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Ion acoustic solitary wave solutions of three-dimensional nonlinear extended Zakharov–Kuznetsov dynamical equation in a magnetized two-ion-temperature dusty plasma

Aly R. Seadawy [a,b,*], Dianchen Lu [c]

[a] Mathematics Department, Faculty of Science, Taibah University, Al-Ula, Saudi Arabia
[b] Mathematics Department, Faculty of Science, Beni-Suef University, Egypt
[c] Department of Mathematics, Faculty of Science, Jiangsu University, China

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ABSTRACT

We consider the propagation of three-dimensional nonlinear magnetized two-ion-temperature dusty plasma. The problem formulation of this mathematical model leads to nonlinear extended Zakharov–Kuznetsov (EZK) dynamical equation in three-dimensional by applying the reductive perturbation theory. We found the families of dust and ion solitary wave solutions of the three-dimensional nonlinear EZK dynamical equation using the auxiliary equation mapping method and direct algebraic mapping method.

1. Introduction and problem formulation of derivation EZK equation

Dusty plasmas are the ionized gases containing small particles of solid matter since their discovery in laboratory as well as in space. The nonlinear propagation of such waves can give rise to the formation of solitons with negative or positive wave amplitudes, which has potential applications in astrophysical and space environments as well as in laboratory and technological studies [1]. Furthermore, electrostatic solitary waves have been observed in several regions, including the Earths magnetotail, solar wind, and polar magnetosphere [1]. Solitary waves and solitons represent one of the interesting and famous aspects of nonlinear phenomena in spatially extended systems. They appear as specific types of localized solutions of various nonlinear partial differential equations and possess several important properties [2]. Extended Zakharov–Kuznetsov equation used to describe the nonlinear dust-ion-acoustic waves in the magnetized two-ion-temperature dusty plasmas [3–6], or the propagation of the low-frequency ion-acoustic waves in a dense quantum magneto-plasma, or the obliquely propagating higher-order dispersion electron-acoustic solitary waves in a magnetized interactionless plasma. Some numerical and analytical studies have been conducted on extended Zakharov–Kuznetsov equation [7–10]: the explosive and periodic solutions have been obtained, existence and instability of the propagating solitary wave solutions have been simulated numerically, the conserved quantities and one-soliton solutions have been given via the mapping and Ansatz methods and Lie Group analysis, the symmetry solutions and reductions have been investigated, and some analytical solutions have been obtained [11–18].

The nonlinear propagation of dust-ion-acoustic solitary waves and shocks in a four component dusty plasma consisting of dust particles which are extremely massive and usually negatively charged, electrons, high-temperature ions and low temperature ions in the presence of an external magnetic filed M. The dynamics of the propagation dust-ion-acoustic waves such as plasma are governed by the fluid equations (the normalized fluid equations of continuity and motion and system is closed by Poisson's equation) as

$$\nabla \cdot \nabla \phi = 4\pi e\left(\sum_{j=1}^{N} n_{dj} Z_{dj} + n_e - n_{il} - n_{ih}\right), \tag{1}$$

$$\frac{\partial n_{dj}}{\partial t} + \nabla \cdot (n_{dj}\mathbf{u}_{dj}) = 0, \tag{2}$$

* Corresponding author at: Mathematics Department, Faculty of Science, Taibah University, Al-Ula, Saudi Arabia.
E-mail addresses: Aly742001@yahoo.com (A.R. Seadawy), dclu@ujs.edu.cn (D. Lu).

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$$n_{dj}m_{dj}\left(\frac{\partial \mathbf{u}_{dj}}{\partial t}+(\mathbf{u}_{dj}\cdot\nabla)\mathbf{u}_{dj}\right)=Z_{dj}en_{dj}\nabla\phi-Z_{dj}n_{dj}e(\mathbf{u}_{dj}\times\mathbf{M}), \tag{3}$$

$$n_e=vexp(s\beta_1\phi),\quad n_{il}=\mu_l exp(-s\phi),\quad n_{ih}=v_h exp(-s\beta_2\phi), \tag{4}$$

