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High-energy elementary amplitudes from quenched and full QCD

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Abstract

Making use of the Stochastic Vacuum Model and the gluon gauge-invariant two-point correlation function, determined by numerical simulation on the lattice in both quenched approximation and full QCD, we calculate the elementary (quark–quark) scattering amplitudes in the momentum transfer space and at asymptotic energies. Our main conclusions are the following: (1) the amplitudes decrease monotonically as the momentum transfer increases; (2) the decreasing is faster when going from quenched approximation to full QCD (with decreasing quark masses) and this effect is associated with the increase of the correlation lengths; (3) dynamical fermions generate two components in the amplitude at small momentum transfer and the transition between them occurs at momentum transfer near 1 GeV^2. We also obtain analytical parametrizations for the elementary amplitudes, that are suitable for phenomenological uses, and discuss the effects of extrapolations from the physical regions investigated in the lattice.

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1. Introduction

Soft hadronic scattering, characterized by long distance phenomena, is one of the great challenges in high-energy physics. The difficulty arises from the fact that perturbative QCD cannot be applied to these processes and, presently, we do not know even how to calculate elastic hadron–hadron scattering amplitudes from a pure non-perturbative QCD formalism. However, progresses have been achieved through the approach introduced by Landshoff and Nachtmann [1], developed by Nachtmann [2] and connected with the Stochastic Vacuum Model (SVM) [3]. In that formalism the low frequencies contributions in the functional integral of QCD are described in terms of a stochastic process, by means of a cluster expansion. The most general form of the lowest cluster is the gauge invariant two-point field strength correlator [3,4]

$$\begin{aligned}\left\langle \mathbf{F}^{C}_{\mu\nu}(x)\mathbf{F}^{D}_{\rho\sigma}(y)\right\rangle &= \delta^{CD}\frac{g^2\langle FF\rangle}{12(N_c^2-1)}\Big\{(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})\kappa D\big(z^2/a^2\big)\\ &\quad+\frac{1}{2}[\partial_\mu(z_\rho\delta_{\nu\sigma}-z_\sigma\delta_{\nu\rho})+\partial_\nu(z_\sigma\delta_{\mu\rho}-z_\rho\delta_{\mu\sigma})]\\ &\quad\times(1-\kappa)D_1\big(z^2/a^2\big)\Big\},\end{aligned}\tag{1}$$

where $z = x - y$ is the two-point distance, a is a characteristic correlation length, κ a constant, $g^2\langle FF\rangle$

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the gluon condensate and N_c the number of colours ($\mathrm{C}, \mathrm{D} = 1, \ldots, N_c^2 - 1$). The two scalar functions D and D_1 describe the correlations and they play a central role in the application of the SVM to high energy scattering [4]. Once one has information about D and D_1, the SVM leads to the determination of the elementary quark–quark scattering amplitude, which constitutes important input for models aimed to construct hadronic amplitudes. Numerical determinations of the above correlation functions, in limited interval of physical distances, exist from lattice QCD in both quenched approximation (absence of fermions) [5,6] and full QCD (dynamical fermions included) [7].

In a previous paper, we determined the elementary amplitudes from lattice QCD in the quenched approximation [8], using as framework the SVM. In this communication, we apply the same procedure, now taking into account the lattice results in full QCD. Our main goal is to investigate the differences in the elementary amplitudes associated with quenched theory and full QCD and also the effect of different bare quark masses. In addition, we obtain analytical parametrizations for the amplitudes that are suitable for phenomenological uses, and discuss in some detail the region of validity of all the results.

The Letter is organized as follows. In Section 2 we recall the main formulas related with the elementary amplitudes in the context of the SVM and in Section 3 we review the parametrizations for the correlators from numerical simulations on the lattice. In Section 4 we present the results for the elementary amplitudes from full QCD with different quark masses and discuss the similarities and differences with our previous result in the quenched approximation. The conclusions and some final remarks are the contents of Section 5.

2. Stochastic vacuum model

In this section we briefly review the main steps of the calculation scheme that allows the determination of the elementary amplitudes through the SVM [3,4]. We refer the reader to [8] for more details concerning specific calculation.

