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ScienceDirect
Procedia Engineering 144 (2016) 841 – 848
Procedia Engineering
www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Dynamic Characteristics of a Finite-Width Journal Bearing Lubricated with Powders

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Abstract

This paper attempts to investigate the dynamic characteristics of a powder lubricated journal bearing. The stiffness and damping coefficients are obtained using finite perturbation method. The stability limit of the rotor speed is obtained for a system consisting of a single rotor disc in the middle of a flexible shaft having identical plain cylindrical journal bearings at the ends. The threshold speed of instability for a rotor supported on powder lubricated journal bearing is compared with that of oil lubricated journal bearing. The numerical results show that a rotor supported on powder lubricated journal bearings remains stable for a speed limit much higher than that for oil lubricated bearings.

Peer-review under responsibility of the organizing committee of ICOVP 2015
Keywords: Powder lubrication; Journal bearing; Grain size; Stability of rigid rotor

1. Introduction

Bearings lubricated with conventional oils are unable to support load at high temperatures due to molecular breakdown caused by oxidation of oil [1]. Therefore it is a topic of interest for various researchers to explore the suitability of powders as alternate lubricants for hostile environments where liquid lubricants cannot perform [2–4]. Heshmat [5] was the first to demonstrate that powder lubricants can exhibit hydrodynamic behaviour similar to oil lubricants. It was verified experimentally that the pressure profile obtained with TiO_2 powders in a slider bearing was remarkably similar to that of an oil lubricant. Heshmat termed this behaviour of the powder lubricant as quasi-hydrodynamic. Heshmat [6] conducted a series of experiments to develop a Powder-Lubricated Quasi-Hydrodynamic (PLQH) journal bearing for high-temperature and hostile environments where the use of liquid

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.097

lubricants is impractical. It was successfully demonstrated that the bearing was capable to operate at speeds up to 58,000 rpm. Heshmat and Brewe [7,8] performed experiments on a three pad journal bearing using Molybdenum disulphide (MoS_2) and Tungsten disulphide (WS_2) powders respectively. Thermal stability was achieved up to 400°C and 600°C for MoS_2 and WS_2 powders respectively. A comparative evaluation of MoS_2 and WS_2 powders in three pad journal bearings is presented by Higgs et al. [9]. Kaur and Heshmat [10] developed a self-contained solid/powder lubricated auxiliary hydrodynamic bearing, capable of supporting significant rotor loads of 445N, operating at 815°C and 30,000 rpm.

The theoretical modelling of powder flow in the interface is complicated because both physicochemical and mechanical conditions greatly influence powder flow behaviour [11]. Iordanoff et al. [11] described several approaches to model solid third bodies (substances in the interface that are not integral with either surface). Generally, the modelling techniques fall into two categories; (a) discrete model and (b) continuum model. The continuum model is computationally efficient and exhibits good agreement with experimental results. Using continuum approach Haff [12] developed a grain theory considering individual grain as a molecule of granular fluid. Simple 'microscopic' kinetic model was used to derive the expression for 'coefficients' of viscosity, thermal diffusivity and energy absorption due to collisions. Haff obtained a closed-form solution for the velocity and pressure distribution in Couette flow. Using Haff's theory Dai et al. [13] and McKeague and Khonsari [14] developed an analytical model for one dimensional slider bearing. Tsai and Jeng [15] derived a closed-form average lubrication equation for thin film grain flow with the effects of surface roughness. In another work [16], the authors studied the effect of particle size and inelasticity during grain-grain collision on the performance parameters. Their results were consistent with the experimental findings of Heshmat and Brewe [7,8].

In the last two decades, there is a significant progress in the study of powder lubricated journal bearings. Several experimental and numerical investigations are carried out to study the behaviour of powder lubricants. However, the dynamic characteristics still need to be explored. In the present work, an attempt is made to obtain the bearing coefficients and study the stability characteristics of a flexible and a rigid rotor supported on powder lubricated journal bearings. Numerical results for bearing coefficients, eigenvalues, and response of the rotor in time domain are obtained for a rotor-bearing system. The results obtained for the rotor-bearing system in which both the supporting bearings are lubricated with powders are compared with those obtained with a system having oil lubricated bearings.