where ϕ is the electrostatic potential; $\mathbf{u}_{dj}$ is the velocity of the jth dust grain; u_{dj}, v_{dj} and w_{dj} are the velocities of the dust grain flow along x, y and z directions, respectively; t refers to the time; n_{dj} is the number density of the jth dust grain; n_e, n_{il} and n_{ih} represent the number density of electrons, low-temperature ions and high-temperature ions, respectively; at equilibrium, the charge neutrality can be given as $n_{il0}+n_{ih0}=\sum_{j=1}^{N}n_{d0j}Z_{dj}+n_{e0}$, where n_{d0j} is the equilibrium number density of the jth dust grain; Z_{dj} is the charge of the jth dust grain divided by the electron charge e, n_{e0}, n_{ilo} and n_{iho} refer to the equilibrium number densities of electrons, low-temperature ions and high-temperature ions, respectively; m_{dj} are masses of N different species of dust grain. And $v=n_{e0}/(\overline{Z_{d0}}N_{tot})$; $\beta_1=T_{il}/T_e$; $\beta_2=T_{ih}/T_e$; $s=T_{eff}/T_e$; $T_{eff}=\overline{Z_{d0}}N_{tot}(n_{e0}/T_e+n_{ilo}/T_{il}+n_{iho}/T_{ih})$; $\mu_l=n_{il0}/(\overline{Z_{d0}}N_{tot})$; $\mu_h=n_{ih0}/(\overline{Z_{d0}}N_{tot})$; T_e, T_{il} and T_{ih} refer to the temperatures for electrons, lower and higher temperature for ions; $\overline{Z_{d0}}$ is the average charge number residing on the dust grain; $N_{tot}=\sum_{j=1}^{N}n_{d0j}$.

Assumed that the linear dispersion relation as $u_{dj}=u_0e^{k_1x+k_2y+k_3z-\omega t}$; $v_{dj}=v_0e^{k_1x+k_2y+k_3z-\omega t}$; $w_{dj}=w_0e^{k_1x+k_2y+k_3z-\omega t}$; where k_1, k_2 and k_3 are the wave numbers in the x, y, z directions, respectively, *om* refers the frequency of the linear wave. Using the reductive perturbation technique and transformations, the new stretching coordinates (space and time) of the scale are given as

$$x_1=\epsilon^{1/2}x,\quad y_1=\epsilon^{1/2}y,\quad z_1=\epsilon^{1/2}(z-v_0t),\quad t_1=\epsilon^{3/2}t, \tag{5}$$

where u_0 is the phase velocity of the wave along the x-direction; ϵ is a small expansion parameter proportional to the amplitude of the perturbation which characterizes the strength of the nonlinearity of the system. To obtain the dimensional nonlinear EZK dynamical equation, expand the fluid velocity, density and electrostatic potential in power series of ϵ. By applying the reduction perturbation method from Eqs. (1)–(4) and new scaling (5), then the collecting coefficients of lowest order of ϵ, by eliminating the second order quantities from Eqs. (1)–(4), and using the expressions for the first order quantities (5), can be reduced the three-dimensional nonlinear EZK equation as

$$\frac{\partial\phi}{\partial t}+A\phi\frac{\partial\phi}{\partial x}+B\frac{\partial^3\phi}{\partial x^3}+C\left(\frac{\partial^3\phi}{\partial x\partial y^2}+\frac{\partial^3\phi}{\partial x\partial z^2}\right)=0, \tag{6}$$

where

$$A=\frac{3}{2u_0}\sum_{j=1}^{N}\left(\frac{n_{d0j}Z_{dj}^3}{m_{dj}^2}\right)-\frac{1}{2}u_0s^2(v\beta_1-\mu_l-\mu_h\beta_2^2),$$

$$B=\frac{u_0}{2},\quad C=B\left(1+\frac{1}{B^2}\sum_{j=1}^{N}(n_{d0j}m_{dj})\right),$$

where x, y, z and t represent the partial derivatives, A is the nonlinear coefficient, B and C are the dispersion coefficients. Consider the traveling wave solutions as

$$\phi(x,y,z,t)=\phi(\theta),\quad\text{and}\quad\theta=k_1x+k_2y+k_3z-\omega t, \tag{7}$$

where k_1, k_2, k_3 and ω are wave numbers and frequency. Then Eq. (6) becomes

$$-\omega\phi'+Ak_1\phi\phi'+Bk_1^3\phi^{(3)}+Ck_1(k_2^2+k_3^2)\phi^{(3)}=0. \tag{8}$$

