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Complete electroweak chiral Lagrangian with a light Higgs at NLO

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Abstract

We consider the Standard Model, including a light scalar boson h, as an effective theory at the weak scale $v = 246$ GeV of some unknown dynamics of electroweak symmetry breaking. This dynamics may be strong, with h emerging as a pseudo-Goldstone boson. The symmetry breaking scale Λ is taken to be at $4\pi v$ or above. We review the leading-order Lagrangian within this framework, which is nonrenormalizable in general. A chiral Lagrangian can then be constructed based on a loop expansion. A systematic power counting is derived and used to identify the classes of counterterms that appear at one loop order. With this result the complete Lagrangian is constructed at next-to-leading order, $\mathcal{O}(v^2/\Lambda^2)$. This Lagrangian is the most general effective description of the Standard Model containing a light scalar boson, in general with strong dynamics of electroweak symmetry breaking. Scenarios such as the SILH ansatz or the dimension-6 Lagrangian of a linearly realized Higgs sector can be recovered as special cases.

Keywords: Electroweak chiral Lagrangian; Light Higgs; Effective field theory

1. Introduction

The recent discovery of a scalar sector in the Standard Model has been one of the most important breakthroughs of the last decades in particle physics. The additional confirmation, as more

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and more experimental evidence is piling up [1–4], that the scalar particle closely resembles the Higgs boson is even more remarkable, meaning that the Standard Model provides a rather successful description of electroweak symmetry breaking. In particular, recent experimental results strengthen the evidence for a particle with spin 0 and positive parity [5].

However, the Standard Model solution to electroweak symmetry breaking is extremely fine-tuned and should be deemed unsatisfactory. More natural solutions typically call for new physics states at the TeV scale, for which unfortunately there is no evidence so far. However, their eventual existence would typically induce deviations from the Standard Model Higgs parameters, which, even if only slight, would be of profound significance for the renormalizability and unitarization of the theory and, more generally, for our understanding of the dynamics of electroweak symmetry breaking.

There exists a large number of alternatives to the Higgs model, which provide different dynamical explanations of electroweak symmetry breaking. From a phenomenological viewpoint it is however more efficient to test these potential deviations from the Standard Model with a broader framework and then particularize to specific models, the Standard Model being one of them. Given the large energy gap between the electroweak scale $v = 246$ GeV and the expected new physics scale $\Lambda \sim$ few TeV, this broader framework can be most easily cast in an effective field theory (EFT) language. This EFT should provide, by construction, the most general description of the electroweak interactions in the presence of a light scalar h, and therefore provide the right framework to test its dynamical nature. As a result, the EFT we are after is actually the most general EFT description of the electroweak interactions with the presently known particle content.

The starting point for such an EFT requires a parameterization of the minimal coset $SU(2)_L \times U(1)_Y/U(1)_{em}$, which can be done using a nonlinear realization [6]. The resulting Goldstone bosons provide the longitudinal components of the gauge bosons. The new scalar h is then introduced in full generality as a singlet under $SU(2)_L \times U(1)_Y$. This path has been pursued before, and partial sets of the resulting effective-theory operators have been listed [7,8] and their phenomenological consequences explored [9–11]. However, the previous papers lacked a careful discussion of the foundations of the EFT, including essential aspects in the construction of the operator basis such as power-counting arguments. In this paper we want to fill this gap and put the EFT on a more systematic basis. A large part of this effort was already done in [12], where the systematics of the nonlinear EFT of electroweak interactions was spelled out. In this paper we show how to extend those results when a scalar singlet h is included.

This paper is organized as follows: in Section 2 we review the Standard Model chiral Lagrangian at leading order as the most general description of electroweak symmetry breaking. In Section 3 we discuss how to organize the EFT expansion in powers of v^2/Λ^2 with a consistent power-counting. Section 4 is devoted to working out the most general basis of operators at next-to-leading order (NLO). In Section 5 we extend our discussion to include generic scenarios of partial compositeness as interpolations between the purely strongly-coupled and weakly-coupled limits. A comparison with the previous literature is provided in Section 6. For illustration, in Section 7 we include two particular model realizations, namely the $SO(5)/SO(4)$ composite Higgs model and a Higgs-portal model, showing how they reduce to particular parameter choices of the general EFT. Conclusions are given in Section 8, while technical details are collected in Appendix A.

2. SM chiral Lagrangian at leading order

In this section we summarize the leading-order (LO) electroweak chiral Lagrangian of the Standard Model including a light Higgs field h. Further comments on the systematics behind its construction can be found in Appendix A.

The leading-order Lagrangian can be written as

$$\mathcal{L}_{\rm LO} = \mathcal{L}_4 + \mathcal{L}_{Uh} \tag{1}$$

The first term, $\mathcal{L}_4$, represents the unbroken, renormalizable part, built from the left-handed doublets q, l and right-handed singlets u, d, e of quarks and leptons, together with the gauge fields G, W, B of $SU(3)_C$, $SU(2)_L$, $U(1)_Y$:

$$\begin{aligned}\mathcal{L}_4 = &-\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\\ &+\bar{q}i\not{D}q + \bar{l}i\not{D}l + \bar{u}i\not{D}u + \bar{d}i\not{D}d + \bar{e}i\not{D}e\end{aligned} \tag{2}$$

Generation indices have been omitted. Here and in the following the trace of a matrix M is denoted by $\langle M\rangle$. The covariant derivative of a fermion field $\psi_{L,R}$ is defined as

$$D_\mu\psi_L = \partial_\mu\psi_L + igW_\mu\psi_L + ig'Y_{\psi_L}B_\mu\psi_L, \qquad D_\mu\psi_R = \partial_\mu\psi_R + ig'Y_{\psi_R}B_\mu\psi_R \tag{3}$$

dropping the QCD part for simplicity. The Higgs-sector Lagrangian reads

$$\begin{aligned}\mathcal{L}_{Uh} = &\frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\big(1+F_U(h)\big) + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h)\\ &- v\Bigg[\bar{q}\left(\hat{Y}_u + \sum_{n=1}^{\infty}\hat{Y}_u^{(n)}\left(\frac{h}{v}\right)^n\right)UP_+r + \bar{q}\left(\hat{Y}_d + \sum_{n=1}^{\infty}\hat{Y}_d^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-r\\ &+\bar{l}\left(\hat{Y}_e + \sum_{n=1}^{\infty}\hat{Y}_e^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-\eta + \text{h.c.}\Bigg]\end{aligned} \tag{4}$$

where

$$F_U(h) = \sum_{n=1}^{\infty} f_{U,n}\left(\frac{h}{v}\right)^n, \qquad V(h) = v^4\sum_{n=2}^{\infty} f_{V,n}\left(\frac{h}{v}\right)^n \tag{5}$$

Here the right-handed quark and lepton fields are written as $r = (u,d)^T$ and $\eta = (\nu,e)^T$, respectively. In general, different flavor couplings $\hat{Y}_{u,d,e}^{(n)}$ can arise at every order in the Higgs field h^n, in addition to the usual Yukawa matrices $\hat{Y}_{u,d,e}$. We define

$$P_\pm \equiv \frac{1}{2} \pm T_3, \qquad P_{12} \equiv T_1 + iT_2, \qquad P_{21} \equiv T_1 - iT_2 \tag{6}$$

where P_{12} and P_{21} will be needed later on.

Under $SU(2)_L \times SU(2)_R$ the Goldstone boson matrix U and the Higgs-singlet field h transform as

$$U \to g_L U g_R^\dagger, \qquad h \to h, \qquad g_{L,R} \in SU(2)_{L,R} \tag{7}$$

The transformations g_L and the $U(1)_Y$ subgroup of g_R are gauged, so that the covariant derivatives are given by

$$D_\mu U = \partial_\mu U + igW_\mu U - ig' B_\mu U T_3, \qquad D_\mu h = \partial_\mu h \tag{8}$$

The explicit relation between the matrix U and the Goldstone fields φ^a is

$$U = \exp(2i\Phi/v), \quad \Phi = \varphi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\varphi^0}{\sqrt{2}} & \varphi^+ \\ \varphi^- & -\frac{\varphi^0}{\sqrt{2}} \end{pmatrix} \tag{9}$$

where $T^a = T_a$ are the generators of $SU(2)$.

3. Power counting

The leading-order Lagrangian (1) is nonrenormalizable in general. A consistent effective field theory can be constructed order by order in the loop expansion. The next-to-leading order terms can be classified according to the counterterms that appear at one loop. The corresponding classes of operators are determined by standard methods of power counting. For the case of the chiral Lagrangian in (1) without the Higgs scalar h this procedure has been discussed in [12], where further details can be found. The generalization to include the light Higgs scalar is straightforward and will be summarized in the following. We will omit ghost fields, which insure manifest gauge independence, but do not affect the power counting.

Without h, a generic L-loop diagram $\mathcal{D}$, built from (1), contains n_i φ^{2i}-vertices and ν_k Yukawa interactions $\bar\psi_{L(R)}\psi_{R(L)}\varphi^k$, a number m_l of gauge-boson-Goldstone vertices $X_\mu\varphi^l$, r_s such vertices of the type $X_\mu^2\varphi^s$, x quartic gauge-boson vertices X_μ^4, u triple-gauge-boson vertices X_μ^3, and z_L (z_R) fermion-gauge-boson interactions $\bar\psi_{L(R)}\psi_{L(R)}X_\mu$. Here ψ_L (ψ_R), φ and X_μ denote left-handed (right-handed) fermions, Goldstone bosons and gauge fields, respectively.