2. Mathematical model

Haff [12] derived governing equations for grain flow using conservation of mass, momentum, and energy by considering grain flow as a fluid mechanics problem with continuum perspective. The theory assumes that grain particles are identical spheres and the average separation (s) is much lesser than the grain diameter (d) which implies that the bulk density (ρ) is nearly constant throughout the flow domain. In addition to the bulk velocity, each grain particle moves randomly in its cell with an average fluctuation velocity ($\bar{v}$) which is also known as thermal velocity or granular pseudo-temperature. During collision between two grain particles, momentum is exchanged and energy is lost due to inelastic collision. Using microscopic kinetic model, expressions for pressure (p), viscosity (η), thermal diffusivity (K), and energy dissipation (I) are obtained as:

$$p = td\rho\frac{\bar{v}^2}{s}; \qquad \eta = ad^2\rho\frac{\bar{v}}{s}; \qquad K = kd^2\frac{\bar{v}}{s}; \qquad I = w\rho\frac{\bar{v}^3}{s} \tag{1}$$

where, t, q, k, and w are dimensionless coefficients.

The schematic of a journal bearing is shown in Fig. 1. The expression for film thickness (h) is given by,

$$h = C_r + x_0\cos\theta + y_0\sin\theta \tag{2}$$

where C_r is the radial clearance and (x_0, y_0) is the position of the center of the journal.

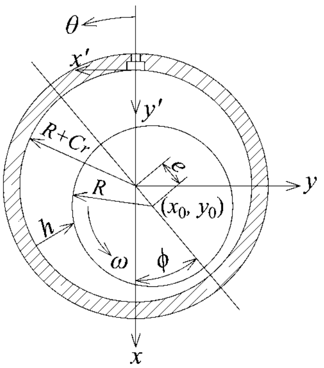

Fig. 1. Schematic of a journal bearing

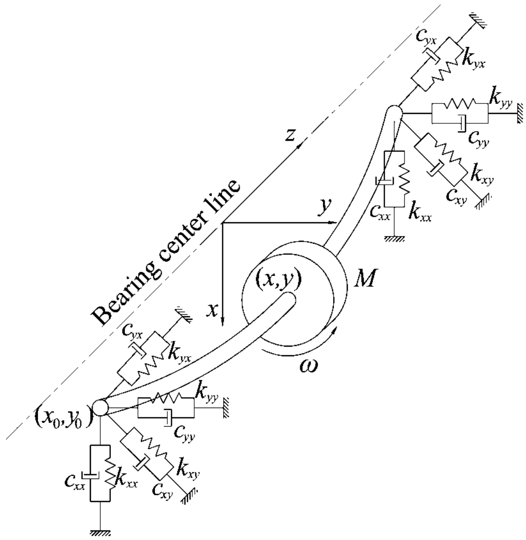

Fig. 2. Schematic of a flexible rotor

Considering the order of magnitude analysis, Dai et al. [13] obtained governing equations which are written as:

$$\frac{\partial p}{\partial x'} = \frac{\partial}{\partial y'}\left(\eta \frac{\partial u_{x'}}{\partial y'}\right) \tag{3}$$

$$\frac{\partial p}{\partial y'} = 0 \tag{4}$$

$$\frac{\partial p}{\partial z'} = \frac{\partial}{\partial y'}\left(\eta \frac{\partial u_{z'}}{\partial y'}\right) \tag{5}$$

$$\frac{\partial^2 \overline{v}}{\partial y'^2} - \frac{w}{kd^2}\overline{v} = 0 \tag{6}$$

