

Available online at www.sciencedirect.com**ScienceDirect**

Procedia Engineering 66 (2013) 474 – 488

**Procedia
Engineering**www.elsevier.com/locate/procedia

5th Fatigue Design Conference, Fatigue Design 2013

The mobility of principal stress directions in Crossland criterion

Bertin Soh Fotsing^a, Bienvenu Kenmeugne^b, Médard Fogue^a,Kevin M. Tsapi Tchoupou^{a*}*a*Department of Mechanical Engineering, IUT Fotso Victor – University of Dschang.*P.O. Box 134 Bandjoun – Cameroon.**b*Department of Mechanical and industrial Engineering, National Polytechnic Advanced School –
University of Yaounde 1. *P.O. Box 8390 Yaoundé – Cameroon.*

Abstract

This paper is a new proposal of a multiaxial fatigue criterion which takes better into account the mobility of principal directions of stress tensor. The model proposed is based on the Crossland Criterion and start by defining an equivalent stress with zero out-of-phase angles for the stress components with a phase difference, which apply to Crossland criterion gives the Crossland* criterion. Poor predictions are obtained from Crossland criterion for stress states with mobile principal stress directions. We impute this discrepancy not on the capacity of parameters appearing in the criterion to account for the complexity of the loading or to account the additional damage generate by the mobility of principal stress directions, but instead on the procedure of evaluation of parameters appearing in the criterion and also on the formulation of linear combinations of two scalar parameters proposed by Crossland. Assuming that the second invariant of the stress deviator and maximum hydrostatic pressure can account of the additional damage generate by the mobility of principal stress directions.

The re-formulated Crossland criterion gives good predictions of experimental data when stress states present mobility of principal stress directions. The results show that the predictions obtained using in general the equivalent stress should be more satisfactory than the initial Crossland criterion and Crossland.

© 2013 The Authors. Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/4.0/).
Selection and peer-review under responsibility of CETIM

* Corresponding author.
E-mail address: sohfotsing@aol.fr

1. Introduction

Poor predictions are obtained from Crossland criterion for stress states with mobile principal stress directions [1]. We impute this discrepancy not on the capacity of parameters appearing in the criterion to account for the complexity of the loading or to account the additional damage generate by the mobility of principal stress directions, but instead on the procedure of evaluation of parameters appearing in the criterion and also on the formulation of linear combinations of two scalar parameters proposed by Crossland [2], [3]. Assuming that the second invariant of the stress deviator and maximum hydrostatic pressure can account of the additional damage generate by the mobility of principal stress directions.

Experimental fatigue limit data are used in this work to validate the current fatigue criterion [4]. The experimental data was generated under multiaxial fatigue loading with fixed and mobile principal stress directions: presence of non-zero mean stresses as well as non-zero out-of-phase angles. The accuracy in estimating fatigue limit of the multiaxial fatigue criterion has been quantified according to the fatigue strength error index (ΔI) [4].

The only available data are those of Mielke, reported by, Heindenreich et al [5], Mc Diarmid [6], [7], Froustey [8], and Dietmann et al [9].

Mielke noted that the fatigue limit decreased for frequency ratios higher than $\lambda_Y = 1$ and $\lambda_{XY} = 1$. Similar results were obtained by Heindenreich et al [5].

MacDiarmid results showed decreasing fatigue limits for increasing frequency ratios.

Dietmann et al. [9] reported results for biaxial sinusoidal, triangular and trapezoidal loadings, observed in all cases a decrease of the fatigue limit with respect to the in-phase loadings at the same frequency.

Multiaxial fatigue test results presented in the data bases are those conducted with loads having different frequencies are presented by Bernabes [10], Banvilet [11] and A. Bernasconi et al, [12].

The comparison between the experimental fatigue limit and the calculated ones obtained from Crossland fatigue criteria showed clearly his poor predictions capacity for stress states presenting non-zero out-of-phase angles and non-zero mean stresses.