The presence of h introduces into $\mathcal{D}$ a number σ_{ja} of Goldstone–Higgs vertices $\varphi^{2j}h^a$, τ_{tb} Yukawa vertices with t Goldstone and b Higgs lines, as well as ω_q h^q-interactions.

Following the steps discussed in [12], the power-counting for the diagram $\mathcal{D}$ can be summarized by the formula

$$\mathcal{D} \sim \frac{(yv)^\nu (gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2-2\omega}} \frac{p^d}{\Lambda^{2L}} \bar\psi_L^{F_L^1} \psi_L^{F_L^2} \bar\psi_R^{F_R^1} \psi_R^{F_R^2} \left(\frac{X_{\mu\nu}}{v}\right)^V \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H \tag{10}$$

where the power of external momenta p is

$$d \equiv 2L + 2 - \frac{F_L + F_R}{2} - V - \nu - m - 2r - 2x - u - z - 2\omega \tag{11}$$

Here $F_L = F_L^1 + F_L^2$, $F_R = F_R^1 + F_R^2$ and V is the number of external left-handed fermion, right-handed fermion and gauge-boson lines, respectively. g is a generic gauge coupling, and we have used $\nu \equiv \sum_k \nu_k + \sum_{t,b} \tau_{tb}$, $m \equiv \sum_l m_l$, $r \equiv \sum_s r_s$, $z \equiv z_L + z_R$, $\omega \equiv \sum_q \omega_q$. An exponent $d \geqslant 0$ in (10) indicates a divergence by power counting, as well as the number of derivatives in the corresponding counterterm. The expression (11) for d is useful, because F_L, F_R and V, as well as the numbers of vertices, all enter with a negative sign. This implies that the number of divergent diagrams at a given order in L is finite. We also note that the numbers of both external Goldstone and Higgs boson lines, B and H, enter the power counting formula (10) only through the factors $(\varphi/v)^B$ and $(h/v)^H$. They are irrelevant in particular for the exponent d, which counts the powers of momentum. This indicates explicitly that, at any given order in the effective theory, the counterterms contain an arbitrary number of Goldstone fields $U = U(\varphi/v)$, as well as Higgs fields h/v. Both φ and h are therefore on the same footing. This result of power counting is in agreement with the discussion in Appendix A.

Since (10) and (11) are very similar to the case without h discussed in [12], the generalization to the scenario that includes h follows immediately. The NLO counterterms are found by enumerating the classes of diagrams that give rise to a degree of divergence $d \geqslant 0$ with $L = 1$ in (11). Denoting by Uh the presence of any number of Goldstone fields U (or $U^\dagger$) and Higgs singlets h, and by D^n, ψ^F, X^k the numbers n, F, k, respectively, of derivatives, fermion fields and gauge-boson field-strength tensors, these classes are schematically given by

$$UhD^4, \quad X^2Uh, \quad XUhD^2, \quad \psi^2UhD, \quad \psi^2UhD^2, \quad \psi^4Uh \tag{12}$$

The next section will be devoted to constructing the full set of basis operators in each class.

4. Effective Lagrangian at next-to-leading order

The NLO operators are conveniently expressed using the definitions

$$L_\mu \equiv iUD_\mu U^\dagger, \qquad \tau_L \equiv UT_3U^\dagger \tag{13}$$

Both L_μ and τ_L are hermitean and traceless. They obey the identities

$$D_\mu L_\nu - D_\nu L_\mu = gW_{\mu\nu} - g' B_{\mu\nu}\tau_L + i[L_\mu, L_\nu] \tag{14}$$

$$D_\mu \tau_L = -i[\tau_L, L_\mu] \tag{15}$$

The NLO operators can be constructed using elementary building blocks, as reviewed in [12] for the case without h field. In the Goldstone–Higgs sector the required building blocks are

$$\langle L_\mu L_\nu\rangle, \quad \langle \tau_L L_\mu\rangle, \quad \langle L_\mu L_\nu L_\lambda\rangle, \quad \langle \tau_L L_\mu L_\nu\rangle, \quad \partial_\mu h, \quad F(h) \tag{16}$$

where $F(h)$ denotes a generic function of h/v. Five additional building blocks arise when the electroweak field strengths are included

$$\langle W_{\mu\nu} L_\lambda\rangle, \quad \langle \tau_L W_{\mu\nu}\rangle, \quad \langle W_{\mu\nu} L_\lambda L_\rho\rangle, \quad \langle \tau_L W_{\mu\nu} L_\lambda\rangle, \quad B_{\mu\nu} \tag{17}$$

Together with the terms in the LO Lagrangian, these elements are sufficient to construct the NLO operators in the purely bosonic sector. Operators with fermions can be obtained along similar lines [12]. Note that apart from the functions $F(h)$, which enter each operator as an overall factor, the only new building block in comparison to [12] is $\partial_\mu h$.

Using integration by parts, the identities (14) and (15), and the leading-order equations of motion, certain operators can be shown to be redundant. To proceed in a systematic way, we eliminate a given operator, if possible, in favor of operators with fewer derivatives.

The next-to-leading-order effective Lagrangian of the Standard Model with dynamically broken electroweak symmetry, including a light Higgs scalar, can then be written as

$$\mathcal{L} = \mathcal{L}_{\rm LO} + \mathcal{L}_{\beta_1} + \sum_i c_i \frac{v^{6-d_i}}{\Lambda^2}\mathcal{O}_i \tag{18}$$

Here $\mathcal{L}_{\rm LO}$ is the leading order Lagrangian (1) and $\mathcal{L}_{\beta_1}$ the custodial-symmetry breaking, dimension-2 operator

$$\mathcal{L}_{\beta_1} = -\beta_1 v^2 \langle \tau_L L_\mu\rangle \langle \tau_L L^\mu\rangle F_{\beta_1}(h), \quad F_{\beta_1}(h) = 1 + \sum_{n=1}^\infty f_{\beta_1,n}\left(\frac{h}{v}\right)^n \tag{19}$$

As discussed in Appendix A this operator can be treated as a next-to-leading order correction. Apart from this term, the NLO operators are denoted by $\mathcal{O}_i$ in (18). They come with a suppression by two powers of the symmetry-breaking scale $\Lambda \approx 4\pi v$ and have dimensionless coefficients c_i, which are naturally of order unity. d_i is the canonical dimension of the operator $\mathcal{O}_i$. Conservation of baryon and lepton number will be assumed in the present context, since their violation is expected to arise only at scales much above the few TeV range. Further remarks can be found in [12].

In the following we list the NLO operators $\mathcal{O}_i$ according to the classification introduced at the end of Section 3.

4.1. UhD^4 terms

The operators of this class generalize the $\mathcal{O}(p^4)$ chiral-Lagrangian terms UD^4 already given in [13], now including arbitrary powers of h/v. There is a total of 15 independent operators, of which 11 are CP-even and 4 are CP-odd.

The CP-even operators can be written as

$$\begin{aligned}
\mathcal{O}_{D1} &= \langle L_\mu L^\mu \rangle^2 F_{D1}(h) \\
\mathcal{O}_{D2} &= \langle L_\mu L_\nu \rangle \langle L^\mu L^\nu \rangle F_{D2}(h) \\
\mathcal{O}_{D3} &= \left(\langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \right)^2 F_{D3}(h) \\
\mathcal{O}_{D4} &= \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle L_\nu L^\nu \rangle F_{D4}(h) \\
\mathcal{O}_{D5} &= \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \langle L^\mu L^\nu \rangle F_{D5}(h)
\end{aligned} \tag{20}$$

$$\mathcal{O}_{D6} = i \langle \tau_L L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \frac{\partial^\nu h}{v} F_{D6}(h) \tag{21}$$

$$\begin{aligned}
\mathcal{O}_{D7} &= \langle L_\mu L^\mu \rangle \frac{\partial_\nu h \partial^\nu h}{v^2} F_{D7}(h) \\
\mathcal{O}_{D8} &= \langle L_\mu L_\nu \rangle \frac{\partial^\mu h \partial^\nu h}{v^2} F_{D8}(h) \\
\mathcal{O}_{D9} &= \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \frac{\partial_\nu h \partial^\nu h}{v^2} F_{D9}(h) \\
\mathcal{O}_{D10} &= \langle \tau_L L_\mu \rangle \langle \tau_L L_\nu \rangle \frac{\partial^\mu h \partial^\nu h}{v^2} F_{D10}(h)
\end{aligned} \tag{22}$$

$$\mathcal{O}_{D11} = \frac{(\partial_\mu h \partial^\mu h)^2}{v^4} F_{D11}(h) \tag{23}$$

The CP-odd operators are

$$\begin{aligned}
\mathcal{O}_{D12} &= \langle L_\mu L^\mu \rangle \langle \tau_L L_\nu \rangle \frac{\partial^\nu h}{v} F_{D12}(h) \\
\mathcal{O}_{D13} &= \langle L_\mu L_\nu \rangle \langle \tau_L L^\mu \rangle \frac{\partial^\nu h}{v} F_{D13}(h) \\
\mathcal{O}_{D14} &= \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle \langle \tau_L L_\nu \rangle \frac{\partial^\nu h}{v} F_{D14}(h) \\
\mathcal{O}_{D15} &= \langle \tau_L L_\mu \rangle \frac{\partial^\mu h \partial_\nu h \partial^\nu h}{v^3} F_{D15}(h)
\end{aligned} \tag{24}$$

We have defined

$$F_{Di}(h) \equiv 1 + \sum_{n=1}^{\infty} f_{Di,n}\left(\frac{h}{v}\right)^n \tag{25}$$

The four subclasses in (20), (21), (22) and (23) correspond, respectively, to terms with zero, one, two and four derivatives acting on h. The subclass of CP-odd operators has terms with one derivative acting on h and contains the only operator with three derivatives on h. Note that all operators are written with only single derivatives on either U or h fields. In the absence of the field h the basis reduces to the five operators in (20) with $F_{Di} = 1$, known from [13].