The expression for thermal velocity ($\overline{v}$) is obtained by solving eq. (6) with the boundary conditions $\overline{v}_{(y'=0)} = B_0$ and $\overline{v}_{(y'=h)} = B_h$, which is written as:

$$\overline{v} = \left(\frac{B_h - B_0 e^{-\lambda h}}{e^{\lambda h} - e^{-\lambda h}}\right) e^{\lambda y'} + \left(\frac{B_0 e^{\lambda h} - B_h}{e^{\lambda h} - e^{-\lambda h}}\right) e^{-\lambda y'} \tag{7}$$

where $\lambda = (1/d)\sqrt{w/k}$. The grain flow viscosity is treated as the average viscosity across the film [16] and is expressed as:

$$\eta = \frac{a}{t}\frac{d}{\overline{v}_{av}}\, p \tag{8}$$

It can be noted that the grain flow viscosity is a function of pressure and grain diameter. The average thermal velocity is obtained from eq. (7) as,

$$\overline{v}_{av} = \frac{1}{h}\int_0^h \overline{v} dy = \frac{2B}{h\lambda}\left(\frac{R_2 - 2}{R_1}\right) \tag{9}$$

where $B = \left(B_0 + B_h\right)/2$, $R_1 = e^{\lambda h} - e^{-\lambda h}$, and $R_2 = e^{\lambda h} + e^{-\lambda h}$. Integrating eqs. (3), and (5) twice with respect to *y'*, the expressions for flow velocities in *x'*, and *z'* directions respectively are obtained. Applying conservation of mass to a control volume of the grain flow system, the Reynolds type lubrication equation for grain flow and is obtained as:

$$\frac{\partial}{R\partial\theta}\left(\frac{1}{6}\frac{t}{a}\psi h^2 \frac{\partial(\ln p)}{R\partial\theta}\right) + \frac{\partial}{\partial z'}\left(\frac{1}{6}\frac{t}{a}\psi h^2 \frac{\partial(\ln p)}{\partial z'}\right) = \frac{U}{2}\frac{\partial h}{R\partial\theta} + \frac{\partial h}{\partial t} \tag{10}$$

where $\psi = \overline{v}_{av} h/2d$. Film force acting on the journal is resolved in vertical (*x*) and horizontal (*y*) directions as below:

$$F_x = \int_0^L \int_0^{2\pi} pR\cos\theta \, d\theta \, dz \tag{11}$$

$$F_y = \int_0^L \int_0^{2\pi} pR\sin\theta \, d\theta \, dz \tag{12}$$

The lubricating film is modelled to have both direct and cross coupled stiffness and damping coefficients. Stiffness and damping coefficients are computed using the finite perturbation approach [17]. For the displacement and velocity perturbations $\left(\Delta x, \Delta y, \Delta\dot{x}, \Delta\dot{y}\right)$ from equilibrium, normalized stiffness (*K*) and damping coefficients (*C*) are given by:

$$K_{xx} = \left(C_r/W\right)\left[F_x(x_0 + \Delta x, y_0, 0, 0) - F_x(x_0 - \Delta x, y_0, 0, 0)\right]/\left(2\Delta x\right) \tag{13}$$

$$C_{xx} = \left(C_r\omega/W\right)\left[F_x(x_0, y_0, \Delta\dot{x}, 0) - F_x(x_0, y_0, -\Delta\dot{x}, 0)\right]/\left(2\Delta\dot{x}\right) \tag{14}$$

where *W* is the total load on the bearing. Other coefficients may be obtained in a similar way.

For a flexible rotor of shaft stiffness (k_s) and disc mass (*M*) as shown in Fig. 2, the equation of motion for free vibration can be written as,

$$\begin{aligned}
&M\ddot{x} + k_s\left(x - x_0\right) = 0 \\
&M\ddot{y} + k_s\left(y - y_0\right) = 0 \\
&k_s\left(x - x_0\right) = 2k_{xx}x_0 + 2k_{xy}y_0 + 2c_{xx}\dot{x}_0 + 2c_{xy}\dot{y}_0 \\
&k_s\left(y - y_0\right) = 2k_{yx}x_0 + 2k_{yy}y_0 + 2c_{yx}\dot{x}_0 + 2c_{yy}\dot{y}_0
\end{aligned} \tag{15}$$

