LASER FREQUENCY STABILIZATION BY POLARIZATION SPECTROSCOPY OF A REFLECTING REFERENCE CAVITY *

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Received 2 September 1980

We propose a new scheme for locking the frequency of a laser to a resonant reference cavity. A linear polarizer or Brewster plate is placed inside the reference cavity, so that the reflected light acquires a frequency-dependent elliptical polarization. A simple polarization analyzer detects dispersion shaped resonances which can provide the error signal for electronic frequency stabilization without any need for modulation techniques.

1. Introduction

The cavity of a laser is generally subject to various perturbations, and the stability of a single-mode laser can be improved by electronically locking its frequency to some passive reference cavity [1,2]. We are proposing a new method for laser frequency locking which utilizes a reference cavity with internal linear polarizer or Brewster plate and monitors changes in the polarization of the reflected light. A simple analyzer for elliptical polarization provides dispersion-shaped resonances, which immediately give the error signal for the servo loop. In this way, the laser can be locked to the center of a cavity resonance without any need for modulation techniques. Moreover, the error signal in the wings, far away from resonance, remains strong enough to permit automatic re-locking even after larger accidental frequency jumps.

Most previous locking schemes monitor the light intensity transmitted by the reference cavity although it has been pointed out that the reflected intensity can offer a better signal to noise ratio [3]. The transmitted intensity versus frequency (or, equivalently, versus the phase difference δ between waves in successive cavity roundtrips) exhibits the familiar periodic resonant maxima or fringes as described by the Airy function and illustrated in fig. 1A.

In one popular approach [1], the laser frequency is servo-locked to the side of one fringe, using a fast dif-

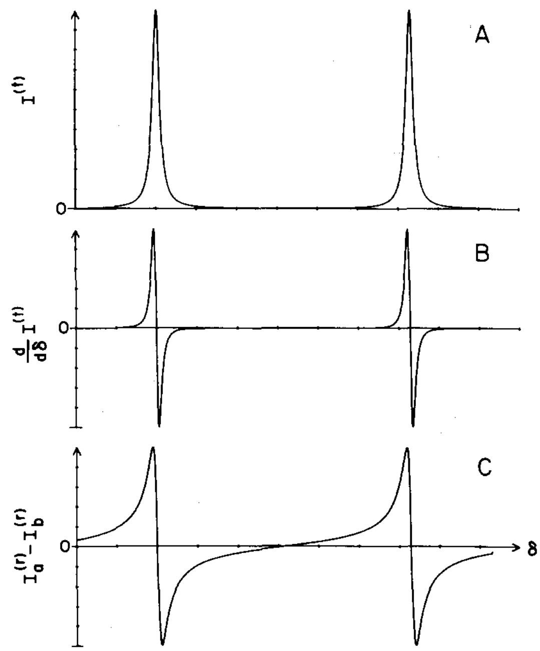

Fig. 1. A: spectrum of the intensity transmitted by a passive cavity. B: first derivative of the transmission spectrum. C: dispersive resonances obtained by polarization spectroscopy.

* Work supported by the U.S. Office of Naval Research under Contract ONR N00014-78-C-0403.
** C.N.R.S. Postdoctoral Fellow.

ferencing technique. In this case, an accidental frequency jump of as little as half as fringe width can throw the system out of lock. A reference cavity of low finesse reduces this problem, but then the laser frequency becomes sensitive to small changes in the finesse or to drifts in the photodetectors or associated amplifiers.

In another approach [2], the frequency of the laser or reference cavity is modulated, and the amplitude modulation of the transmitted light is monitored with a phase sensitive detector. The signal corresponds to the derivative of the Airy function (fig. 1B) and permits locking to the fringe center. However, the method requires more elaborate electronics and it is inherently slow. Moreover, the signal drops quickly to zero a few fringe widths away from resonance, making recovery from large frequency jumps difficult.