If custodial symmetry is respected by the UhD^4 terms, the basis reduces to the five operators $\mathcal{O}_{Di}$ with $i = 1, 2, 7, 8$ and 11, all of which are CP-even. The custodial-symmetry violating UhD^4 operators are not generated as one-loop counterterms if the leading-order Goldstone–Higgs sector is custodial symmetric. They might still appear as finite contributions at NLO.

4.2. X^2Uh and $XUhD^2$ terms

The CP-even operators are

$$\begin{aligned}
\mathcal{O}_{Xh1} &= g'^2 B_{\mu\nu} B^{\mu\nu} F_{Xh1}(h) \\
\mathcal{O}_{Xh2} &= g^2 \langle W_{\mu\nu} W^{\mu\nu}\rangle F_{Xh2}(h) \\
\mathcal{O}_{Xh3} &= g_s^2 \langle G_{\mu\nu} G^{\mu\nu}\rangle F_{Xh3}(h)
\end{aligned} \tag{26}$$

$$\begin{aligned}
\mathcal{O}_{XU1} &= g' g B_{\mu\nu} \langle W^{\mu\nu} \tau_L\rangle \big(1 + F_{XU1}(h)\big) \\
\mathcal{O}_{XU2} &= g^2 \langle W_{\mu\nu} \tau_L\rangle^2 \big(1 + F_{XU2}(h)\big) \\
\mathcal{O}_{XU3} &= g \varepsilon_{\mu\nu\lambda\rho} \langle W^{\mu\nu} L^\lambda\rangle \langle \tau_L L^\rho\rangle \big(1 + F_{XU3}(h)\big) \\
\mathcal{O}_{XU7} &= i g' B_{\mu\nu} \langle \tau_L [L^\mu, L^\nu]\rangle F_{XU7}(h) \\
\mathcal{O}_{XU8} &= i g \langle W_{\mu\nu} [L^\mu, L^\nu]\rangle F_{XU8}(h) \\
\mathcal{O}_{XU9} &= i g \langle W_{\mu\nu} \tau_L\rangle \langle \tau_L [L^\mu, L^\nu]\rangle F_{XU9}(h)
\end{aligned} \tag{27}$$

In correspondence to (26) and (27) there are also nine CP-odd operators:

$$\begin{aligned}
\mathcal{O}_{Xh4} &= g'^2 \varepsilon_{\mu\nu\lambda\rho} B^{\mu\nu} B^{\lambda\rho} F_{Xh4}(h) \\
\mathcal{O}_{Xh5} &= g^2 \varepsilon_{\mu\nu\lambda\rho} \langle W^{\mu\nu} W^{\lambda\rho}\rangle F_{Xh5}(h) \\
\mathcal{O}_{Xh6} &= g_s^2 \varepsilon_{\mu\nu\lambda\rho} \langle G^{\mu\nu} G^{\lambda\rho}\rangle F_{Xh6}(h)
\end{aligned} \tag{28}$$

$$\begin{aligned}
\mathcal{O}_{XU4} &= g' g \varepsilon_{\mu\nu\lambda\rho} \langle \tau_L W^{\mu\nu}\rangle B^{\lambda\rho} \big(1 + F_{XU4}(h)\big) \\
\mathcal{O}_{XU5} &= g^2 \varepsilon_{\mu\nu\lambda\rho} \langle \tau_L W^{\mu\nu}\rangle \langle \tau_L W^{\lambda\rho}\rangle \big(1 + F_{XU5}(h)\big) \\
\mathcal{O}_{XU6} &= g \langle W_{\mu\nu} L^\mu\rangle \langle \tau_L L^\nu\rangle \big(1 + F_{XU6}(h)\big) \\
\mathcal{O}_{XU10} &= i g' \varepsilon_{\mu\nu\lambda\rho} B^{\mu\nu} \langle \tau_L [L^\lambda, L^\rho]\rangle F_{XU10}(h) \\
\mathcal{O}_{XU11} &= i g \varepsilon_{\mu\nu\lambda\rho} \langle W^{\mu\nu} [L^\lambda, L^\rho]\rangle F_{XU11}(h) \\
\mathcal{O}_{XU12} &= i g \varepsilon_{\mu\nu\lambda\rho} \langle W^{\mu\nu} \tau_L\rangle \langle \tau_L [L^\lambda, L^\rho]\rangle F_{XU12}(h)
\end{aligned} \tag{29}$$

Here

$$F_{Xi}(h) = \sum_{n=1}^{\infty} f_{Xi,n} \left(\frac{h}{v}\right)^n \tag{30}$$

The terms $\mathcal{O}_{XUi}$, $i = 1, \dots, 6$, remain independent operators in the limit $h \to 0$, while all other operators become redundant. For this reason the former operators are multiplied by $(1 + F_{Xi}(h))$. Omitting the functions F_{Xi}, the operators $\mathcal{O}_{XUi}$ reduce to those listed already in [13,14].

4.3. $\psi^2 UhD$ terms

The operators in this class are given by

$$\begin{aligned}
&\mathcal{O}_{\psi V1} = -\bar{q}\gamma^\mu q \langle \tau_L L_\mu \rangle F_{\psi V1}(h) \\
&\mathcal{O}_{\psi V2} = -\bar{q}\gamma^\mu \tau_L q \langle \tau_L L_\mu \rangle F_{\psi V2}(h) \\
&\mathcal{O}_{\psi V3} = -\bar{q}\gamma^\mu U P_{12} U^\dagger q \langle L_\mu U P_{21} U^\dagger \rangle F_{\psi V3}(h) \\
&\mathcal{O}^\dagger_{\psi V3} \\
&\mathcal{O}_{\psi V4} = -\bar{u}\gamma^\mu u \langle \tau_L L_\mu \rangle F_{\psi V4}(h) \\
&\mathcal{O}_{\psi V5} = -\bar{d}\gamma^\mu d \langle \tau_L L_\mu \rangle F_{\psi V5}(h) \\
&\mathcal{O}_{\psi V6} = -\bar{u}\gamma^\mu d \langle L_\mu U P_{21} U^\dagger \rangle F_{\psi V6}(h) \\
&\mathcal{O}^\dagger_{\psi V6} \\
&\mathcal{O}_{\psi V7} = -\bar{l}\gamma^\mu l \langle \tau_L L_\mu \rangle F_{\psi V7}(h) \\
&\mathcal{O}_{\psi V8} = -\bar{l}\gamma^\mu \tau_L l \langle \tau_L L_\mu \rangle F_{\psi V8}(h), \\
&\mathcal{O}_{\psi V9} = -\bar{l}\gamma^\mu U P_{12} U^\dagger l \langle L_\mu U P_{21} U^\dagger \rangle F_{\psi V9}(h) \\
&\mathcal{O}^\dagger_{\psi V9} \\
&\mathcal{O}_{\psi V10} = -\bar{e}\gamma^\mu e \langle \tau_L L_\mu \rangle F_{\psi V10}(h)
\end{aligned} \tag{31}$$

where

$$F_{\psi Vi}(h) = 1 + \sum_{n=1}^{\infty} f_{\psi Vi,n} \left(\frac{h}{v}\right)^n \tag{32}$$

They generalize the terms first listed for the case without h in [15]. The minus signs on the r.h.s. of (31) have been introduced to be consistent with the notation of [12] in the limit $F_{\psi Vi}(h) \to 1$. In the sector with left-handed quarks q, the four operators $\mathcal{O}_{\psi V1}$, $\mathcal{O}_{\psi V2}$, $\mathcal{O}_{\psi V3}$ and $\mathcal{O}^\dagger_{\psi V3}$ are equivalent to the four terms $\bar{q}\gamma^\mu q \langle \tau_L L_\mu \rangle F$, $\bar{q}\gamma^\mu \tau_L q \langle \tau_L L_\mu \rangle F$, $\bar{q}\gamma^\mu L_\mu q F$ and $\bar{q}\gamma^\mu i[\tau_L, L_\mu] q F$, obtained as the independent structures formed directly with the building blocks τ_L, L_μ and a (generic) $F(h)$. We prefer to work with $\mathcal{O}_{\psi V3}$ and $\mathcal{O}^\dagger_{\psi V3}$ in (31) since in unitary gauge these operators simply correspond to charged-current interactions with $W^\pm$. Taking into account the remaining building block $\partial_\mu h$, two further operators may be written down, $\bar{q}\gamma^\mu q \partial_\mu h F$ and $\bar{q}\gamma^\mu \tau_L q \partial_\mu h F$. These are seen to be redundant upon integrating by parts, and using the fermion equations of motion and the identity in (15). Similar comments apply to the operators with right-handed quarks and with leptons. The operators in class $\psi^2 UhD$ are therefore identical to those in class $\psi^2 UD$ of [12], up to overall factors of $F(h)$.