Assuming a solution of the form, $x_i = X_i e^{\lambda t}$ where $x_i = x, y, x_0, y_0$, and $X_i = X, Y, X_0, Y_0$. Substituting x_i in eq. (15), following set of equations are obtained,

$$\begin{bmatrix} M\lambda^2 + k_s & 0 & -k_s & 0 \\ 0 & M\lambda^2 + k_s & 0 & -k_s \\ -k_s & 0 & 2k_{xx} + 2c_{xx}\lambda + k_s & 2k_{xy} + 2c_{xy}\lambda \\ 0 & -k_s & 2k_{yx} + 2c_{yx}\lambda & 2k_{yy} + 2c_{yy}\lambda + k_s \end{bmatrix} \begin{Bmatrix} X \\ Y \\ X_0 \\ Y_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

Eq. (16) can be written as,

$$[C]\{X\} = \{0\} \tag{17}$$

For a non-trivial solution, the determinant of the matrix [C] must be equal to zero. Setting $|C| = 0$ gives a polynomial equation in 'λ' known as characteristic equation. The roots of the characteristic equations are the eigenvalues of the system. These eigenvalues are generally complex whose real part indicates the stability of the system. If the real part of the eigenvalue is negative, the system will be stable because the response (x_i) will be an exponentially decaying function of time. The threshold of instability will occur when the real part of eigenvalue becomes zero.

3. Computational procedure

Eq. (10) is discretized using finite difference method and Gauss-Seidel method is used to obtain pressure iteratively. The convergence criteria for pressure is given by,

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left|\left(p_{i,j}\right)^{k+1} - \left(p_{i,j}\right)^{k}\right|}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left|\left(p_{i,j}\right)^{k+1}\right|} \le 10^{-5} \tag{18}$$

where i, j are the grid points in θ and z directions respectively, m, n are the associated number of nodes, and k is the iteration number. Bearing forces (F_x and F_y) are then evaluated by integrating the pressure over the surface of the bearing using Simpson's 1/3 rule. First, bearing forces are obtained for steady state equilibrium position of the journal center. Then, bearing forces are obtained for perturbed position of journal center from the equilibrium. Perturbation amplitudes of $0.001C_r$ and $0.001\omega C_r$ are chosen for position and velocity perturbations respectively. Stiffness and damping coefficients are calculated using eqs. (13)-(14). Once the stiffness and damping coefficients are calculated, the eigenvalues of the system shown in Fig. 2 are obtained using eq. (16). To obtain the response of the rotor in time domain the equations of motion given in eq. (15) are solved numerically using 4th order Runge-Kutta method.

All the results are obtained for a finite width journal bearing whose width to diameter ratio is unity (L/D=1). To compare the performances of a bearing lubricated by powder and oil the following data are assumed as given in Table-1.

4. Results and discussion

The variation of stiffness and damping coefficients with Sommerfeld number (S) is shown in Fig. 3. Sommerfeld number is defined as $S = \left(\eta N_s LD/W\right)\left(R/C_r\right)^2$. It may be noted that for powder lubricated bearing the cross stiffness (K_{xy} and K_{yx}) gets negatively coupled at a higher value of S as in comparison with oil lubricated bearings. A higher value of S implies a higher speed of rotation for a fixed load and bearing geometry. Thus, a powder lubricated bearing will remains stable for a speed limit much higher than that of oil lubricated bearings. The values of direct stiffness (K_{xx} and K_{yy}) and all the damping coefficients of a powder lubricated bearing are higher in comparison with oil lubricated bearing and this implies that, given a perturbation about equilibrium a rotor supported on powder lubricated bearings will regain its equilibrium position more quickly than the situation when it is supported on oil lubricated bearings.