It has long been recognised that changing of the principal stress directions influences fatigue phenomena. The account of mobility of principal stress directions is the object of many works in the literature [10], [11], [12], [13], [3], [14]. Many parameters are the causes of change over time of principal stress directions the fatigue loading. Among fatigue tests, these parameters we name: discontinuity of variation of principal stress directions, triaxiality, out-of-phase loading, loads with different frequency, and mean stresses.

2. Material and methods

This work was carried out by using the tests found in the literature's databases.

The data collected on sinusoidal multiaxial fatigue experiments for a number of different materials and reported were considered to assess the proposed criterion in predicting fatigue strength error index under a high number of cycles for various values of parameter n . We also assess Crossland criterion when using the equivalent stress, relation (2) for the real stress acting on a material point M, assumed to be the critical one for the component integrity [4].

The study of the influence of defined parameter in predicting fatigue strength error index trough Crossland* criterion and the re-formulated Crossland criterion is done for several values of parameter.

The predictions of fatigue strength provided by Crossland, Crossland*, re-formulated Crossland criteria, for several materials under in-phase and out-of-phase alternated bending and torsion loading are reported in tables V-IX.

3. Mobility of principal stress directions theory

3.1. Predictions of Crossland criterion

Table I, table II and table III report predictions of fatigue strength error index provided by Crossland criterion, for various materials under in-phase and out-of-phase traction-torsion, flexion-torsion, traction-compression loadings [4]. Analysis of these data revealed:

- When the mobility of principal stress directions is caused by the presence of non-zero mean stresses, Crossland criterion show, in general, satisfactory estimate of fatigue damage evolution in a material. The fatigue strength error index is close to zero ;
- When the mobility of principal stress directions is caused by the presence non-zero out-of-phase angles or the combine effect of non-zero mean stresses and non-zero out-of-phase angles, Crossland criterion yielded quite poor predictions ; in general the results are non conservative ;
- When the principal stress directions are fixed, Crossland criterion rendered excellent results as reported in table I.

Table I. Predictions of Crossland criterion for loadings with fixed principal stress directions (20 tests).

$ \Delta I $	Crossland
5%	80%
10%	100%

Table II. Predictions of Crossland criterion for loadings with mobile principal stress directions (41 tests)

Cause of mobility	$ \Delta I $	Crossland
Non-zero mean stress (7 tests)	5%	62.5%
	10%	75%
	15%	87.5%
Non -zero out-of-phase (22 tests)	5%	40.91%
	10%	63.64%
	15%	81.82%
Combine effect of non-zero mean stresses and non-zero out-of-phase (12 tests)	5%	8.33%
	10%	25%
	15%	58.33%

Table III. Recapitulative of Crossland predictions for loadings with mobile principal stress directions (41 tests)

$ \Delta I $	Crossland
5%	36.59%
10%	56.10%
15%	76.61%

3.2. First proposal taking into account the mobility of principal directions of stress

We propose a new procedure for the evaluation of parameters appearing in Crossland criterion in such a way that the poor predictions, as reported in table IV should be more conservative; with a fatigue strength error index close to zero.

Thus, a new procedure is proposed by defining an equivalent stress with zero out-of-phase angles for the stress components with a phase difference. The equivalent stress is normalised in such a way that Crossland predictions are identical in case of in phase loadings. Therefore, we associate to the shear stress τ , defined at the generic material point M an equivalent shear stress τ_{eq} defined such as:

$$\tau = \tau_m + \tau_a \sin(\omega t - \varphi) \tag{1}$$

$$\tau_{eq} = \tau_m + \tau_a (|\cos \beta + \sin \beta|)^n \sin(\omega t) \tag{2}$$

With n a real and

$$\beta = (\delta_{0\varphi} - 1 + \varphi) \tag{3}$$

Where δ is Kronecker symbol

Later on, Crossland* criterion shall represent Crossland criterion when applied using the above equivalent stress.