4.4. $\psi^2 UhD^2$ and $\psi^2 UhX$ terms

The class $\psi^2 UhD^2$ contains fermion bilinears with Lorentz-scalar or tensor structure. The scalar operators are (hermitean conjugate versions will not be listed separately in this section)

$$\begin{aligned}
\mathcal{O}_{\psi S1} &= \bar{q} U P_+ r \langle L_\mu L^\mu \rangle F_{\psi S1} \\
\mathcal{O}_{\psi S2} &= \bar{q} U P_- r \langle L_\mu L^\mu \rangle F_{\psi S2} \\
\mathcal{O}_{\psi S3} &= \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle F_{\psi S3} \\
\mathcal{O}_{\psi S4} &= \bar{q} U P_- r \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle F_{\psi S4} \\
\mathcal{O}_{\psi S5} &= \bar{q} U P_{12} r \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle F_{\psi S5} \\
\mathcal{O}_{\psi S6} &= \bar{q} U P_{21} r \langle \tau_L L_\mu \rangle \langle U P_{12} U^\dagger L^\mu \rangle F_{\psi S6} \\
\mathcal{O}_{\psi S7} &= \bar{l} U P_- \eta \langle L_\mu L^\mu \rangle F_{\psi S7} \\
\mathcal{O}_{\psi S8} &= \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \langle \tau_L L^\mu \rangle F_{\psi S8} \\
\mathcal{O}_{\psi S9} &= \bar{l} U P_{12} \eta \langle \tau_L L_\mu \rangle \langle U P_{21} U^\dagger L^\mu \rangle F_{\psi S9} \\
\mathcal{O}_{\psi S10} &= \bar{q} U P_+ r \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S10} \\
\mathcal{O}_{\psi S11} &= \bar{q} U P_- r \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S11} \\
\mathcal{O}_{\psi S12} &= \bar{q} U P_{12} r \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S12} \\
\mathcal{O}_{\psi S13} &= \bar{q} U P_{21} r \langle U P_{12} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S13} \\
\mathcal{O}_{\psi S14} &= \bar{q} U P_+ r \left(\partial_\mu \frac{h}{v} \right) \left(\partial^\mu \frac{h}{v} \right) F_{\psi S14} \\
\mathcal{O}_{\psi S15} &= \bar{q} U P_- r \left(\partial_\mu \frac{h}{v} \right) \left(\partial^\mu \frac{h}{v} \right) F_{\psi S15} \\
\mathcal{O}_{\psi S16} &= \bar{l} U P_- \eta \langle \tau_L L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S16} \\
\mathcal{O}_{\psi S17} &= \bar{l} U P_{12} \eta \langle U P_{21} U^\dagger L_\mu \rangle \left(\partial^\mu \frac{h}{v} \right) F_{\psi S17} \\
\mathcal{O}_{\psi S18} &= \bar{l} U P_- \eta \left(\partial_\mu \frac{h}{v} \right) \left(\partial^\mu \frac{h}{v} \right) F_{\psi S18}
\end{aligned} \tag{33}$$

The list of operators with a tensor current is

$$\begin{aligned}
\mathcal{O}_{\psi T1} &= \bar{q} \sigma_{\mu\nu} U P_+ r \langle \tau_L L_\mu L_\nu \rangle F_{\psi T1} \\
\mathcal{O}_{\psi T2} &= \bar{q} \sigma_{\mu\nu} U P_- r \langle \tau_L L_\mu L_\nu \rangle F_{\psi T2} \\
\mathcal{O}_{\psi T3} &= \bar{q} \sigma_{\mu\nu} U P_{12} r \langle \tau_L L^\mu \rangle \langle U P_{21} U^\dagger L^\nu \rangle F_{\psi T3} \\
\mathcal{O}_{\psi T4} &= \bar{q} \sigma_{\mu\nu} U P_{21} r \langle \tau_L L^\mu \rangle \langle U P_{12} U^\dagger L^\nu \rangle F_{\psi T4} \\
\mathcal{O}_{\psi T5} &= \bar{l} \sigma_{\mu\nu} U P_{12} \eta \langle \tau_L L^\mu \rangle \langle U P_{21} U^\dagger L^\nu \rangle F_{\psi T5}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\psi T6} &= \bar{l}\sigma_{\mu\nu} U P_- \eta \langle \tau_L L_\mu L_\nu \rangle F_{\psi T6} \\
\mathcal{O}_{\psi T7} &= \bar{q}\sigma_{\mu\nu} U P_+ r \langle \tau_L L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T7} \\
\mathcal{O}_{\psi T8} &= \bar{q}\sigma_{\mu\nu} U P_- r \langle \tau_L L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T8} \\
\mathcal{O}_{\psi T9} &= \bar{q}\sigma_{\mu\nu} U P_{21} r \langle U P_{12} U^\dagger L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T9} \\
\mathcal{O}_{\psi T10} &= \bar{q}\sigma_{\mu\nu} U P_{12} r \langle U P_{21} U^\dagger L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T10} \\
\mathcal{O}_{\psi T11} &= \bar{l}\sigma_{\mu\nu} U P_- \eta \langle \tau_L L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T11} \\
\mathcal{O}_{\psi T12} &= \bar{l}\sigma_{\mu\nu} U P_{12} \eta \langle U P_{21} U^\dagger L^\mu \rangle \left(\partial^\nu \frac{h}{v} \right) F_{\psi T12}
\end{aligned} \tag{34}$$

Here we have used

$$F_{\psi S(T)i} \equiv F_{\psi S(T)i}(h) = 1 + \sum_{n=1}^{\infty} f_{\psi S(T)i,n} \left(\frac{h}{v} \right)^n \tag{35}$$

For completeness, we also quote the terms of the form $\psi^2 U h X$:

$$\begin{aligned}
\mathcal{O}_{\psi X1} &= \bar{q}\sigma_{\mu\nu} U P_+ r B^{\mu\nu} F_{\psi X1} \\
\mathcal{O}_{\psi X2} &= \bar{q}\sigma_{\mu\nu} U P_- r B^{\mu\nu} F_{\psi X2} \\
\mathcal{O}_{\psi X3} &= \bar{q}\sigma_{\mu\nu} U P_+ r \langle \tau_L W^{\mu\nu} \rangle F_{\psi X3} \\
\mathcal{O}_{\psi X4} &= \bar{q}\sigma_{\mu\nu} U P_- r \langle \tau_L W^{\mu\nu} \rangle F_{\psi X4} \\
\mathcal{O}_{\psi X5} &= \bar{q}\sigma_{\mu\nu} U P_{12} r \langle U P_{21} U^\dagger W^{\mu\nu} \rangle F_{\psi X5} \\
\mathcal{O}_{\psi X6} &= \bar{q}\sigma_{\mu\nu} U P_{21} r \langle U P_{12} U^\dagger W^{\mu\nu} \rangle F_{\psi X6} \\
\mathcal{O}_{\psi X7} &= \bar{q}\sigma_{\mu\nu} G^{\mu\nu} U P_+ r F_{\psi X7} \\
\mathcal{O}_{\psi X8} &= \bar{q}\sigma_{\mu\nu} G^{\mu\nu} U P_- r F_{\psi X8} \\
\mathcal{O}_{\psi X9} &= \bar{l}\sigma_{\mu\nu} U P_- \eta B^{\mu\nu} F_{\psi X9} \\
\mathcal{O}_{\psi X10} &= \bar{l}\sigma_{\mu\nu} U P_- \eta \langle \tau_L W^{\mu\nu} \rangle F_{\psi X10} \\
\mathcal{O}_{\psi X11} &= \bar{l}\sigma_{\mu\nu} U P_{12} \eta \langle U P_{21} U^\dagger W^{\mu\nu} \rangle F_{\psi X11}
\end{aligned} \tag{36}$$

where

$$F_{\psi Xi} \equiv F_{\psi Xi}(h) = 1 + \sum_{n=1}^{\infty} f_{\psi Xi,n} \left(\frac{h}{v} \right)^n \tag{37}$$

The operators $\psi^2 U h X$ are not required as NLO counterterms, since the one-loop diagrams inducing these structures in the effective theory are finite. These operators are expected to contribute at NNLO. Also the tensor operators in (34) are not generated as one-loop counterterms. The genuine counterterms in the class $\psi^2 U h D^2$ are then those with the scalar fermion currents given in (33).

4.5. $\psi^4 Uh$ terms

The 4-fermion operators of the class $\psi^4 U$ have been listed in [12]. Since no derivatives are involved, the generalization to the case including the h field simply amounts to a multiplication of each of these operators with a general function

$$F_{4\psi i}(h) = 1 + \sum_{n=1}^{\infty} f_{4\psi i,n}\left(\frac{h}{v}\right)^n \tag{38}$$

The operators in the class $\psi^4 Uh$ are then given by

$$\mathcal{O}_{4\psi Uh,i} = \mathcal{O}_{4\psi U,i} F_{4\psi i}(h) \tag{39}$$

where $\mathcal{O}_{4\psi U,i}$ are the 4-fermion operators listed in Section 4.5 of [12].

Not all of these operators need actually appear as counterterms at one loop. While for instance operators of the form $\bar{\psi}_L U \psi_R \bar{\psi}_L U \psi_R F(h)$ are required as counterterms, the operators $\bar{\psi}_L \gamma^\mu \psi_L \bar{\psi}_L \gamma_\mu \psi_L F(h)$ are not. Still the latter could arise as finite contributions at NLO through the tree-level exchange of TeV-scale resonances.