Table 1. Input data

Parameter	For powder lubrication	For oil Lubrication
Bearing diameter (*D*), m	0.04	0.04
Bearing length (*L*), m	0.04	0.04
Radial clearance (C_r), m	0.1 x10^{-3}	0.1 x10^{-3}
Reference viscosity (η_0), Pa.s	0.067	0.067
Disc mass (*M*), kg	51	51
Shaft stiffness (K_s)	4.13x10^7	4.13x10^7
Grain diameter (*d*), m	10^{-6}	-
t/a	1	-
w/k	0.0004	-
B/U	4	-

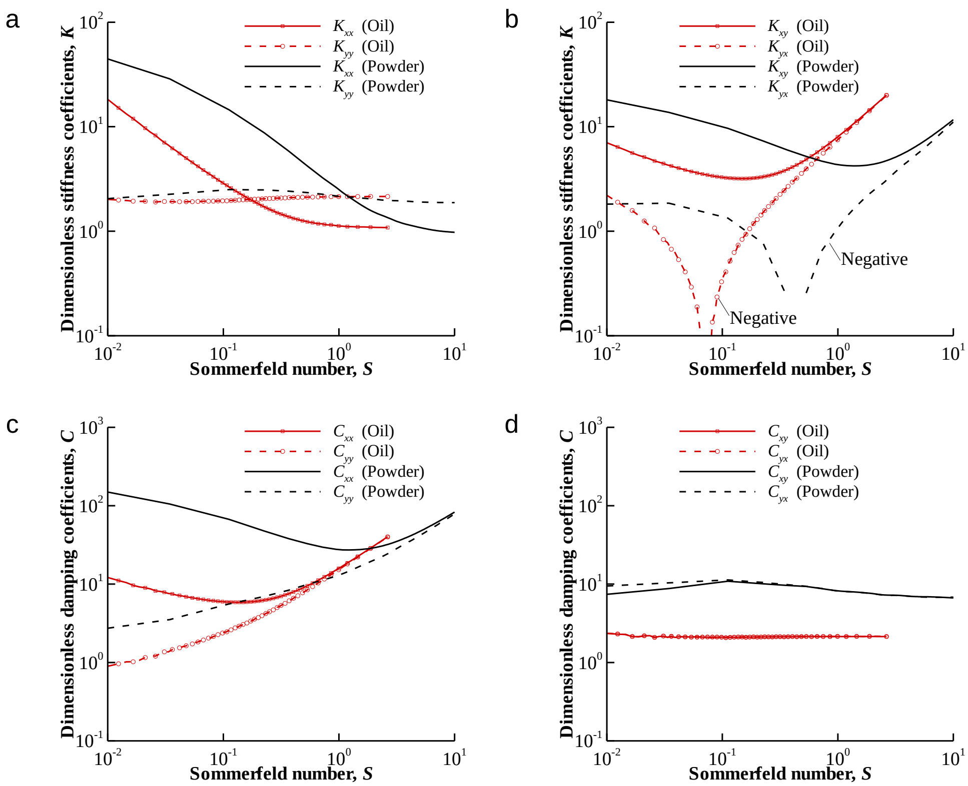

Fig. 3. Variation of stiffness and damping coefficients with Sommerfeld number

A plot of maximum real part of all the eigenvalue versus rotor speed is shown in Fig. 4 for rigid as well as flexible rotors. It may be seen that the maximum real part goes from negative to positive with increase in rotor speed. The point where it crosses the horizontal axis is the threshold speed of instability. It may be seen that the threshold speeds for a flexible and a rigid rotor supported on oil lubricated bearing are 6939 rpm and 7585 rpm respectively, whereas the same for powder lubricated bearings are 23434 rpm and 25313 rpm respectively. The threshold speed for rotor supported on powder lubricated bearings is more than three times of that when the bearings are oil lubricated. Also, the threshold speed is more for rigid rotor than that of a flexible rotor as observed by Rao [18].

The response obtained by moving the journal center to the bearing center and then releasing it reflect the stability of the system known as position perturbation [19]. For position perturbation, the time response and the trajectory of the journal center for a flexible shaft supported on powder lubricated bearings are shown in Fig. 5. Two cases are shown; (a) a stable system at 20000 rpm, and (b) an unstable system at 27000 rpm. For a stable system the amplitude of displacement decreases with time and the journal goes back to its steady state position. For an unstable system the amplitude of displacement increases with time and the journal goes far and far away from the equilibrium position.