2. Families of solitary wave solutions

We found the families of solitary wave solutions for the nonlinear three-dimensional extended ZK dynamical equation by applying the extended direct algebraic mapping and extended sech-tanh function methods. The different values for the electrostatic potential ϕ give different analytic solutions of Eq. (6), which gives the following families:

Families I: By applying the auxiliary equation mapping method, the nonlinear three-dimensional extended ZK dynamical equation has general solution in series as:

$$\phi(\theta)=\sum_{i=0}^{n}a_iF^i(\theta)+\sum_{i=-1}^{-n}b_{-i}F^i(\theta)+\sum_{i=2}^{n}c_iF^{i-2}(\theta)F'(\theta)+\sum_{i=-1}^{-n}d_{-i}F^i(\theta)F'(\theta), \tag{9}$$

where $a_0, a_1, \ldots a_n, b_1, \ldots b_n, c_2, \ldots c_n, d_1, \ldots d_n$ are arbitrary constants, the value of $F(\theta)$ and $F'(\theta)$ satisfy the following

$$\begin{aligned}&F'(\theta)=\sqrt{pF^2(\theta)+qF^3(\theta)+rF^4(\theta)};\\&F''(\theta)=pF(\theta)+\tfrac{3}{2}qF^2(\theta)+2rF^3(\theta);\\&F'''(\theta)=\left(p+3qF(\theta)+6rF^2(\theta)\right)F'(\theta);\\&F^{(4)}(\theta)=\tfrac{1}{2}F(\theta)\big(2p^2+15pqF(\theta)+5(3q^2+8pr)\\&F^2(\theta)+60qrF^3(\theta)+48r^2F^4(\theta)\big),\end{aligned} \tag{10}$$

Balancing the highest order nonlinear term and the highest order linear partial derivative term in Eq. (8) yields the value of $m=2$. The solution of Eq. (6) takes the form

$$\phi(\theta)=a_0+a_1F(\theta)+a_2F^2(\theta)+\frac{b_1}{F(\theta)}+\frac{b_2}{F^2(\theta)}+c_2F'(\theta)+d_1\frac{F'(\theta)}{F(\theta)}+d_2\frac{F'(\theta)}{F^2(\theta)}. \tag{11}$$

Substitute Eq. (11) into Eq. (8) and collect coefficients of $F'^j(\theta)F^i(\theta)(j=0,1; i=0,1,2,3,\ldots n)$, then set each coefficient equal to zero to derive a set of over-determined algebraic equations. By solving this system, the parameters $a_0, a_1, a_2, b_1, b_2, c_2, d_1, d_2$ can be determined as

$$\begin{aligned}&a_0=\tfrac{\omega-Bpk_1^3-BCpk_1k_2^2-BCpk_1k_3^2}{Ak_1},\quad a_1=-\tfrac{6Bq(k_1^2+Ck_2^2+Ck_3^2)}{A},\\&a_2=-\tfrac{3Bq^2(k_1^2+Ck_2^2+Ck_3^2)}{Ap},\quad b_1=b_2=c_2=d_1=d_2=0,\quad r=\tfrac{q^2}{4p}\end{aligned} \tag{12}$$

Substituting from Eqs. (12) into (11), the electrostatic potential of Eq. (6) can be obtained as a ion-acoustic solitary wave solutions as:

$$\phi_1(x,y,z,t)=\tfrac{1}{Ak_1}\Big(2Bk_1(k_1^2+C(k_2^2+k_3^2))p+\omega-3Bk_1(k_1^2+C(k_2^2+k_3^2))ps^2\,tanh^2\Big[\tfrac{\sqrt{p}}{2}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0\Big]\Big) \tag{13}$$

$$\phi_2(x,y,z,t)=\tfrac{1}{A}\left(\tfrac{\omega-Bpk_1^3-BCpk_1k_2^2-BCpk_1k_3^2}{k_1}\right)+12Bq(k_1^2+Ck_2^2+Ck_3^2)\sqrt{\tfrac{p}{r}}\left(1+\tfrac{s\ sinh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}{\rho+cosh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}\right)-\tfrac{3B}{r}(k_1^2+Ck_2^2+Ck_3^2)\left(q+\tfrac{qs\ sinh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}{\rho+cosh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}\right)^2 \tag{14}$$