Table IV. Results with quite poor predictions

Tests	2-3	2-9	2-11	3-2	3-6	3-9	4-2	4-6	4-9
$\Delta I^*(-1)$	22.93	23.7	25.5	28.14	28.89	23.99	27.27	25.12	14.97

3.3. Re-formulation of Crossland criterion

As underlined when interpreting the results of the assessment of Crossland criterion, we now propose a re-formulation of Crossland criterion, by combining the two scalar parameters such that, predictions obtained using in general the equivalent stress should be more satisfactory with the re-formulated criterion than those obtained using criteria Crossland and Crossland*. The following expression of the re-formulated Crossland fatigue strength is then proposed:

$$E_{N.F.Cr} = \frac{\sqrt{|J_{2a} + a (P_{\max})^2 \text{sign}(P_{\max})|}}{b} \tag{4}$$

$E_{N.F.Cr}$ is the fatigue strength defining the New Formulation of Crossland criterion.

3.3.1. Determination of material parameters

1st case: under fully-reversed torsional fatigue loading (fatigue limit τ_{-1})

$$\sigma(t) = \begin{bmatrix} 0 & \tau_{-1} \sin(\omega t) & 0 \\ \tau_{-1} \sin(\omega t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

In the coordinate system of the principal stresses, the above stress state can be rewritten as:

$$\sigma_p(t) = \begin{bmatrix} \tau_{-1} \sin(\omega t) & 0 & 0 \\ 0 & -\tau_{-1} \sin(\omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

$$Max_t[P(t)] = Max_t \left(\frac{(\tau_{-1} - \tau_{-1}) \sin \omega t}{3} \right) = 0 \quad (7)$$

$$J_{2a} = \frac{1}{6} \left[(2\tau_{-1})^2 + (\tau_{-1})^2 + (-\tau_{-1})^2 \right] = (\tau_{-1})^2 \quad (8)$$

Thus: when $E_{N.F.Cr} = 1$, we obtained $b = \tau_{-1}$.

2nd case: under fully-reversed uniaxial fatigue loading (fatigue limit σ_{-1})

$$\sigma(t) = \begin{bmatrix} \sigma_{-1} \sin(\omega t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$Max_t[P(t)] = Max_t \left(\frac{(\sigma_{-1}) \sin \omega t}{3} \right) = \frac{\sigma_{-1}}{3} \quad (10)$$

$$J_{2a} = \frac{1}{6} \left[2(\sigma_{-1})^2 \right] = \frac{(\sigma_{-1})^2}{3} \quad (11)$$

Thus: when $E_{N.F.Cr} = 1$, we then obtained: $a = 3 \left[-1 + 3(\tau_{-1}/\sigma_{-1})^2 \right]$.

3.3.2. Definition domain of the criterion

Conditions $sign(P_{max})$ and $a > 0$ were introduced in other to conserve the positive effect of a negative hydrostatic pressure. The definition domain of a criterion is determined by the interval where the ratio $\tau_{-1}(N)/\sigma_{-1}(N)$ has positive values. With this condition, the proposed criterion is valid when τ_{-1}/σ_{-1} is greater than $1/\sqrt{3}$.

4. Results

The quality of the predictions deduced by the criteria examined can be evaluated through fatigue strength error index, ΔI . The absolute value of the error index ($|\Delta I|$) is defined and the number of experimental tests whose error index falls in each interval of 5% range is considered with respect to the values of parameter n and the total number of tests. The results of such an analysis are shown in table V to table IX.

Table V. Predictions for loadings with fixed principal stress directions (20 tests)

$ \Delta I $	Crossland	Crossland*	N.F. of. Crossland Criterion
5%	80%	80%	85%
10%	100%	100%	100%

The results for loadings with mobile principal stress directions are reported in the tables bellow with respect to some causes of mobility of principal stress directions like non-zero out-of-phase angles, non-zero mean stresses or the combine effect of non-zero mean stresses and non-zero out-of-phase angles.