The elementary amplitude f in the momentum transfer space may be expressed in terms of the elementary profile γ in the impact parameter space, through the symmetrical two-dimensional Fourier transform

$$f(q^2) = \int_0^\infty b\,db\, J_0(qb)\gamma(b), \tag{2}$$

where q^2 is the momentum transfer, b the impact parameter and J_0 is a Bessel function.

In the Nachtmann approach [2], the study of the elementary scattering is based on the amplitude of quarks moving on lightlike paths in an external field, picking up an eikonal phase in traveling through the nonperturbative QCD vacuum. In order to have gauge invariant Dirac's wave function solutions a Wilson loop is proposed to represent each quark. In this context the no-colour exchange parton–parton (loop–loop) amplitude can be written as [2]

$$\gamma = \Big\langle \mathrm{Tr}\big[\mathcal{P}e^{-ig\int_{\mathrm{loop}\ 1} d\sigma_{\mu\nu}\, F_{\mu\nu}(x;w)} - 1\big] \\ \times \mathrm{Tr}\big[\mathcal{P}e^{-ig\int_{\mathrm{loop}\ 2} d\sigma_{\rho\sigma}\, F_{\rho\sigma}(y;w)} - 1\big]\Big\rangle,$$

where $\langle\cdot\rangle$ means the functional integration over the gluon fields (the integrations are over the respective loop areas), and w is a common reference point from which the integrations are performed.

This expression is simplified in the Krämer and Dosch description by taking the Wilson loops on the light-cone. In the SVM the *leading order* contribution to the amplitude is given by [4]

$$\gamma(b) = \eta\epsilon^2(b), \tag{3}$$

where η is a constant depending on normalizations and

$$\epsilon(b) = g^2 \iint d\sigma_{\mu\nu}\, d\sigma_{\rho\sigma}\, \mathrm{Tr}\langle F_{\mu\nu}(x;w)F_{\rho\sigma}(y;w)\rangle.$$

Here $\langle g^2 F_{\mu\nu}(x;w)F_{\rho\sigma}(y;w)\rangle$ is the Minkowski version of the gluon correlator.

After a two-dimensional integration, $\epsilon(b)$ may be expressed in terms of the correlation functions in (1) by [4]

$$\epsilon(b) = \epsilon_{\mathrm{I}}(b) + \epsilon_{\mathrm{II}}(b), \tag{4}$$

where

$$\epsilon_{\mathrm{I}}(b) = \kappa\langle g^2 FF\rangle \int_b^\infty db'\,(b'-b)\mathcal{F}_2^{-1}\big[D(k^2)\big](b'), \tag{5}$$

$$\epsilon_{\mathrm{II}}(b) = (1-\kappa)\langle g^2 FF\rangle \mathcal{F}_2^{-1}\left[\frac{d}{dk^2}D_1(k^2)\right](b). \tag{6}$$

For $\mathcal{D} = D$ or D_1, $\mathcal{D}(k^2) = \mathcal{F}_4[\mathcal{D}(z^2)]$, where $\mathcal{F}_n$ denotes a n-dimensional Fourier transform.

With the above formalism, once one has inputs for the correlation functions $D(z)$ and $D_1(z)$, the elementary amplitude in the momentum transfer space, Eq. (2), may, in principle, be evaluated through Eqs. (3)–(6). It is important to stress that, as constructed, this approach is intended for the high energy limit and small momentum transfer region, namely, $s \to \infty$, where $\sqrt{s}$ is the c.m. energy and $q^2 \lesssim \mathcal{O}(1)$ GeV^2.

3. Lattice parametrizations

The determination of the correlation functions through numerical simulation on a lattice is made by means of the cooling technique, a procedure that removes the effects of short-range fluctuations on large distance correlators. The numerical results with the associated error are usually parametrized with the functions [6,7]

$$D(z) = A_0 \exp\left(-\frac{|z|}{\lambda_A}\right) + \frac{a_0}{|z|^4} \exp\left(-\frac{|z|}{\lambda_a}\right), \tag{7}$$

$$D_1(z) = A_1 \exp\left(-\frac{|z|}{\lambda_A}\right) + \frac{a_1}{|z|^4} \exp\left(-\frac{|z|}{\lambda_a}\right), \tag{8}$$

where λ_A in the nonperturbative exponential terms is the correlation length of the gluon field strengths. Discussions on these choices, including the perturbative-like divergence at short distances, may be found in Refs. [6] and [8].