4.6. $X^3 Uh$ terms

The operators X^3, built from 3 factors of field-strength tensors, are not required as counterterms at next-to-leading order. There are only four operators of this type [16,17]

$$\mathcal{O}_{X1} = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}, \qquad \mathcal{O}_{X2} = f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu} \tag{40}$$

$$\mathcal{O}_{X3} = \varepsilon^{abc} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}, \qquad \mathcal{O}_{X4} = \varepsilon^{abc} \tilde{W}_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu} \tag{41}$$

where f^{ABC} and ε^{abc} are the structure constants of color $SU(3)$ and weak $SU(2)$, respectively. They are dimension-6 operators and therefore suppressed by two powers of the heavy mass scale Λ. A loop suppression brings the coefficients further down to the NNLO level $\mathcal{O}(v^4/\Lambda^4)$ [14,18,19] (see [12] for additional comments). Similar arguments hold for the entire class of terms $X^3 Uh$, that is including Goldstone and Higgs fields, which we do not consider further here.

5. Partial compositeness and the linear realization

The power-counting formula we have derived and applied in the preceding sections assumed that the scale of electroweak symmetry breaking $4\pi v$ and the cut-off scale $4\pi f$ were of comparable size. This situation includes nondecoupling scenarios, where there is only one relevant scale v and the composite Higgs plays the role of a pseudo-Goldstone boson. In these scenarios, the full unitarization of amplitudes (e.g. in $W_L W_L$ scattering) is taken care of by states at the TeV scale. On the opposite end, $v/f \to 0$, we have the Standard Model Higgs, which alone unitarizes the physical amplitudes due to the renormalizability of its interactions. Between these two pictures, there is a continuum of possibilities where heavy resonances and a light Higgs together render the theory unitary. In order to cover the transition between the pure nondecoupling case (TeV-scale new states) and the Standard-Model scenario (infinitely heavy new states), the scales f and v should be distinguished. Theories with vacuum misalignment [20,21], for instance, are examples of how this splitting of scales can be dynamically realized. The vacuum-tilting parameter

$$\xi = \frac{v^2}{f^2} \tag{42}$$

therefore gauges the degree of h-compositeness or, equivalently, the degree of decoupling of the theory: $\xi = 1$ corresponds to purely nondecoupling scenarios, while $\xi \to 0$ is the decoupling limit, i.e. the Standard-Model case.

The relation between the two limits, $\xi = 1$ and $\xi \ll 1$, can be made more explicit. Since for $\xi \to 0$ the theory reduces to the renormalizable Standard Model, with a linearly transforming Higgs doublet ϕ, the effective Lagrangian can be organized for small ξ in terms of operators of increasing canonical dimension d. The coefficients of these operators then scale as $\xi^{(d-4)/2}$.[1] This corresponds to the usual framework, of which the terms up to $d = 6$ have been classified in [16,17]. The restriction $\xi \ll 1$ may be relaxed by considering ξ as a quantity of $\mathcal{O}(1)$. Then the effective theory in powers of ξ has to be reorganized in terms of the chiral Lagrangian. This effectively resums the series in ξ, replacing it by a loop expansion. As a consequence of the reorganization there is no one-to-one correspondence between the terms classified as NLO in the two scenarios, $\xi = \mathcal{O}(1)$ and $\xi \ll 1$. It also implies that (for most operator classes) the chiral Lagrangian formulation is more general than the effective theory based on canonical dimension, as explained in more detail below.

We may rewrite the dimension-6 operators from [16], whose coefficients count as $\mathcal{O}(\xi)$, in polar coordinates for the Higgs field, using

$$\phi = (v+h)U \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \tilde{\phi} = (v+h)U \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{43}$$

The resulting terms can be matched to some of the operators in the chiral Lagrangian. The coefficients of those operators are then seen to start at $\mathcal{O}(\xi)$ in the small-ξ limit. Higher powers of ξ are always present in the expansion of these coefficients. This is because additional factors of $\phi^\dagger \phi = (v+h)^2$, multiplying a given operator, lead to higher-dimensional operators that map onto the same operator in the chiral Lagrangian. Operators in the chiral Lagrangian that cannot be obtained from the dimension-6 basis of [16] derive from operators of dimension $d > 6$. Their coefficients then count as $\mathcal{O}(\xi^{(d-4)/2})$ in the small-ξ expansion.

We illustrate this for the dimension-6 operators in the class $\psi^2\phi^2 D$ of [16]. They have the form

$$\bar{q}\gamma^\mu q \phi^\dagger i \overleftrightarrow{D}_\mu \phi = 2(v+h)^2 \bar{q}\gamma^\mu q \langle \tau_L L_\mu \rangle \tag{44}$$

$$\bar{q}\gamma^\mu T^a q \phi^\dagger i \overleftrightarrow{D}_\mu T^a \phi = -\frac{1}{2}(v+h)^2 \bar{q}\gamma^\mu L_\mu q \tag{45}$$

$$\bar{u}\gamma^\mu d \tilde{\phi}^\dagger i D_\mu \phi = -(v+h)^2 \bar{u}\gamma^\mu d \langle L_\mu U P_{21} U^\dagger \rangle \tag{46}$$

with similar relations for the remaining operators. Recalling that

$$-\bar{q}\gamma^\mu L_\mu q F(h) = \mathcal{O}_{\psi V3} + \mathcal{O}^\dagger_{\psi V3} + 2\mathcal{O}_{\psi V2} \tag{47}$$

we find that all operators in (31) are generated. Their coefficients thus count as $\mathcal{O}(\xi)$. If we had used $\bar{q}\gamma^\mu L_\mu q F$ as a basis element instead of, say, $\mathcal{O}_{\psi V2}$, the operator $\mathcal{O}_{\psi V3}$ would not be generated with an $\mathcal{O}(\xi)$ coefficient, but could only arise at $\mathcal{O}(\xi^2)$. This shows that the order in ξ of the coefficients in the chiral Lagrangian is in general basis dependent.

[1] Further small factors such as couplings or powers of $1/4\pi$, arising e.g. from resonance masses $M_R \sim 4\pi f$, will be ignored in the present context. The resulting suppression of particular coefficients can be separately addressed.

Table 1
Correspondence between classes of NLO operators in the loop expansion of the chiral Lagrangian (present work, first row) and the $1/f$ expansion of the effective Lagrangian based on canonical dimension ([16,17], second row).

$\mathcal{L}_\chi$:	LO	LO	X^2Uh $XUhD^2$	ψ^2UhD	ψ^4Uh	UhD^4	ψ^2UhD^2	NNLO	NNLO
$\mathcal{L}_{\text{BW}}$:	ϕ^6 ϕ^4D^2	$\psi^2\phi^3$	$X^2\phi^2$	$\psi^2\phi^2D$	ψ^4	NNLO	NNLO	X^3	$\psi^2X\phi$

Mapping the entire dimension-6 basis of [16] onto the chiral Lagrangian, leads to the following list of chiral operators with $\mathcal{O}(\xi)$ coefficients:

$$X^2Uh, XUhD^2: \quad \mathcal{O}_{Xhi}, \quad i=1,\dots,6; \quad \mathcal{O}_{XU1}, \mathcal{O}_{XU4} \tag{48}$$

$$\psi^2UhD: \quad \mathcal{O}_{\psi Vi}, \quad i=1,\dots,10 \tag{49}$$

$$\psi^4Uh: \quad \text{all 4-fermion operators without } U\text{-fields} \tag{50}$$

In these classes the chiral basis is more general than its dimension-6 counterpart: Not all chiral operators are generated from the dimension-6 basis, only terms up to second order in h appear, and some of the coefficients are correlated. The operators of classes ϕ^6, ϕ^4D^2 and $\psi^2\phi^3$ in [16] contribute $\mathcal{O}(\xi)$ corrections to leading-order terms in the chiral Lagrangian.[2] The operators X^3 and $\psi^2X\phi$ have $\mathcal{O}(\xi)$ coefficients, but appear only at NNLO.

The remaining NLO operators in our basis for the chiral Lagrangian have coefficients of higher order in ξ. For a complete classification of the various orders in ξ, the lists of higher-dimensional operators in the Standard Model would have to be worked out systematically beyond the dimension-6 level. Since such lists are not yet available, we will content ourselves with commenting on a few typical cases. An important example is given by the terms of class UhD^4 in Section 4.1. The lowest-dimension, nonredundant operators that can generate them are operators in the pure-Higgs sector with four derivatives. The three independent terms in this class are the dimension-8 operators

$$D_\mu\phi^\dagger D^\mu\phi D_\nu\phi^\dagger D^\nu\phi, \quad D_\mu\phi^\dagger D_\nu\phi D^\mu\phi^\dagger D^\nu\phi, \quad D_\mu\phi^\dagger D_\nu\phi D^\nu\phi^\dagger D^\mu\phi \tag{51}$$

Rewriting those in polar coordinates using (43), one finds that all CP-even operators $\mathcal{O}_{Di}$, $i = 1,\dots,11$, are generated with the exception of $\mathcal{O}_{D3}$. We conclude that these 10 operators have coefficients starting at $\mathcal{O}(\xi^2)$.

Another example is given by the 4-fermion operators ψ^4Uh that explicitly include U fields, such as terms of the form $\bar\psi_L U\psi_R\bar\psi_L U\psi_R F(h)$. This term can only come from a dimension-8 operator and thus also counts as $\mathcal{O}(\xi^2)$.

The comparison between the chiral Lagrangian discussed in this work and the usual expansion in terms of canonical dimension, with a linearly transforming Higgs doublet, is summarized in Table 1.

A special case of the small ξ limit is the so-called Strongly-Interacting Light Higgs (SILH) model [22], which considers a scenario where a composite scalar doublet ϕ gets nonstandard interactions, driven by a subset of $d = 6$ operators. With the identification in (43), the SILH Lagrangian can be rewritten in terms of the U and h fields and shown to correspond to a specific choice of the EFT coefficients. This exercise shows that:

[2] One finds a direct correspondence between operators with the exception of the operator $(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$, which in the chiral Lagrangian can be reabsorbed in terms of leading order coefficients (see Appendix A).