Table VI.a Crossland* predictions for stresses with non-zero out-of-phase (22 tests)

$ \Delta I $	Crossland	Values of parameter n in Crossland* criterion				
		1/2	1/4	1/8	1/16	1/32
5%	40.91%	27.27%	31.82%	54.55%	54.55%	63.64%
10%	63.64%	59.09%	95.45%	100%	100%	100%
15%	81.82%	90.91%	100%			

Table VI.b N.F. Crossland predictions for stresses with non-zero out-of-phase (22 tests)

$ \Delta I $	Crossland	Values of parameter n in N.F of Crossland criterion				
		1/2	1/4	1/8	1/16	1/32
5%	40.91%	36.36%	50%	63.64%	77.27%	95.45%
10%	63.64%	59.09%	95.45%	95.45%	95.45%	95.45%
15%	81.82%	90.91%	100%	100%	100%	100%

Table VII. Predictions for non-zero mean stresses (7 tests)

$ \Delta I $	Crossland	Crossland*	N.F of Crossland criterion
5%	71.43%	71.43%	85.71%
10%	85.71%	85.71%	85.71%

Table VIII.a Crossland* predictions for combine effect of non-zero mean stresses and non-zero out-of-phase (12 tests)

$ \Delta I $	Crossland	Values of parameter n in Crossland* criterion				
		1/2	1/4	1/8	1/16	1/32
5%	8.33%	41.67%	50%	58.33%	66.67%	66.67%
10%	25%	91.67%	100%	100%	100%	100%
15%	58.33%	100%				

Table VIII.b N.F. Crossland for combine effect of non-zero mean stresses and non-zero out-of-phase (12 tests)

$ \Delta I $	Crossland	Values of parameter n in N.F of Crossland criterion				
		1/2	1/4	1/8	1/16	1/32
5%	8.33%	66.67	75%	83.33%	83.33%	83.33%
10%	25%	83.33%	100%	100%	100%	100%
15%	58.33%	100				

Tables IX.a-b give recapitulative of predictions for loadings with mobility of principal stress directions at any material point M, assumed to be the critical one for the component integrity, using Crossland criterion, the proposed equivalent stress in predictions with Crossland criterion (Crossland* criterion) and finally the New Formulation of Crossland criterion (N.F. of Crossland criterion) or re-formulated Crossland* criterion. The tables revealed the criteria that show, in general, satisfactory predictions of fatigue strength error index, regardless of the mobility of principal stress directions.

Table IX.a Recapitulative of Crossland* predictions for loadings with mobile principal stress directions (41 tests)

$ \Delta I $	Crossland	Values of parameter n in Crossland* criterion				
		1/2	1/4	1/8	1/16	1/32
5%	36.59%	39.02%	43.58%	58.54%	60.98%	65.85%
10%	56.10%	73.17%	95.12%	97.56%	97.56%	97.56%
15%	76.61%	92.68%	97.56%			

Table IX.b Recapitulative of N.F. Crossland* predictions for loadings with mobile principal stress directions (41 tests)

$ \Delta I $	Crossland	Values of parameter n in N.F of Crossland criterion				
		1/2	1/4	1/8	1/16	1/32
5%	36.59%	53.66%	63.41%	73.17%	68.69%	73.17%
10%	56.10%	70.73%	95.12%	95.12%	97.56%	97.56%
15%	76.61%	96.68%	97.56%	97.56%		

5. Analysis and discussion of results

We now compare graphically for various values of parameter n , predictions obtained from Crossland criterion, Crossland* criterion and re-formulated Crossland criterion. Figures 1-2 give histogram of comparison of the absolute value of fatigue strength error index respectively when $|\Delta I| = 5\%$ and, $|\Delta I| = 10\%$ for $n = 1/2, 1/4, 1/8, 1/16, 1/32$. Figures 3-5 give variations of the of fatigue strength error index with respect to the total number of tests when $n=1/32$.