These correlation functions were first determined in the quenched SU(3) theory and in the interval of physical distances (Euclidean space) between 0.1 and 1 fm [5,6]. After that, the effects of dynamical fermions have also been included (full QCD), for bare quark masses $a.m_q = 0.01$ and $a.m_q = 0.02$ (a is the lattice spacing) and physical distances between 0.3 and 0.9 fm [7]. The above parametrizations are the same in all these cases and the numerical values of the parameters, obtained from Refs. [6] and [7], are displayed in Table 1.

Table 1
Central values of the fit parameters (without statistical errors) for the correlators (7) and (8) [6,7]

Parameters	Full QCD		Quenched
	$m_q.a = 0.01$	$m_q.a = 0.02$	approximation
A_0 (fm^{-4})	14.87	31.04	128.4
a_0	0.71	0.66	0.69
A_1 (fm^{-4})	1.709	4.102	27.23
a_1	0.45	0.39	0.46
λ_A (fm)	0.34	0.29	0.22
λ_a (fm)	4.4	3.0	0.43

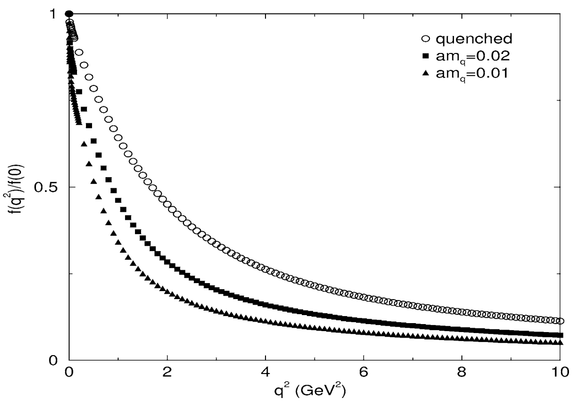

Fig. 1. Elementary amplitudes from full QCD and our previous result in quenched approximation.

4. Results and discussion

With the procedure described in Section 2 (see [8] for all the calculational details), the elementary scattering amplitude in the momentum transfer space may be determined. The numerical results from full QCD, with $a.m_q = 0.01$ and $a.m_q = 0.02$, are shown in Fig. 1, together with our previous result in the quenched approximation. The normalized amplitudes are displayed in the region of high momentum transfer, up to 10 GeV^2.

We see that in all the cases the amplitudes decrease smoothly as the momentum transfer increases and they do not present any zeros in that region. The overall basic effect of the dynamical quarks is to originate a more rapid decrease of the amplitude, an effect that depends on the bare quark mass: the smaller the mass the faster the decrease. This behavior is associated with the correlation lengths λ_A and λ_a, since they are the only parameters that decrease when going from full QCD (with increasing quark masses) to quenched approximation (see Table 1).

In order to obtain analytical expressions, suitable for investigating distinct contributions and also for

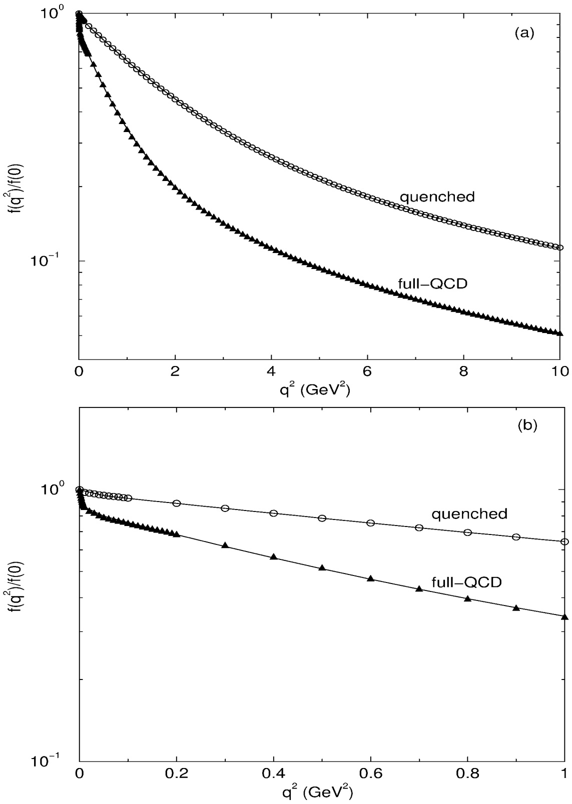

Fig. 2. Fits to numerical points with parametrization (9) in the cases of full QCD for $m_q.a = 0.01$ and quenched approximation, in the region of large (a) and small (b) momentum transfer.