- All the bosonic CP-even operators of Sections 4.1 and 4.2, to linear order in ξ, are present in the SILH Lagrangian with independent coefficients.
- Some of the SILH operators renormalize terms in the leading-order Lagrangian (1).
- SILH does not contain any explicit fermionic operator but includes the combinations $D^\mu W_{\mu\nu}$ and $\partial^\mu B_{\mu\nu}$, which can be reduced to fermionic operators by a straightforward application of the equations of motion for the gauge fields. This hypothesis of universality imposes strong constraints on the fermionic operators. In particular, the model does not contain NLO operators with tensor and scalar fermion bilinears, and only two independent combinations of fermionic vector currents are generated, namely $\sum_f Y_f \mathcal{O}_{\Psi Vf}$ and $2\mathcal{O}_{\Psi V2,8} + \mathcal{O}_{\Psi V3,9} + \mathcal{O}^\dagger_{\Psi V3,9}$. In turn, the four-fermion sector is constrained to three independent combinations of operators, coming from operators like $D^\mu W_{\mu\nu} D_\lambda W^{\lambda\nu}$ after using the equations of motion.
- Two operators of the class X^3 are considered, which strictly speaking should be counted as next-to-next-to-leading order $(1/16\pi^2)v^2/\Lambda^2$.

6. Comparison with previous literature

Traditional effective field theory descriptions of EWSB with underlying strong dynamics have focused mainly on higgsless scenarios [13,23,24]. While the idea of the Higgs as a composite pseudo-Goldstone, resulting from spontaneous breaking of either internal [20] or space–time symmetries [25] was proposed much earlier, only recently these ideas have been cast in the language of EFTs. In most of the cases, effective operators have been constructed according to phenomenological needs, without aiming at completeness.

To the best of our knowledge, the closest to a systematic classification of operators was done in [8,26], where the bosonic CP-even sector and fermion bilinear operators were explored under certain restrictions. In the following we list the main differences between [8,26] and the present paper:

- The Higgs self-interacting operator $\mathcal{O}_{D11}$ in (23) is not discussed in [8]. The CP-odd bosonic operators have been omitted there, based on the assumption of CP invariance in the bosonic sector. Regarding the fermionic terms, Lorentz-vector bilinear operators in [26] are built only from left-handed quark fields. If leptons and the right-handed fermions are also included, the basis gets enlarged from the 4 terms they consider to 13. For the scalar and tensor bilinear sector, operators with derivatives on h are not included. If one considers leptons and quarks, one finds 12 and 18 operators, respectively, instead of the 4 and 6 listed in [26]. Finally, a discussion of four-fermion operators is absent.
- Comparing our basis to the set of 24 bosonic operators $\mathcal{P}_i$ in [8], we note that the 8 operators $\mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_{11}, \mathcal{P}_{12}, \mathcal{P}_{13}, \mathcal{P}_{14}, \mathcal{P}_{16}$ and $\mathcal{P}_{17}$ are redundant in the sense that they can be expressed as fermionic bilinear operators using the equations of motion for the gauge and U fields. From the independent 16 operators in [8], the operators $\mathcal{P}_2$, $\mathcal{P}_3$, $\mathcal{P}_9$ are redundant in the absence of h [12,27–29]. Therefore, they only appear with at least one power of h.
- The assignment of powers of ξ to the various operators given in [8] is not in agreement with the discussion presented in Section 5.

On a more general level, the major difference of [8,26] to our approach is that we rely on a consistent power-counting. This is not a mere technicality, but rather a fundamental issue in order to be able to organize the EFT expansion. In particular, without a power-counting one cannot even define a leading-order Lagrangian, let alone next-to-leading order corrections. One criticism one can raise against [8] is that they seem to use a naive dimensional power-counting, which is known to fail for strongly-coupled expansions. In particular, kinetic and mass terms for the gauge fields would have different power-counting dimensions, which is clearly inconsistent: both terms should instead be homogeneous and stand at the same order in the EFT expansion.

7. Models of UV physics

In this section we briefly discuss the SM effective Lagrangian as a low-energy approximation of two simple models of physics at higher energies. In the first part, we consider the MCHM5 model [30–32] and show how the generic function $F_U(h)$ in (4) emerges in this case. In the second part we take a closer look at a specific UV-completion, based on the Higgs portal, and illustrate which operators of our NLO basis are generated.

7.1. MCHM5

In the MCHM5, the four real Goldstone bosons h_a are described by the vector parametrizing the coset $SO(5)/SO(4)$

$$\vec{\Sigma} = \frac{\sin\frac{|h|}{f}}{|h|}\left(h_1, h_2, h_3, h_4, |h|\cot\frac{|h|}{f}\right)^T \tag{52}$$

where $|h| = \sqrt{\sum_{a=1}^{4}(h_a)^2}$. For the transition from the real 4-component vector $\vec{h}$ to the matrix U, we define

$$\big(\langle h\rangle + h\big)U = i\sum_{a=1}^{3} h_a\sigma^a - h_4\mathbf{1} = \begin{pmatrix} -h_4 + ih_3 & h_2 + ih_1 \\ -h_2 + ih_1 & -h_4 - ih_3 \end{pmatrix} \tag{53}$$

where $(i\sigma^a, -\mathbf{1})$ defines a basis of 2×2 matrices with the Pauli-matrices σ^a, such that the 4 components h_a are related to the 4 real components ϕ_a of the Higgs doublet, $\phi = (\phi_1 + i\phi_2, \phi_3 + i\phi_4)^T$ by an $SO(4)$ transformation. $\langle h\rangle$ is the vacuum expectation value of the scalar $|h| = \langle h\rangle + h$. An $SO(4)$ transformation that leaves $|h|$ invariant is then equivalent to an $SU(2)_L \times SU(2)_R$ transformation of the matrix U, defined in (7). After expanding (9) in terms of φ,

$$U = \cos\frac{|\varphi|}{v} + i\frac{\varphi^a\sigma^a}{|\varphi|}\sin\frac{|\varphi|}{v} \tag{54}$$

where $|\varphi| = \sqrt{(\varphi^1)^2 + (\varphi^2)^2 + (\varphi^3)^2}$, we find

$$h_a = \big(\langle h\rangle + h\big)\frac{\varphi^a}{|\varphi|}\sin\frac{|\varphi|}{v}, \quad a = 1,2,3, \quad \text{and} \quad h_4 = -\big(\langle h\rangle + h\big)\cos\frac{|\varphi|}{v} \tag{55}$$

Now we can write down the kinetic term of $\vec{\Sigma}$ in terms of U and h,

$$\frac{f^2}{2}D_\mu\vec{\Sigma}^T D^\mu\vec{\Sigma} = \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle \sin^2\left(\frac{\langle h\rangle + h}{f}\right) \tag{56}$$

By comparing this to (4) we can identify

$$\xi = \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} \tag{57}$$

The coefficients $f_{U,n}$ in (4), for $n > 0$, are given by

$$f_{U,n} = \frac{2}{n!} \begin{cases} (1-2\xi)(-4\xi)^{\frac{n}{2}-1}, & \text{for } n \text{ even} \\ \sqrt{1-\xi}(-4\xi)^{\frac{n-1}{2}}, & \text{for } n \text{ odd} \end{cases} \tag{58}$$

We see that each additional power of $(h/v)^2$ introduces a factor ξ. For $\xi \approx 1$ the odd powers of h/v are suppressed in $F_U(h)$.

Finally, as an example of a NLO operator we may consider the 4-derivative term $(D_\mu \vec{\Sigma}^T D^\mu \vec{\Sigma})^2$. From (56) we see that in our basis it corresponds to a combination of the operators $\mathcal{O}_{D1}$, $\mathcal{O}_{D7}$, $\mathcal{O}_{D11}$, listed in Section 4.1.

7.2. Higgs portal

As a specific model for a UV completion we consider the Higgs portal (see [33–36] and references therein). This model postulates the existence of a new, Standard-Model singlet scalar particle, which has allowed dimension-4 couplings to the Higgs field. This interaction modifies the scalar potential of Eq. (4) to

$$V = -\frac{\mu_s^2}{2}|\phi_s|^2 + \frac{\lambda_s}{4}|\phi_s|^4 - \frac{\mu_h^2}{2}|\phi_h|^2 + \frac{\lambda_h}{4}|\phi_h|^4 + \frac{\eta}{2}|\phi_s|^2|\phi_h|^2 \tag{59}$$

where ϕ_s refers to the standard scalar doublet and ϕ_h denotes the hidden scalar. Both of them acquire a vacuum expectation value, which can be written as

$$\frac{v_s}{\sqrt{2}} = \sqrt{\frac{\lambda_h \mu_s^2 - \eta \mu_h^2}{\lambda_s \lambda_h - \eta^2}}, \qquad \frac{v_h}{\sqrt{2}} = \sqrt{\frac{\lambda_s \mu_h^2 - \eta \mu_s^2}{\lambda_s \lambda_h - \eta^2}} \tag{60}$$

Expanding both scalars around their vacuum expectation value, i.e. $|\phi_i| = \frac{1}{\sqrt{2}}(v_i + h_i)$, leads to a potential of the form

$$V = \frac{\lambda_s v_s^2}{4} h_s^2 + \frac{\lambda_h v_h^2}{4} h_h^2 + \frac{\eta}{2} v_s v_h h_s h_h + \mathcal{O}(h_i^3) \tag{61}$$