In 1946 Pound [4] has proposed a method for frequency stabilization of a microwave oscillator, which produces an almost truly dispersion shaped error signal by reflecting the wave off a reference cavity and monitoring a component with a phase shift of 90°. In analogy, our scheme monitors an out-of phase component in the light reflected by an optical cavity, as manifested in the occurrence of elliptical polarization.

In the same reference [4], Pound has introduced a related but different microwave stabilization technique which uses modulation sidebands and a.c. detection to determine the imaginary component of a cavity reflection coefficient. An optical analog of this modulation scheme has very recently been proposed by Drever and Hall [5].

2. The stabilization method

To explain and analyze our frequency stabilization method, let us consider the setup illustrated in fig. 2. Linearly polarized light from a tunable single mode laser is reflected by a confocal reference cavity used off-axis so that a small angle between incident and reflected beam avoids feedback into the laser cavity. The linear polarizer inside the cavity is rotated so that its transmission axis forms an angle θ with the polarization axis of the incident beam.

The incoming light can be decomposed into two orthogonal linearly polarized components with the electric field vector parallel and perpendicular to the transmission axis of the intracavity polarizer. Their

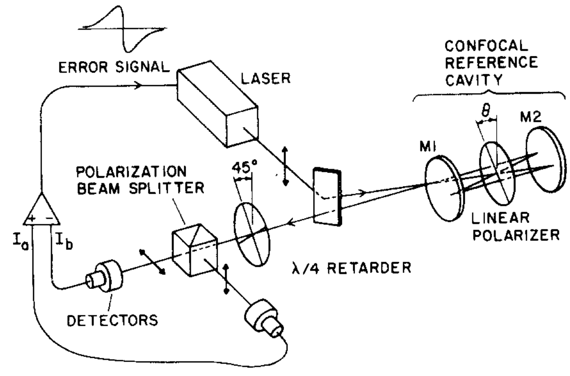

Fig. 2. Scheme for laser frequency stabilization.

field amplitudes in the plane wave approximation are

$$E_{\parallel}^{(i)} = E^{(i)} \cos\theta, \quad E_{\perp}^{(i)} = E^{(i)} \sin\theta, \tag{1}$$

where $E^{(i)}$ is the amplitude of the incident beam. The parallel component sees a cavity of low loss and experiences a frequency-dependent phase shift in reflection. The perpendicular component, simply reflected by mirror M_1, serves as a reference. Any relative phase change between the two reflected components will make the resulting beam elliptically polarized.

We first consider the parallel component. Using the standard approach [6] the complex amplitude of the reflected wave can easily be calculated as

$$E_{\parallel}^{(r)} = E_{\parallel}^{(i)} \left\{ \sqrt{R_1} - \frac{T_1}{\sqrt{R_1}} \frac{R e^{i\delta}}{1 - R e^{i\delta}} \right\}$$

$$= E_{\parallel}^{(i)} \left\{ \sqrt{R_1} - \frac{T_1 R}{\sqrt{R_1}} \frac{\cos\delta - R + i \sin\delta}{(1-R)^2 + 4R \sin^2 \frac{1}{2}\delta} \right\}, \tag{2}$$

where R_1 and T_1 are the reflectivity and transmissivity of the cavity entrance mirror M_1, and $R < 1$ gives the amplitude ratio between successive roundtrips, which determines the cavity finesse $\mathcal{F} = \pi\sqrt{R}/(1-R)$. The ratio R accounts for any attenuation by the internal polarizer and for other losses, including the two extra reflections which are required for one roundtrip in a confocal resonator used off-axis.

The amplitude of the reflected perpendicular component, on the other hand, is to first approximation simply

$$E_{\perp}^{(r)} = E_{\perp}^{(i)} \sqrt{R_1}\,. \tag{3}$$

At exact resonance ($\delta = 2\,m\pi$) both reflection coefficients are real and the reflected wave components remain in phase. The reflected beam remains linearly polarized, even though its polarization axis will be rotated from the original direction. Away from resonance, however, the parallel component acquires a phase shift relative to the perpendicular component, owing to the imaginary part of $E_{\parallel}^{(r)}$, and the reflected beam acquires an elliptical polarization. The handedness of the polarization ellipse depends on the sign of the detuning from resonance.