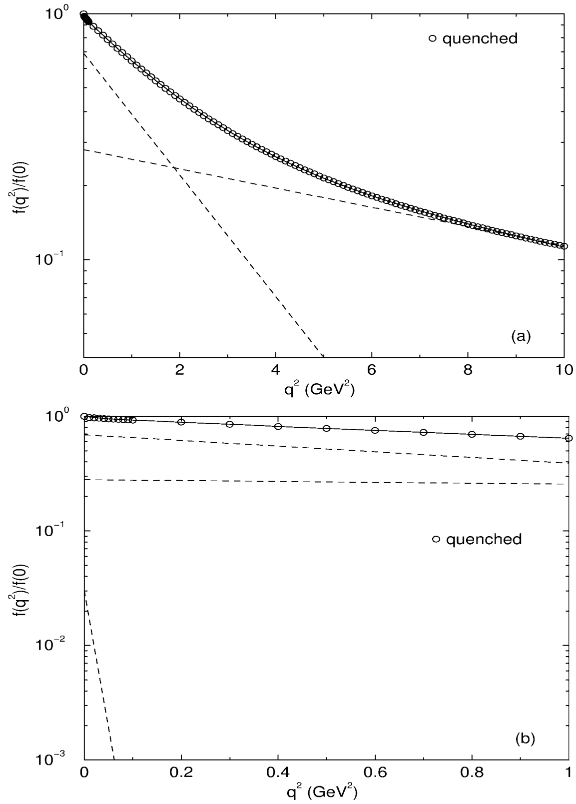

Fig. 3. Exponential components of the fit at large (a) and small (b) momentum transfer in quenched approximation.

phenomenological uses, we have parametrized these numerical points through a sum of exponentials in q^2:

$$\frac{f(q^2)}{f(0)} = \sum_{i=1}^{n} \alpha_i e^{-\beta_i q^2}. \tag{9}$$

By introducting a global uniform error of 0.5% in the numerical points, we fitted the data through the program CERN-Minuit. The results of the fits from full QCD with $m_q.a = 0.01$ (approximation to the chiral limit) and in the quenched approximation are displayed in Table 2 and they are represented by the solid lines in Fig. 2, in the regions of large and small momentum transfer. The corresponding exponential components in each fit are shown in Fig. 3 for the quenched approximation and in Fig. 4 for full QCD.

An immediate conclusion from these results is the presence of an additional component in the case of full QCD. This is a central point that we shall discuss in certain detail in what follows.

Let us start with the components with the highest slopes, which appear in both cases in the small momentum transfer region, namely, below $q^2 \simeq 0.1\ \mathrm{GeV}^2$ (Figs. 3 and 4). As mentioned before, the "real" lattice results correspond to sets of discrete theoretical points with errors, in a finite interval of physical distances, roughly 0.1–1.0 fm. Therefore, the parametrizations (7) and (8) extrapolate this interval down and above. The highest physical distance reached in the simulations was 0.85 fm, which corresponds to $q^2 \simeq 0.24\ \mathrm{GeV}^2$. Therefore, we conclude that the components with the highest slopes in Figs. 3 and 4 ($q^2 \leqslant 0.1\ \mathrm{GeV}^2$) are associated with the extrapolations above the physical region with "real" lattice results. In the same manner, the components with the smallest slopes are connected with extrapolations down the "real" lattice points in physical distances and they are the

Table 2
Values of the fit parameters to the elementary amplitudes, Eq. (9), in the cases of quenched approximation and full QCD with $m_q.a = 0.01$

Parameters:	α_i				β_i (GeV^{-2})			
$i =$	1	2	3	4	1	2	3	4
Quenched	0.03	–	0.69	0.28	55.0	–	0.57	0.09
Full QCD	0.16	0.54	0.20	0.10	170.0	1.52	0.41	0.07