$$\phi_3(x,y,z,t)=\frac{1}{A}\left(\frac{\omega-Bk_1(k_1^2+C(k_2^2+k_3^2))p}{k_1}\right)+6Bp(k_1^2+Ck_2^2+Ck_3^2)\left(1+\frac{s\left(\rho\sqrt{1+\sigma^2}+\cosh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)}{\sigma+\sinh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}\right)-3B(k_1^2+Ck_2^2+Ck_3^2)p\left(1+\frac{s\left(\rho\sqrt{1+\sigma^2}+\cosh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)}{\sigma+\sinh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}\right)^2 \tag{15}$$

Families II: By applying the direct algebraic mapping method, the nonlinear three-dimensional extended ZK dynamical equation has general solution in series as:

$$\phi(\theta)=\sum_{i=0}^{n}a_iF^i(\theta)+\sum_{i=-1}^{-n}b_{-i}F^i(\theta)+\sum_{i=2}^{n}c_iF^{i-2}(\theta)F'(\theta)+\sum_{i=-1}^{-n}d_{-i}F^i(\theta)F'(\theta), \tag{16}$$

where $a_0,a_1,\dots a_n,b_1,\dots b_n,c_2,\dots c_n,d_1,\dots d_n$ are arbitrary constants, the value of $F(\theta)$ and $F'(\theta)$ satisfy the following

$$\begin{aligned}&F'(\theta)=\sqrt{pF^2(\theta)+qF^4(\theta)+rF^6(\theta)};\\&F''(\theta)=pF(\theta)+2qF^3(\theta)+3rF^5(\theta);\\&F'''(\theta)=\left(p+6qF^2(\theta)+15rF^4(\theta)\right)F'(\theta);\\&F^{(4)}(\theta)=F(\theta)\left(p^2+20pqF^2(\theta)+6(4q^2+13pr)F^4(\theta)+120qrF^6(\theta)+105r^2F^8(\theta)\right),\end{aligned} \tag{17}$$

Balancing the highest order nonlinear term and the highest order linear partial derivative term in Eq. (8) yields the value of $m=4$. The solution of Eq. (6) takes the form

$$\phi(\theta)=a_0+a_1F(\theta)+a_2F^2(\theta)+\frac{b_1}{F(\theta)}+\frac{b_2}{F^2(\theta)}+\frac{b_3}{F^3(\theta)}+\frac{b_4}{F^4(\theta)}+c_2F'(\theta)+c_3F(\theta)F'(\theta)+c_4F^2(\theta)F'(\theta)+d_1\frac{F'(\theta)}{F(\theta)}+d_2\times\frac{F'(\theta)}{F^2(\theta)}+d_3\frac{F'(\theta)}{F^3(\theta)}+d_4\frac{F'(\theta)}{F^4(\theta)}. \tag{18}$$

Substitute Eq. (18) into Eq. (16) and collect coefficients of $F'^j(\theta)F^i(\theta)(j=0,1;i=0,1,2,3,\dots n)$, then set each coefficient equal to zero to derive a set of over-determined algebraic equations. By solving this system, the parameters $a_0,a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4,c_2,c_3,c_4,d_1,d_2,d_3,d_4$ can be determined as.

Case I

$$\begin{aligned}&a_0=-\tfrac{\omega}{Ak_1},\quad a_1=0,\quad a_2=-\tfrac{3q\omega}{2Ak_1p},\quad a_3=0,\quad a_4=-\tfrac{3r\omega}{Ak_1p},\\&c_2=c_4=0,\\&c_3=-\tfrac{3\sqrt{r}\omega}{Ak_1p},\quad d_1=d_2=d_3=d_4=0,\quad k_2=\tfrac{\sqrt{\omega-4Bpk_1(k_1^2+Ck_3^2)}}{2\sqrt{BCpk_1}}\end{aligned} \tag{19}$$

Case II

$$\begin{aligned}&a_0=-\tfrac{\omega}{Ak_1},\quad a_1=a_3=0,\quad a_2=-\tfrac{6Bq(k_1^2+C(k_2^2+k_3^2))}{A},\\&b_1=b_2=b_3=b_4=0,\\&a_4=-\tfrac{12Br(k_1^2+C(k_2^2+k_3^2))}{A},\quad c_2=c_4=d_1=d_2=d_3=d_4=0,\\&c_3=\tfrac{12B(k_1^2+C(k_2^2+k_3^2))\sqrt{r}}{A},\quad p=-\tfrac{\omega}{4Bk_1(k_1^2+C(k_2^2+k_3^2))}\end{aligned} \tag{20}$$