The transformation

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} h_s \\ h_h \end{pmatrix} \tag{62}$$

diagonalizes the mass matrix. The rotation angle χ is defined as

$$\tan(2\chi) = \frac{2\eta v_s v_h}{\lambda_h v_h^2 - \lambda_s v_s^2} \tag{63}$$

The masses of the physical states H_1 and H_2 are given by

$$M_{1,2}^2 = \frac{1}{4}\left(\lambda_h v_h^2 + \lambda_s v_s^2\right) \mp \frac{\lambda_h v_h^2 - \lambda_s v_s^2}{4\cos(2\chi)} \tag{64}$$

The Lagrangian relevant for the two scalars then reads

$$\begin{aligned}\mathcal{L}_H = &\frac{1}{2}\partial_\mu H_1 \partial^\mu H_1 + \frac{1}{2}\partial_\mu H_2 \partial^\mu H_2 - V(H_1, H_2)\\ &+ \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\left(1 + \frac{2a_1}{v}H_1 + \frac{2a_2}{v}H_2 + \frac{b_1}{v^2}H_1^2 + \frac{b_{12}}{v^2}H_1 H_2 + \frac{b_2}{v^2}H_2^2\right)\\ &- v(\bar{q}Y_u U P_+ r + \bar{q}Y_d U P_- r + \bar{l}Y_e U P_- \eta + \text{h.c.})\left(1 + \frac{c_1}{v}H_1 + \frac{c_2}{v}H_2\right)\end{aligned} \tag{65}$$

where

$$\begin{aligned}V(H_1, H_2) = &\frac{1}{2}M_1^2 H_1^2 + \frac{1}{2}M_2^2 H_2^2 - \lambda_1 H_1^3 - \lambda_2 H_1^2 H_2 - \lambda_3 H_1 H_2^2 - \lambda_4 H_2^3\\ &- z_1 H_1^4 - z_2 H_1^3 H_2 - z_3 H_1^2 H_2^2 - z_4 H_1 H_2^3 - z_5 H_2^4\end{aligned} \tag{66}$$

The couplings λ_i and z_i depend on μ_s, μ_h, λ_s, λ_h and η. With the parameters of the Higgs-portal model

$$a_1 = \sqrt{b_1} = c_1 = \cos\chi, \qquad a_2 = \sqrt{b_2} = c_2 = \sin\chi, \qquad b_{12} = 2\sin\chi\cos\chi, \tag{67}$$

the theory is renormalizable and unitary. The scalar H_1 is now identified with the light scalar h that was found at the LHC. H_2 is assumed to be heavy such that it can be integrated out. This gives at leading order a special case of the Lagrangian (4), with

$$\begin{aligned}&V = \frac{1}{2}M_1^2 h^2 - \lambda_1 h^3 - z_1 h^4, \qquad F_U = \frac{2a_1}{v}h + \frac{b_1}{v^2}h^2\\ &\hat{Y}^{(1)}_{u,d,e} = c_1 \hat{Y}_{u,d,e}, \qquad \hat{Y}^{(n>1)}_{u,d,e} = 0\end{aligned} \tag{68}$$

Solving the tree-level equations of motion for H_2 yields the effective Lagrangian up to terms of $\mathcal{O}(1/M_2^4)$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{H_2=0} + \frac{A^2}{2M_2^2} + \mathcal{O}\left(\frac{1}{M_2^4}\right) \tag{69}$$

where

$$\begin{aligned}A = &\lambda_2 h^2 + z_2 h^3 + \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\left(\frac{2a_2}{v} + \frac{b_{12}}{v^2}h\right)\\ &- c_2(\bar{q}Y_u U P_+ r + \bar{q}Y_d U P_- r + \bar{l}Y_e U P_- \eta + \text{h.c.})\end{aligned} \tag{70}$$

Operators induced by the exchange of more than one heavy particle are suppressed by additional factors of $1/M_2^2$ and can be neglected. The effective Lagrangian $\mathcal{L}_{\text{eff}}$ contains operators that modify the leading-order Lagrangian (4) as well as a subset of the next-to-leading operators of Section 4. In particular, we have the pure Goldstone-h operator $\mathcal{O}_{D1}$ up to h^2, the fermion bilinears $\mathcal{O}_{\psi S1}$, $\mathcal{O}_{\psi S2}$ and $\mathcal{O}_{\psi S7}$ and their hermitean conjugates up to h^1, and 4-fermion operators coming from the square of the Yukawa terms without additional scalars. The 4-fermion operators that are generated are the same as those in the heavy-Higgs model discussed in [12]. They are

$$\begin{aligned}&\mathcal{O}_{FY1}, \quad \mathcal{O}_{FY3}, \quad \mathcal{O}_{FY5}, \quad \mathcal{O}_{FY7}, \quad \mathcal{O}_{FY9}, \quad \mathcal{O}_{FY10}, \quad \mathcal{O}_{ST5}, \quad \mathcal{O}_{ST9}\\ &\mathcal{O}_{LR1}, \quad \mathcal{O}_{LR3}, \quad \mathcal{O}_{LR8}, \quad \mathcal{O}_{LR9}, \quad \mathcal{O}_{LR10}, \quad \mathcal{O}_{LR12}, \quad \mathcal{O}_{LR17}, \quad \mathcal{O}_{LR18}\end{aligned} \tag{71}$$

and their hermitean conjugates.

This discussion shows explicitly how a subset of our NLO operators is generated in the Higgs-portal scenario. After integrating out the heavy scalar H_2 the theory is nonrenormalizable and our general effective Lagrangian applies. In particular, it is seen that operators of canonical dimension 4 ($\mathcal{O}_{D1}$), 5 ($\mathcal{O}_{\psi Si}$) and 6 (4-fermion terms) contribute at the same (next-to-leading) order $1/M_2^2$. This shows that the effective Lagrangian is not simply organized in terms of canonical dimension.

8. Conclusions

The main results of this paper can be summarized as follows:

- We formulate the most general effective field theory for the Standard Model at the electroweak scale v, which includes a light scalar boson h, singlet under the Standard-Model gauge group. The framework allows for the possibility of dynamical electroweak symmetry breaking and a composite nature of h.
- The leading-order Lagrangian is reviewed, emphasizing the assumptions behind its construction.
- The resulting effective theory is nonrenormalizable in general, with a cutoff at $\Lambda = 4\pi v$ or above. It takes the form of an electroweak chiral Lagrangian, generalized to include the singlet scalar h. A power-counting analysis is used to clarify the systematics of the effective theory beyond the leading order, which is based on a loop expansion, rather than on the canonical dimension of operators.
- The power-counting formula is used to identify the classes of operators that are required as one-loop counterterms. The full set of NLO operators is subsequently worked out.
- We discuss the relation between the chiral Lagrangian and the conventional effective theory with a linearly transforming Higgs, based on operators ordered by increasing canonical dimension. We show that the usual dimension-6 Standard-Model Lagrangian and the SILH framework can be obtained as special cases from our scenario.
- To illustrate some important features of our formulation, we briefly discuss two specific models within the context of the chiral Lagrangian, the composite Higgs model based on $SO(5)/SO(4)$, and a simple, UV complete model based on the Higgs portal mechanism.

The effective Lagrangian of the Standard Model we have constructed through next-to-leading order in the chiral expansion can be used to analyze, in a model-independent way, new-physics effects in processes at the TeV scale. Loop corrections can be systematically included. Of particular interest will be the detailed investigation of Higgs-boson properties, which should ultimately guide us to a deeper understanding of electroweak symmetry breaking.

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Appendix A. Leading-order effective Lagrangian

In this section we review the construction of the leading-order electroweak chiral Lagrangian of the Standard Model including a light Higgs singlet, $\mathcal{L}_{\rm LO}(h)$, as given in (1)–(5). This Lagrangian is nonrenormalizable in general. It defines the starting point for the systematic power counting on which the construction of the complete effective field theory is based. This construction determines in particular the next-to-leading order operators, which are the subject of the present work. Although the form of $\mathcal{L}_{\rm LO}(h)$ is known [10,32], it is worthwhile to discuss in detail the underlying assumptions. We will also emphasize a few features that allow for simplifications in the final form of $\mathcal{L}_{\rm LO}(h)$.

The effective Lagrangian is based on an expansion in powers of v^2/Λ^2, where $v = 246$ GeV is the electroweak scale and $\Lambda = 4\pi v$ the scale of dynamical electroweak symmetry breaking. To leading order the Lagrangian has to contain the unbroken, renormalizable part of the Standard Model (2). It consists of dimension-4 terms, which therefore scale as v^4, for processes at electroweak energies. Electroweak symmetry breaking is introduced to leading order by the Higgs sector Lagrangian $\mathcal{L}_{Uh}$ in (4). The Goldstone sector provides masses to the W and Z bosons through the U-field kinetic term $v^2\langle D_\mu U^\dagger D^\mu U\rangle$, and to the fermions through the Yukawa interactions $v\bar{\psi}_L U\psi_R$. Both scale as v^4, which identifies them as proper leading order terms, as it has to be the case. Note that the latter operators, and those in (2), have canonical dimension two, three and four, respectively. This already implies that dimension alone is not the criterion by which the operators in the effective Lagrangian are ordered.

