S_{(2n-2)}=\beta_{(2n-2)}\int_{M_{2n-1}}\Omega_{2n-1},\\&\Omega_{2n-1}\equiv G_{\Lambda_1\Xi_1}Y^{\Lambda_1}\,dY^{\Xi_1}\wedge\cdots\\&\qquad\wedge G_{\Lambda_n\Xi_n}\,dY^{\Lambda_n}\wedge dY^{\Xi_n}\end{aligned} \tag{56}$$

can be considered as the $OSp(1,2M)$ invariant actions for the new models of super p-branes $(p=2n-2)$ with enhanced supersymmetry. For $n=1$ we get the known action [18] for superparticles, but for $n=2,3$ we find the new actions for the supermembrane

$$S_2 = \beta_2 \int_{M_3} \Omega_3 = \tilde{\beta}_2 \int d\tau\, d^2\sigma\, \varepsilon^{\mu\nu\rho} Y^{\Lambda} \partial_\mu Y_{\Lambda} \partial_\nu Y^{\Xi} \partial_\rho Y_{\Xi}, \tag{57}$$

or a domain wall in the symplectic superspace, and for the super four-brane

$$S_4 = \beta_4 \int_{M_5} \Omega_5 = \tilde{\beta}_4 \int d\tau\, d^4\sigma\, \varepsilon^{\mu\nu\rho\lambda\phi} Y^{\Lambda} \partial_\mu Y_{\Lambda} \partial_\nu Y^{\Xi} \partial_\rho Y_{\Xi} \times \partial_\lambda Y^{\Sigma} \partial_\phi Y_{\Sigma}. \tag{58}$$

We shall analyse these models in another place.

7. The Weyl symmetry restoration for the Wess–Zumino actions

A characteristic feature of the proposed Wess–Zumino actions is the explicit breaking of the Weyl gauge symmetry (13). When the Wess–Zumino terms are considered as the boundary terms generating the Dirichlet boundary conditions for the superstring (36) and super p-branes (54) the breaking of the Weyl symmetry is localized at the boundaries. It shows that the spontaneous breaking of the $OSp(1,2M)$ symmetry on the boundaries is accompanied by the explicit breakdown of the Weyl gauge symmetry on the boundaries. Because the Dirichlet boundary conditions are associated with the Dp-branes attached on their boundaries [30], a question on the action of Dp-branes in the symplectic superspaces considered here appears. It implies the correspondent generalization of the proposed Wess–Zumino actions. One of the possible generalizations is rather natural and is based on the observation that the Weyl invariance of the considered Wess–Zumino actions may be restored by the minimal lengthening of the differentials $d \to D = d - A$, where the worldvolume one-form A is the gauge field associated with the Weyl symmetry. The covariant differentials DY^{Σ} are homogeneously transformed under the Weyl symmetry transformations (13)

$$\big(DY^{\Sigma}\big)' \equiv \big((d-A)Y^{\Sigma}\big)' = e^{\lambda} DY^{\Sigma}, \quad A' = A + d\lambda. \tag{59}$$

Then the generalized $OSp(1,2M)$ invariant two and one-forms

$$\big(e^{\phi} DY^{\Sigma} G_{\Sigma\Xi} DY^{\Xi}\big)' = e^{\phi} DY^{\Sigma} G_{\Sigma\Xi} DY^{\Xi}, \quad \big(e^{\phi} Y^{\Sigma} G_{\Sigma\Xi} DY^{\Xi}\big)' = e^{\phi} Y^{\Sigma} G_{\Sigma\Xi} DY^{\Xi} \tag{60}$$

become the invariants of the Weyl symmetry also, where the compensating scalar field ϕ, with the transformation low

$$\phi' = \phi - 2\lambda, \tag{61}$$

was introduced. Then the closed $2n$-differential form $\Omega_{2n} = (G_{\Lambda\Xi}\, dY^{\Lambda} \wedge dY^{\Xi})^n$ may be changed by the Weyl invariant $2n$-differential form $\tilde{\Omega}_{2n} = (e^{\phi} G_{\Lambda\Xi} \times DY^{\Lambda} \wedge DY^{\Xi})^n$

$$\tilde{\Omega}_{2n} \equiv e^{n\phi} G_{\Lambda_1\Xi_1} DY^{\Lambda_1} \wedge DY^{\Xi_1} \wedge \cdots \wedge G_{\Lambda_n\Xi_n} DY^{\Lambda_n} \wedge DY^{\Xi_n}, \tag{62}$$

and Ω_{2n-1} by $\tilde{\Omega}_{2n-1}$

$$\tilde{\Omega}_{2n-1} \equiv e^{n\phi} Y^{\Lambda_1} \wedge DY_{\Lambda_1} \wedge \cdots \wedge DY^{\Lambda_n} \wedge DY_{\Lambda_n}. \tag{63}$$

As a result, the actions (53) is transformed to the new super $(2n-1)$-brane action

$$\tilde{S}_{(2n-1)} = \beta_{(2n-1)} \int_{M_{2n}} \tilde{\Omega}_{2n} = \beta_{(2n-1)} \int e^{n\phi} G_{\Lambda_1\Xi_1} DY^{\Lambda_1} \wedge DY^{\Xi_1} \wedge \cdots \wedge G_{\Lambda_n\Xi_n} DY^{\Lambda_n} \wedge DY^{\Xi_n} \tag{64}$$

invariant under the $OSp(1,2M)$ and Weyl symmetries. Respectively, the action

$$\tilde{S}_{(2n-2)} = \beta_{(2n-2)} \int_{M_{2n-1}} \tilde{\Omega}_{2n-1} = \beta_{(2n-2)} \int e^{n\phi} Y^{\Lambda_1} \wedge DY_{\Lambda_1} \wedge \cdots \wedge DY^{\Lambda_n} \wedge DY_{\Lambda_n} \tag{65}$$

will describe a new $OSp(1,2M)$ and Weyl invariant super-$(2n-2)$-brane. These actions may be presented in the Dp-brane like form, e.g.,

$$\tilde{S}_p = \tilde{\beta}_p \int d\tau\, d^p\sigma\, e^{\frac{(p+1)}{2}\phi} \times \sqrt{\big|\det\big[(\partial_\mu - A_\mu) Y^{\Lambda} G_{\Lambda\Xi} (\partial_\nu - A_\nu) Y^{\Xi}\big]\big|} \quad (p = 2n-1), \tag{66}$$

where $\tilde{\beta}_p$ is the Dp-brane tension.

8. Conclusion

We considered the general solutions of the equations of motion in the simple model of closed and open tensionless superstring and super p-branes and found that these static solutions spontaneously break the $OSp(1,2M)$ symmetry and $N=1$ supersymmetry. Next, we generalized this model to the higher orders in the derivatives of the Goldstone fields and constructed the new Wess–Zumino like actions supposed to describe tensile super p-branes. These actions generate the Dirichlet boundary conditions which, in particular, break the Weyl gauge symmetry. The introduction of additional vector and scalar fields restores the Weyl symmetry and results in the Weyl and $OSp(1,2M)$ invariant Dp-brane like actions. The open problem is to find supersymmetric YM field theories having the considered superbranes as vacuum states spontaneously breaking the $OSp(1,2M)$ symmetry. One can conjecture that these branes appear as supersymmetric solutions of $D=11$ supergravity [1,2], where the $OSp(1,64)$ symmetry is also spontaneously broken [20]. Then a connection between the R^{31} holonomy and space–time symmetries [1,2] with the local Abelian shifts of the space–time and TCC brane coordinates by the null multivectors [26] may appear. We will study these problems in another place.

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