Substituting Eqs. 19,20 into (9), the electrostatic potential of Eq. (6) can be obtained as a ion-acoustic solitary wave solutions as:

Case I

$$\phi_1(x,y,z,t)=\frac{\omega}{2Ak_1q^2}\Big(q^2-6pr-3q\sqrt{pr}\,sech^2\left[\tfrac{\sqrt{p}}{2}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0\right]+3(q^2-4pr)tanh\left[\tfrac{\sqrt{p}}{2}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0\right]-6prtanh^2\left[\tfrac{\sqrt{p}}{2}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0\right]\Big) \tag{21}$$

$$\phi_2(x,y,z,t)=\frac{\omega}{2Ak_1p(4pr-q^2)\left(\frac{q}{\sqrt{q^2-4pr}}-cosh[2\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)^2}\Big(3pq^2+20rp^2+\tfrac{24rp^2q^2-6pq^4}{q^2-4pr}+2pq\sqrt{q^2-4pr}\,cosh[2\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]+p(q^2-4\,pr)cosh[4\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0](48p^2r-12pq^2)\sqrt{\tfrac{pr}{q^2-4pr}}sinh[2\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\Big) \tag{22}$$

$$\phi_3(x,y,z,t)=-\frac{\omega}{Ak_1\left(q-\sqrt{q^2-4pr}sin[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)^2}(-2(q^2-6pr)+6\sqrt{pr(4pr-q^2)}cos[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]+q\sqrt{q^2-4pr}\,sin[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0](q^2-4pr)sin^2[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]) \tag{23}$$

Case II

$$\phi_1(x,y,z,t)=-\frac{\omega}{Ak_1}-\frac{6Bp}{Aq}(k_1^2+C(k_2^2+k_3^2))\sqrt{pr}\,sech^2[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]+6Bp(k_1^2+C(k_2^2+k_3^2))(1+tanh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0])-\frac{12}{q^2}Brp^2(k_1^2+C(k_2^2+k_3^2))(1+tanh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0])^2 \tag{24}$$

$$\phi_2(x,y,z,t)=-\frac{\omega}{Ak_1}-\frac{48Brp^2(k_1^2+C(k_2^2+k_3^2))}{A(q^2-4pr)\left(\frac{q}{\sqrt{q^2-4pr}}-cosh[2\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)^2}+\frac{12Bpq(k_1^2+C(k_2^2+k_3^2))}{A\left(q-\sqrt{q^2-4pr}cosh[2\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)}+\frac{48Bp(k_1^2+C(k_2^2+k_3^2))\sqrt{pr}tanh[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]}{A\sqrt{q^2-4pr}\left(-2+\left(1+\frac{q}{\sqrt{q^2-4pr}}\right)sech^2[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)^2}\times sech^2[\sqrt{p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0] \tag{25}$$

$$\phi_3(x,y,z,t)=\frac{1}{Ak_1\left(q-\sqrt{q^2-4pr}sin[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\right)^2}\Big(12Bpk_1(k_1^2+C(k_2^2+k_3^2))(q^2-4pr)+\omega q^2-24pBk_1(k_1^2+C(k_2^2+k_3^2))\sqrt{p(4pr-q^2)}\,cos[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]-2q\sqrt{q^2-4pr}(6Bpk_1(k_1^2+C(k_2^2+k_3^2))p+\omega)\,sin[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]+(q^2-4pr)\omega\,sin^2[2\sqrt{-p}(k_1x+k_2y+k_3z-\omega\tau)+\theta_0]\Big) \tag{26}$$

The electrostatic potentials defined in the pervious cases are a Hamiltonian system for which the momentum is given by

$$M = \lim_{s\to\infty} \frac{1}{2} \int_0^s \phi_i^2 d\theta, \tag{27}$$

where $i = 1, 2, 3$. The sufficient condition for ion acoustic solitary wave solutions stability is

$$\frac{\partial M}{\partial \omega} > 0. \tag{28}$$

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