To detect the ellipticity, the reflected light is sent into an analyzer assembly consisting of a $\lambda/4$ retarder and a linear polarization beam splitter. The fast axis of the retarder is rotated by 45° relative to the polarization axis of beam splitter output a. The light intensities I_a and I_b at the two outputs are monitored by two photodetectors connected to a differential amplifier.

To understand the function of this analyzer we consider elliptically polarized light as a superposition of two counterrotating circularly polarized components of different amplitudes. The $\lambda/4$ retarder transforms these circular components into orthogonal lineraly polarized waves, which are separated by the beamsplitter so that their intensities can be measured individually. If the incoming light is linearly polarized, the two circular components have equal intensities. This fact can be very useful for balancing the sensitivity of the two photodetectors. We note that the signal $I_a - I_b$ depends only on the magnitude and handedness of the ellipticity, but not on the azimuth angle, i.e. the entire analyzer assembly can be rotated around the beam axis without affecting the signal.

In order to calculate the signal easily we assume that the assembly is rotated so that the fast axis of the $\lambda/4$ retarder is parallel to the polarization axis of the intracavity polarizer. Using the Jones calculus [7] we find the field amplitudes of the reflected beam after passing through the retarder and polarization beamsplitter:

$$E_{a,b} = \tfrac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} E_{\parallel}^{(r)} \\ E_{\perp}^{(r)} \end{pmatrix}. \qquad (4)$$

The corresponding intensities at the two outputs are

$$I_{a,b} = \tfrac{1}{2}c\epsilon |E_{a,b}|^2 = \tfrac{1}{2}c\epsilon |\tfrac{1}{2}(E_{\parallel}^{(r)} \pm iE_{\perp}^{(r)})|^2, \qquad (5)$$

and with equations (5), (3), (2), and (1) we calculate a signal

$$I_a - I_b = I^{(i)}\, 2\cos\theta \sin\theta \,\frac{T_1 R \sin\delta}{(1-R)^2\, 4R \sin^2\tfrac{1}{2}\delta}, \qquad (6)$$

where $I^{(i)} = \tfrac{1}{2}c\epsilon|E^{(i)}|^2$ is the intensity of the incident laser beam.

The function (6) has been plotted in fig. 1c. It combines a steep resonant slope with far reaching wings and provides an ideal error signal for servo locking of a laser frequency. The signal is maximized if $\theta = 45°$, so that $2\cos\theta \sin\theta = 1$. However, the total intensity of the reflected light at resonance is smallest near $\theta = 0$, and operation at smaller angles θ can offer a better signal to noise ratio, if laser intensity fluctuations are the dominant source of noise. Any birefringence due to stress in the dielectric mirror coatings or other optical elements has been ignored in our analysis. Such residual birefringence can produce line asymmetries and should be avoided or compensated.

3. Experiment

We have performed some preliminary experimental tests of the proposed method, using a Coherent Radiation Model 699-21 ring cavity cw dye laser operating with rhodamine 6G. A Spectra Physics Model 470 spectrum analyzer with 2 GHz free spectral range served as the reference cavity. A small glass piece cut from a microscope slide was inserted at Brewster's angle into this cavity to act as the linear polarizer. A $f = 5$ cm lens was placed in front of the cavity in order to reduce the divergence of the reflected beam to about 10 mrad. The elliptic polarization analyzer consisted of a glass Fresnel rhomb and calcite polarization beam splitter, followed by two silicon photodiodes.