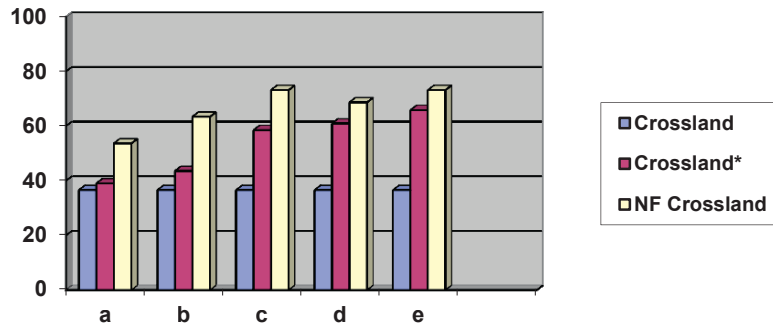


Figure 1: Histogram of comparison of fatigue strength error index ($|\Delta L| = 5\%$) when n takes respectively values: $a=1/2$, $b=1/4$, $c=1/8$, $d=1/16$, $e=1/32$.

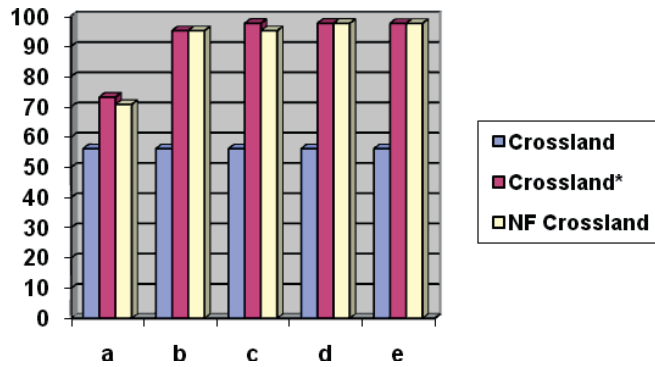


Figure 2: Histogram of comparison of fatigue strength error index ($|\Delta L| = 10\%$) when n takes respectively values: $a=1/2$, $b=1/4$, $c=1/8$, $d=1/16$, $e=1/32$.

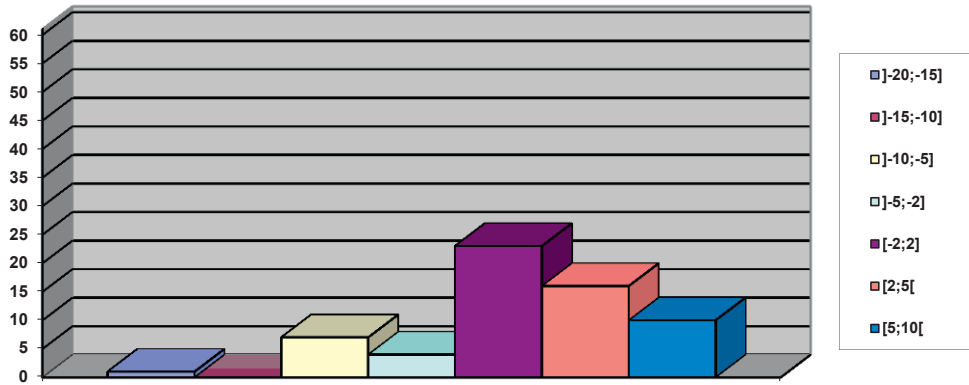


Figure 3: Total number of tests of the fatigue strength error index according to Crossland* criterion (n=1/32)

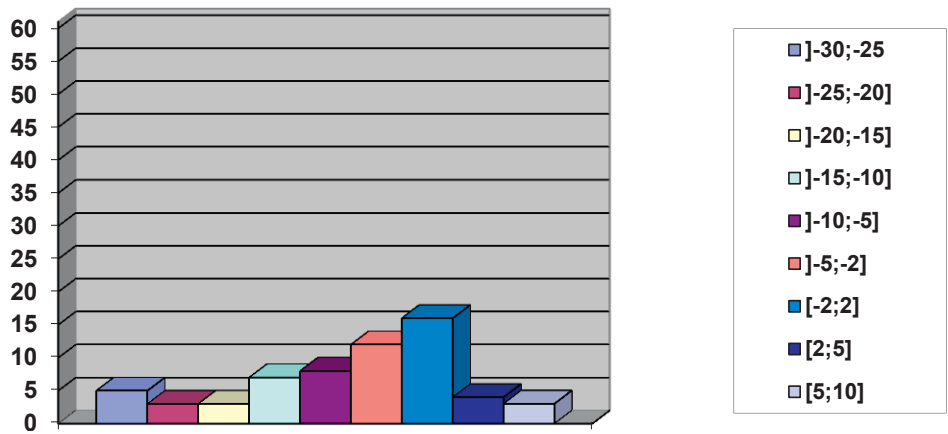


Figure 4: Total number of tests of the fatigue strength error index according to Crossland criterion (n=1/32)