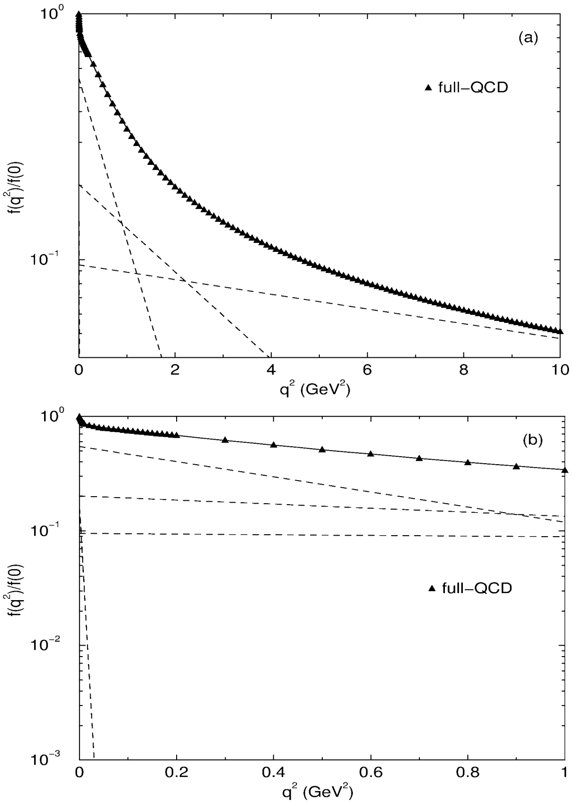

Fig. 4. Exponential components of the fit at large (a) and small (b) momentum transfer in full QCD for $m_q.a = 0.01$.

responsible for the amplitudes in high momentum region ($q^2 \geqslant$ 6–7 GeV^2 in Figs. 3 and 4); therefore, they are outside the region of validity of the SVM, namely, $q^2 \lesssim \mathcal{O}(1)$ GeV^2.

We conclude that in this context only the intermediate components, predominant in the interval, let us say, $0.5 \lesssim q^2 \lesssim 2.0$ GeV^2, can have physical meaning in the sense of being in agreement with the SVM and the "real" lattice results in both full QCD and quenched approximation. From Figs. 3 and 4 this "secure" region is characterized by only one component in the case of the quenched approximation and two components in full QCD. The transition between these two components occurs at $q^2 \simeq 1$ GeV^2 (Fig. 4(a)), a limit region for which the SVM is intended for.

5. Conclusions and final remarks

In this work we have obtained analytical parametrizations for the quark–quark scattering amplitudes in a nonperturbative QCD framework (SVM) and using as inputs the correlation functions, determined from numerical simulation on a lattice, in both quenched approximation and full QCD. The formalism is intended for small momentum transfer ($q^2 \lesssim \mathcal{O}(1)$ GeV^2), asymptotic energies $s \to \infty$ and physical distances between 0.1 and ~ 0.9 fm. As discussed in Section 4, these conditions put some restrictions in the physical interpretations and, therefore, in the practical phenomenological uses of these amplitudes.

However, even under the above strictly conditions we can surely extract some novel results: (1) the amplitudes decrease smoothly as the momentum transfer increases and they do not present zeros; (2) the decreasing is faster when going from quenched approximation to full QCD (with decreasing quark masses), and this effect is associated with the increase of the correlation lengths (λ_A and λ_a); (3) the dynamical fermions generate two contributions in the region of small momentum transfer, which are of the same order at $q^2 \sim 1$ GeV^2 (only one contribution is present in the case of quenched approximation).

We understand that result (3) may suggest some kind of change in the dynamics at the elementary level, near $q^2 \sim 1$ GeV^2 and at asymptotic energies. If that is true, some signal could be expected at the hadronic level. One possibility is that this effect can be associated with the position of the dip (or beginning of the "shoulder") in the hadronic (elastic) differential cross section data. The asymptotic condition embodied in our result indicates that $q^2 \sim 1$ GeV^2 seems in agreement with limit of the shrinkage of the diffraction peak, empirically verified when the energy increases in the region 23 GeV $\leqslant \sqrt{s} \leqslant$ 1.8 TeV.

At last it should be noted that if there is no new effect above the physical distances presently investigated on the lattice ($\sim$ 0.9 fm), the extrapolations can be considered as a good representation of the lattice results. In this case our analytical parametrizations may be useful inputs for phenomenological uses in the region of small momentum transfer and asymptotic energies.

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