In a first test, the laser frequency was scanned rapidly over several free spectral ranges of the reference cavity, and the two photodiodes were connected to the differential input of an oscilloscope. Fig. 3 shows a trace of the resulting signal. For comparison, the intensity of the light transmitted by the cavity is displayed above. The cavity finesse is rather low, owing to low reflectivity of the cavity mirrors and to the poor optical quality of the internal Brewster plate. The polarization signal agreed well with the predictions after the analyzer system had been balanced with the help of a variable attenuator in front of one of the detectors.

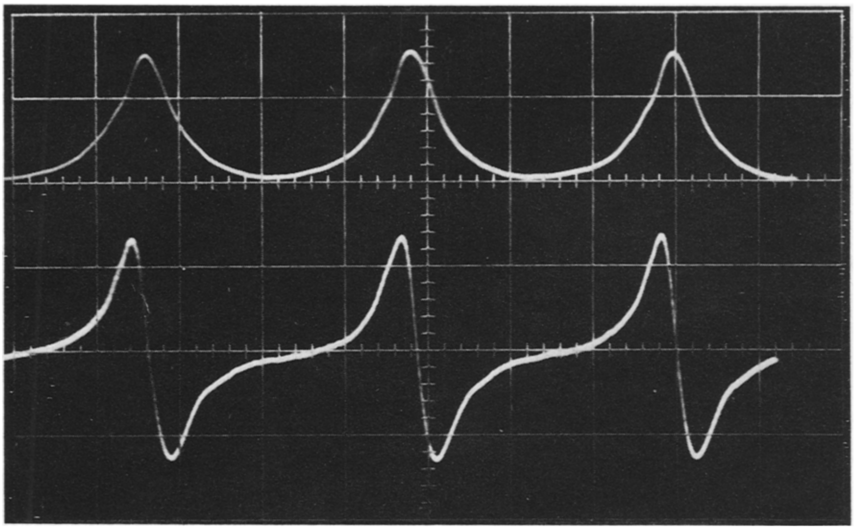

Fig. 3. Experimental response of a confocal reference cavity of 2 GHz free spectral range during a rapid laser frequency scan. Top trace: transmitted intensity. Bottom trace: output of the polarization analyzer for the reflected beam.

The dye laser frequency could be locked without problems to the reference cavity by simply directing the two output beams of the polarization beamsplitter onto the photodiodes of the standard servo-control system of the laser as provided by the factory. When the reference cavity was tuned piezoelectrically, the laser frequency remained in resonance, as evidenced by the strong transmitted intensity. A check with a second spectrum analyzer confirmed that the linewidth of the laser thus stabilized was less than a few MHz, although the resolution was insufficient for a more accurate measurement.

4. Conclusions

We have shown theoretically and experimentally that polarization spectroscopy of a reflecting anisotropic cavity can provide dispersion-shaped resonances well suited for locking a laser frequency to a fringe center. If an adjustable Brewster plate is used as frequency tuning element inside the reference cavity, no separate linear polarizer is required. To conclude we would like to point out that any jump in the phase of the incident laser radiation relative to the light field stored inside the cavity will temporarily alter the polarization of the reflected light, and the resulting error signal should make it possible, with the help of suitable fast electronic circuitry, to phase-lock the laser to the passive reference cavity. We expect that extremely narrow laser linewidths can be achieved in this way.

References

[1] R.L. Barger, M.S. Sorem and J.L. Hall, Appl. Phys. Letters 22 (1973) 573.
[2] A.D. White, IEEE J. Quant. Electronics QE-1 (1965) 349.
[3] Yu.V. Troitskii, Sov. J. Quantum Electron. (USA) 8 (1978) 628.
[4] R.V. Pound, Rev. Sci. Instr. 17 (1946) 490.
[5] R.W.P. Drever and J.L. Hall, private communication.
[6] M. Born and E. Wolf, Principles of optics (Pergamon, Oxford, 1970) Chapter 7.6.
[7] R.C. Jones, J. Opt. Soc. Am. 46 (1956) 126, and references therein.