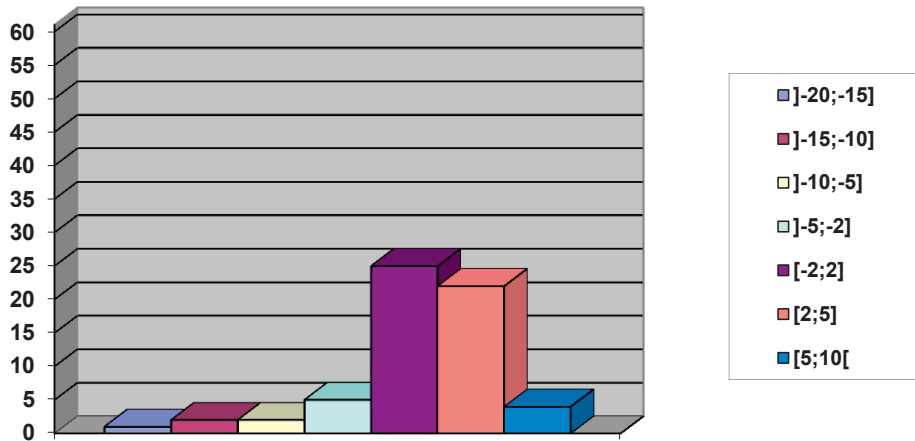


Figure 5: Total number of tests of the fatigue strength error index according to the N.F. of Crossland criterion ($n=1/32$)

From the recapitulative, we can make the following remarks:

- The influence of parameter n is meaningful for loadings with mobile principal stress directions, especially when the cause of mobility of principal stress directions is non-zero out-of-phase angles.
- The closest value to zero of fatigue strength error index for both mobile and fixed principal stress directions are obtained when $n = 1/32$; independently of the value of parameter n , the re-formulated Crossland criterion always provided best predictions estimates than all the other criteria considered in the present study.
- Dependent upon the value of parameter n , application of the proposed equivalent stress led Crossland criterion to yield good predictions when ever the principal stress directions are mobile.
- Analysis of results presented in Appendix shows that parameter n should be correlated to the material of study.
- We think that optimal predictions should be obtained by defining for each material, parameter n which yields closest results to experimental data.
- For stress states with mobile principal stress directions, Crossland* criterion yielded better predictions estimates than Crossland criterion; however the best predictions estimates were obtained with the re-formulate Crossland criterion; especially when $|\Delta I| = 5\%$ as reported in table IX.

6. Conclusion

Implementation of Crossland criterion through an equivalent stress, defined in the proposed procedure to represent the real stress acting on a material point M was studied in the present chapter. Taking inspiration from Crossland criterion, a new formulation Crossland criterion was proposed. The re-formulated Crossland criterion gives good predictions of experimental data when stress states present mobility of principal stress directions.

For two criteria formulated from the same parameters, fatigue predictions can highly be ameliorated.

There is a correlation between the fatigue predictions and the procedure that was chosen to evaluate parameters appearing in the criterion. The quality of results obtained from the proposed procedure depend closely on the value of parameter n .