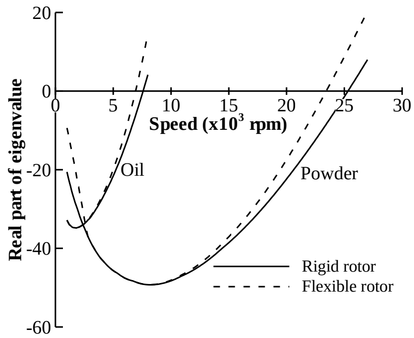

Fig. 4. Plot of maximum value of the real part of the eigenvalues against speed of rotor

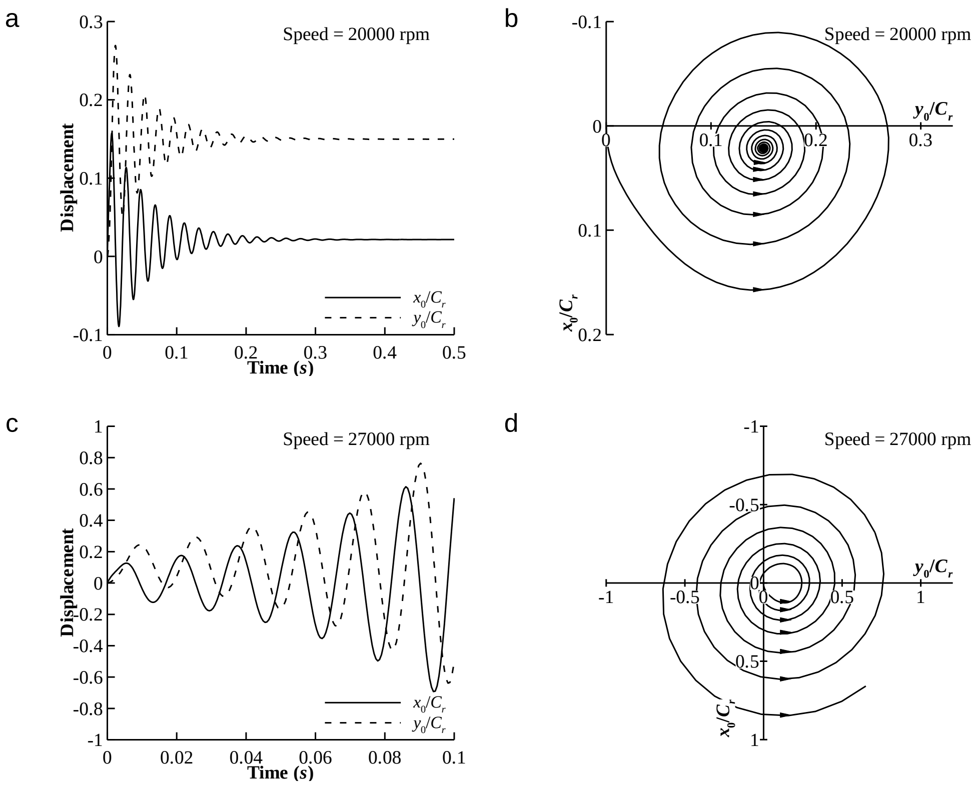

Fig. 5. Time response and orbit plots of a stable and unstable system

5. Conclusions

A powder lubricated journal bearing is investigated for the evaluation of its dynamic characteristics. Stiffness and damping coefficients are obtained and stability limits of spin speed of a rigid as well as flexible rotor consisting of a central disc are investigated and compared with the case when the bearings are oil lubricated. Based on the present investigations for a powder lubricated journal bearing

1. The cross stiffness (K_{xy} and K_{yx}) get negatively coupled at speed higher as compared with the case of oil lubricated bearing.
2. The values of direct stiffness and all the damping coefficients are higher as compared with the case of oil lubricated bearing.
3. The threshold speed of instability of the rotor is higher as compared with the case when the bearing is oil lubricated.

The present work is limited to the study of simple rotor bearing system. Future scope includes the studies on more complex rotor bearing system.

Acknowledgement

The authors of this paper are grateful to the management of Indian Institute of Technology Delhi, India, for providing the infrastructure facilities to carry out the research.

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