We assume that the new strong dynamics respects the global custodial symmetry $U \to g_L U g_R^\dagger$, $g_{L(R)} \in SU(2)_{L(R)}$, to leading order. This singles out the term $v^2\langle D_\mu U^\dagger D^\mu U\rangle$ for the pure Goldstone-boson LO Lagrangian. The only further possible Goldstone term with two derivatives that respects the SM gauge symmetry

$$v^2\langle U^\dagger D_\mu U T_3\rangle^2 \tag{A.1}$$

breaks the custodial symmetry and will be treated as a next-to-leading order correction. This assumption is in line with the empirical fact that there are no $\mathcal{O}(1)$ corrections to the electroweak T-parameter, to which (A.1) contributes. Custodial symmetry is still violated at leading order by the Yukawa couplings and by weak hypercharge. These effects introduce violations of custodial symmetry through one-loop corrections, which also count as NLO terms.

We next include the Higgs singlet h, considered as a light (pseudo-Goldstone) particle of the strong dynamics. The field h is strongly coupled to the Goldstone sector. This introduces interactions with arbitrary powers h^k that multiply the Goldstone Lagrangian. Standard power counting (see e.g. [37] for a review) then implies

$$\mathcal{L} = \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\left(1 + \sum_{k=1}^{\infty} f_k\left(\frac{gh}{\Lambda}\right)^k\right) \tag{A.2}$$

where the canonical form of the Goldstone kinetic term fixes the overall normalization. In the present context g stands for the generic Higgs-sector coupling. Additional derivatives scale as $\partial/\Lambda \sim v/\Lambda$ and are of higher order. For strong coupling $g \approx 4\pi$ the new factor in (A.2) then becomes a general function $1 + F_U(h/v)$. Since h scales as v, higher powers are not suppressed. This is similar to the field $U = \exp(2i\varphi^a T^a/v)$ containing all powers of φ^a/v. However, since h is a singlet, the coefficients f_k are not further restricted. The infinite number of f_k reflects the composite nature of the Higgs, whose internal structure cannot be fully described by a finite number of terms. This limits the predictive power of the effective theory to some extent. Nevertheless, the theory still retains predictivity, since for processes with a given number of external h, and to a given loop order, only a finite number of terms in the Lagrangian contributes.

For the reasons just discussed, interactions with arbitrary powers of h/v are also included into the Yukawa terms in (4).

A kinetic term for h has to be added to the Lagrangian, which may be written as

$$\mathcal{L}_{h,kin} = \frac{1}{2}\partial_\mu h \partial^\mu h\big(1 + F_h(h/v)\big) \tag{A.3}$$

Interactions described by a general function $F_h(h/v)$ have been added to the pure kinetic term, following the same considerations that led to the function $F_U(h/v)$ above. The Lagrangian in (A.3) is the most general expression containing two derivatives and only h fields. It turns out, however, that the function F_h can be removed by the field redefinition

$$\tilde{h} = \int_0^h \sqrt{1 + F_h(s/v)}\, ds \tag{A.4}$$

which transforms (A.3) into

$$\mathcal{L}_{h,kin} = \frac{1}{2}\partial_\mu \tilde{h} \partial^\mu \tilde{h} \tag{A.5}$$

Dropping the tilde, the kinetic term for h takes the simple form used in (4).

There are two further terms with two derivatives that can be built from U and h fields. The first is the operator in (A.1) multiplied by a function $F(h)$, the second is $\langle U^\dagger D_\mu U T_3\rangle \partial^\mu F(h)$. Since they violate custodial symmetry in the sector built only from U and h fields, we do not include them in the leading-order Lagrangian. As a contribution at next-to-leading order the second term can be eliminated using the leading-order equations of motion, which are given below. The first term remains as an operator at NLO.

Lorentz invariant operators with U, h and just a single derivative cannot be formed. This leaves us to consider terms without derivatives, constructed from U and h fields. Since $\langle U^\dagger U\rangle$ is a constant, and no other invariants can be obtained from U alone, the zero-derivative contribution in the scalar sector reduces to the h-field potential $V(h)$. For the pseudo-Goldstone h this potential would be forbidden by shift symmetry, but it can be generated at the one-loop level (see [32] for a review). Standard power counting for strong coupling, but including an overall loop factor $1/16\pi^2$, then gives

$$V(h) = \frac{1}{16\pi^2}\frac{\Lambda^4}{g^2}\sum_k f_{V,k}\left(\frac{gh}{\Lambda}\right)^k = v^4 \sum_k f_{V,k}\left(\frac{h}{v}\right)^k \tag{A.6}$$

which again scales as a leading-order contribution. This implies in particular that the physical Higgs mass is light, of order v^2, rather than Λ^2, as it would be the case for a typical strong-sector resonance. We remark that a linear term ($k = 1$) in (A.6), which will arise for instance from tadpole diagrams, can always be eliminated by shifting the field h and renormalizing other fields and parameters (such as v). Accordingly, $n \geqslant 2$ has been adopted for $V(h)$ in Eq. (5) of the main text.

In principle one might consider the coupling of powers of h/v also to the fermionic terms in (2), expressed through a generic function $f(h)$ as

$$\mathcal{L}_\psi = \frac{i}{2}\bar{\psi}'\overleftrightarrow{\not{D}}\psi'\big(1 + f(h)\big)^{-2} \tag{A.7}$$

The fermionic term has to be written here in its manifestly hermitean form, since the h-dependent factor prevents one from performing the usual simplification via integration by parts. A field redefinition

$$\psi' = \psi\bigl(1 + f(h)\bigr) \tag{A.8}$$

brings (A.7) back to its conventional form $\mathcal{L}_\psi = \bar\psi i\not{D}\psi$, up to a total derivative. This would redefine the Yukawa couplings $\hat{Y}^{(n)}$, but would leave the structure of (4) unchanged. The h-dependent prefactors in (A.7) can therefore be omitted.

Finally, the possibility remains to dress the gauge-field terms by Higgs-dependent functions, as in

$$\langle X_{\mu\nu} X^{\mu\nu}\rangle F_X(h) \tag{A.9}$$

with $X_{\mu\nu}$ a field-strength tensor and $F_X(0) = 0$. We assume that the gauge field strengths are not strongly coupled to the Higgs sector. The operators in (A.9) can arise at one loop with a coefficient $\sim 1/16\pi^2$, but not necessarily with any further suppression in $1/\Lambda$. We therefore count them as terms of next-to-leading order. This completes the explanation of the leading-order Lagrangian in (2) and (4).

For convenience we quote the equations of motions implied by the leading-order Lagrangian (1) in the electroweak sector. They play an important role in simplifying the basis of operators at NLO and are given as follows:

$$\partial^\mu B_{\mu\nu} = g'\left[Y_\psi \bar\psi\gamma_\nu\psi - \frac{i}{2}v^2\langle U^\dagger D_\nu U T_3\rangle\bigl(1 + F_U(h)\bigr)\right] \tag{A.10}$$

$$D^\mu W^a_{\mu\nu} = g\left[\bar\psi_L\gamma_\nu T^a\psi_L + \frac{i}{2}v^2\langle U^\dagger T^a D_\nu U\rangle\bigl(1 + F_U(h)\bigr)\right] \tag{A.11}$$

$$\begin{aligned}\partial^2 h + V'(h) = {} & \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle F'_U(h) \\ & - \sum_{n=0}^{\infty}(n+1)\bigl(\bar q \hat{Y}_u^{(n+1)} U P_+ r + \bar q \hat{Y}_d^{(n+1)} U P_- r + \bar l \hat{Y}_e^{(n+1)} U P_-\eta + \text{h.c.}\bigr) \\ & \times \left(\frac{h}{v}\right)^n \end{aligned} \tag{A.12}$$

$$\begin{aligned} & \frac{v}{2}\bigl[D_\mu\bigl(U^\dagger D^\mu U\bigl(1 + F_U(h)\bigr)\bigr)\bigr]_{ij} \\ & = \left[\hat{Y} + \sum_{n=1}^{\infty}\hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right]_{st}(\bar\psi_{L,s}U)_j(P\psi_{R,t})_i \\ & \quad - \left[\hat{Y} + \sum_{n=1}^{\infty}\hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right]^\dagger_{ts}(\bar\psi_{R,t}P)_j\bigl(U^\dagger\psi_{L,s}\bigr)_i \\ & \quad - \frac{1}{2}\delta_{ij}\left(\left[\hat{Y} + \sum_{n=1}^{\infty}\hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right]_{st}\bar\psi_{L,s}UP\psi_{R,t}\right. \\ & \quad \left. - \left[\hat{Y} + \sum_{n=1}^{\infty}\hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right]^\dagger_{ts}\bar\psi_{R,t}PU^\dagger\psi_{L,s}\right) \end{aligned} \tag{A.13}$$

Here i, j are $SU(2)$ indices, s, t are flavor indices, and the quantities $(\hat{Y}, \hat{Y}^{(n)}, P, \psi_L, \psi_R)$ are summed over $(\hat{Y}_u, \hat{Y}_u^{(n)}, P_+, q, r)$, $(\hat{Y}_d, \hat{Y}_d^{(n)}, P_-, q, r)$ and $(\hat{Y}_e, \hat{Y}_e^{(n)}, P_-, l, \eta)$. In a similar notation, the equations of motion for fermions can be written as

$$i\not{D}\psi_L = v\left[\hat{Y} + \sum_{n=1}^{\infty} \hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right] U P \psi_R$$
$$i\not{D}\psi_R = v\left[\hat{Y} + \sum_{n=1}^{\infty} \hat{Y}^{(n)}\left(\frac{h}{v}\right)^n\right]^{\dagger} P U^{\dagger} \psi_L \tag{A.14}$$

where a summation over the appropriate terms on the right-hand sides is understood.

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