References

- [1] Fogue M., Critère de fatigue a longue durée de vie des états multiaxiaux de contraintes sinusoïdales en phase et hors phase. Thèse de doctorat Institut National des Sciences Appliquées de Lyon. 1987.
- [2] Crossland B., Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel, Proceedings of the International Conference on Fatigue of Metals, Institution of Mechanical Engineers, London, 1956.
- [3] Weber B., Fatigue multiaxiale des structures industrielles sous chargement quelconque, Thèse de Doctorat, Institut National des Sciences Appliquées de Lyon, 1999.
- [4] Kevin Martial Tsapi Tchoupou. Prise en compte des variations des directions principales des contraintes par les critères de fatigue multiaxiaux. Thèse de Master. Université de Dschang. 2011. 75 P.
- [5] R. Heidenreich, I. Richter and H. Zenner, Schubspannungsintensitätshypothese – weitere experimentelle und theoretische Untersuchungen. Konstruktion, Vol. 36, 1984, pp. 99-104.
- [6] D.L. McDiarmid. “Fatigue under out-of-phase biaxial stresses of different frequencies”. In: Multiaxial Fatigue, ASTM STP 853, K.M. Miller and M.W. Brown editors, ASTM, Philadelphia, 1985, pp. 606-621.
- [7] D.L. McDiarmid. “Mean stress effects in biaxial fatigue where the stresses are out-of-phase and at different frequencies”. In: Fatigue under Biaxial and Multiaxial Loadings, ESIS10. K. Kussmaul, D.L. McDiarmid and D. Socie editors, Mechanical Engineering Publications, London, 1991, pp. 321-335.
- [8] C. Froustey. “Fatigue multiaxiale en endurance de l’acier 30 NCD 16”. Thèse de l’Ecole Nationale Supérieure d’Arts et Métiers, Bordeaux, 1987.
- [9] H. Dietmann, T. Bhonghibhat, A. Schmid. “Multiaxial fatigue behaviour of steels under in-phase and out-of-phase loading, including different wave forms and frequencies”. In: Fatigue under Biaxial and Multiaxial Loadings, ESIS 10. K. Kussmaul, D.L. McDiarmid and D. Socie editors, Mechanical Engineering Publications, London, 1991, pp. 449-464.
- [10] Jérôme Benabes, Approche énergétique non locale du calcul de durée de vie de structures en fatigue multiaxiale sous chargement d’amplitude variable : application à une roue de train ferroviaire, Thèse de Doctorat, École Nationale Supérieure D’arts et Métiers, Spécialité Mécanique, 2006.
- [11] Alexis Banvillet, Prévion de durée de Vie en Fatigue Multiaxiale Sous Chargements Réels : vers des essais accélérés, Thèse de Doctorat, École Nationale Supérieur D’arts Métiers, Spécialité : Mécanique, 2001.
- [12] A. Bernasconi, S. Foletti I.V. Papadopoulos, Multiaxial fatigue tests under combined torsion and axial load with different frequencies, XXXIV Convegno Nazionale – 14-17 Septembre 2005, Politecnico Di Milano.
- [13] Khaldoun Nasreddine, effet de la rotation des contraintes sur le comportement des sols argileux, Thèse de Doctorat, École Nationale des ponts et chaussées, spécialité géotechnique, 2004.
- [14] Kenmeugne Bienvenu, Contribution à la modélisation du comportement en fatigue sous sollicitations multiaxiales d’amplitude variable, Thèse de Doctorat, Institut National des Sciences Appliquées de Lyon, 1996.

APPENDIX

Presentation of the multiaxial tests of the databases.

For each group of tests, there are presented both the tests with the mobile and fixed principal directions of stress. The later in bold.

Tests number with the mobility of principal directions due to out of phase load. (22 tests) : 1-2, 1-3, 1-4, 1-6, 1-7, 1-8, 1-10, 1-13, 1-16, 1-17, 1-18, 1-19, 1-20, 1-22, 2-2, 2-3, 2-4, 2-5, 2-6, 3-2, 3-4, 4-2.

Tests number with the mobility of principal directions due to combined effect of out of phase load and mean stress. (12 tests) : 2-8, 2-9, 2-11, 2-12, 3-6, 3-7, 3-9, 4-4, 4-5, 4-6, 4-8, 4-9.

Tests number with the mobility of principal directions due to mean stress. (7 tests) : 2-7, 2-10, 3-5, 3-8, 4-3, 4-7, 4-10.

A.1

Material : Hard steel					
Characteristics					
$\sigma_{-1} = 313.9MPa$					
$\tau_{-1} = 196.2MPa$					
$Rm = 680MPa$					
Author : Nishihara T, Kawamoto M. from [5]					
Type of test : bending - torsion					
Series 1	σ_{11a}	σ_{11m}	σ_{12a}	σ_{12m}	ϕ_{12}
1-1	138.1	0	167.1	0	0
1-2	140.4	0	169.9	0	30
1-3	145.7	0	176.3	0	60
1-4	150.2	0	181.7	0	90
1-5	245.3	0	122.6	0	0
1-6	249.7	0	124.6	0	30
1-7	252.4	0	126.2	0	60
1-8	258.0	0	129.0	0	90
1-9	299.6	0	62.8	0	0
1-10	304.5	0	63.9	0	90
1-11	327.7	0	0.0	0	0
1-12	308.0	0	63.9	0	0
1-13	255.1	0	127..5	0	0
1-14	141.9	0	171.3	0	0
1-15	0.0	0	201.1	0	0
1-16	255.1	0	127.5	0	30
1-17	142.0	0	171.3	0	30
1-18	255.1	0	127.5	0	60
1-19	147.2	0	177.6	0	60
1-20	308.0	0	63.9	0	90
1-21	264.9	0	132.4	0	90
1-22	152.5	0	182.2	0	90

A. 2

Authors : Heidenreich R, Zenner H. Richter I. from [5]		Material : 34Cr4 Characteristics $\sigma_{-1} = 410MPa$ $\tau_{-1} = 256MPa$ $Rm = 710MPa$			
Type of test : Bending - torsion					
Series 2	σ_{11a}	σ_{11m}	σ_{12a}	σ_{12m}	ϕ_{12}
2-1	314	0	157	0	0
2-2	315	0	158	0	60
2-3	316	0	158	0	90
2-4	315	0	158	0	120
2-5	224	0	224	0	90
2-6	80	0	95	0	90
2-7	316	0	158	158	0
2-8	314	0	157	157	60
2-9	315	0	158	158	90
2-10	279	279	140	0	0
2-11	284	284	142	0	90
2-12	212	212	212	0	90

A. 3

Authors : Lempp W. from [5]		Material : 45 Mo4 Characteristics $\sigma_{-1} = 398MPa$ $\tau_{-1} = 260MPa$			
Type of test : tension - torsion					
Series 3	σ_{11a}	σ_{11m}	σ_{12a}	σ_{12m}	ϕ_{12}
3-1	328	0	157	0	0
3-2	286	0	137	0	90
3-3	233	0	224	0	0
3-4	213	0	205	0	90
3-5	266	0	128	128	0
3-6	283	0	136	136	90
3-7	333	0	160	160	120
3-8	280	280	134	0	0
3-9	271	271	130	0	90

A. 4

Material : 30NCD16					
Authors : Froustey C, Lasserre S. from [5]			Characteristics		
Type of test : bending - torsion			$\sigma_{-1} = 660MPa$		
			$\tau_{-1} = 410MPa$		
			$R_m = 1160MPa$		
Series 4	σ_{11a}	σ_{11m}	σ_{12a}	σ_{12m}	ϕ_{12}
4-1	485	0	280	0	0
4-2	480	0	277	0	90
4-3	480	300	277	0	0
4-4	480	300	277	0	45
4-5	470	300	270	0	60
4-6	473	300	273	0	90
4-7	590	300	148	0	0
4-8	565	300	141	0	45
4-9	540	300	135	0	90
4-10	211	300	365	0	0

A. 5

Material : St60					
Authors : El Magd, Mielke [11]			Characteristics		
Type of test : Biaxial bending			$\sigma_{-1} = 294MPa$		
			$\tau_{-1} = 176MPa$		
			$R_m = 765MPa$		
Series 5	σ_{11a}	σ_{11m}	σ_{22a}	σ_{22m}	ϕ_{12}
5-1	284	76			
5-2	290			306	
5-3	259			459	
5-4	286	76		306	
5-5	259	76		459	
5-6	290	153		153	
5-7	279	153		306	
5-8	263	153		4569	