# Scotogenic inverse seesaw model of neutrino mass 

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## A R T I C L E I N F O

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#### Abstract

A variation of the original 2006 radiative seesaw model of neutrino mass through dark matter is shown to realize the notion of inverse seesaw naturally. The dark-matter candidate here is the lightest of three real singlet scalars which may also carry flavor.


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In 1998, the simplest realizations of the dimension-five operator [1] for Majorana neutrino mass, i.e. $\left(v_{i} \phi^{0}\right)\left(v_{j} \phi^{0}\right)$, were discussed systematically [2] for the first time. Not only was the nomenclature for the three and only three tree-level seesaw mechanisms established: (I) heavy singlet neutral Majorana fermion $N$ [3], (II) heavy triplet Higgs scalar $\left(\xi^{++}, \xi^{+}, \xi^{0}\right)$ [4], and (III) heavy triplet Majorana fermion $\left(\Sigma^{+}, \Sigma^{0}, \Sigma^{-}\right)$[5], the three generic one-loop irreducible radiative mechanisms involving fermions and scalars were also written down for the first time. Whereas one such radiative mechanism was already well-known since 1980, i.e. the Zee model [6], a second was not popularized until eight years later in 2006, when it was used [7] to link neutrino mass with dark matter, called scotogenic from the Greek scotos meaning darkness. The third remaining unused mechanism is the subject of this paper. It will be shown how it is a natural framework for a scotogenic inverse seesaw model of neutrino mass, as shown in Fig. 1. The new particles are three real singlet scalars $s_{1,2,3}$, and one set of doublet fermions $\left(E^{0}, E^{-}\right)_{L, R}$, and one Majorana singlet fermion $N_{L}$, all of which are odd under an exactly conserved discrete symmetry $Z_{2}$. This specific realization was designated T1-3-A with $\alpha=0$ in the compilation of Ref. [8]. Note however that whereas $\left(E^{0}, E^{-}\right)_{L}$ is not needed to complete the loop, it serves the dual purpose of (1) rendering the theory to be anomaly-free and (2) allowing $E$ to have an invariant mass for the implementation of the inverse seesaw mechanism.

The notion of inverse seesaw [9-11] is based on an extension of the $2 \times 2$ mass matrix of the canonical seesaw to a $3 \times 3$ mass matrix by the addition of a second singlet fermion. In the space spanned by $(v, N, S)$, where $v$ is part of the usual lepton doublet $(\nu, l)$ and $N, S$ are singlets, all of which are considered left-handed, the most general $3 \times 3$ mass matrix is given by

[^0]

Fig. 1. One-loop generation of inverse seesaw neutrino mass.
$\mathcal{M}_{\nu}=\left(\begin{array}{ccc}0 & m_{2} & 0 \\ m_{2} & m_{N} & m_{1} \\ 0 & m_{1} & m_{S}\end{array}\right)$.
The zero $v-S$ entry is justified because there is only one $v$ to which $N$ and $S$ may couple through the one Higgs field $\phi^{0}$. The linear combination which couples may then be redefined as $N$, and the orthogonal combination which does not couple is $S$. If $m_{S, N}$ is assumed much less than $m_{1}$, then the induced neutrino mass is
$m_{\nu} \simeq \frac{m_{2}^{2} m_{S}}{m_{1}^{2}}$.
This formula shows that a nonzero $m_{v}$ depends on a nonzero $m_{S}$, and a small $m_{v}$ is obtained by a combination of small $m_{S}$ and $m_{2} / m_{1}$. This is supported by the consideration of an approximate symmetry, i.e. lepton number $L$, under which $v, S \sim+1$ and $N \sim-1$. Thus $m_{1,2}$ conserve $L$, but $m_{S}$ breaks it softly by 2 units. Note that there is also a finite one-loop contribution from $m_{N}$ [12,13].

Other assumptions about $m_{1}, m_{S}, m_{N}$ are also possible [14]. If $m_{2}, m_{N} \ll m_{1}^{2} / m_{S}$ and $m_{1} \ll m_{S}$, then a double seesaw occurs with the same formula as that of the inverse seesaw, but of course with


Fig. 2. One-loop generation of seesaw neutrino mass with heavy Majorana $N$.
a different mass hierarchy. If $m_{1}, m_{2} \ll m_{N}$ and $m_{1}^{2} / m_{N} \ll m_{S} \ll$ $m_{1}$, then a lopsided seesaw [14] occurs with $m_{v} \simeq-m_{2}^{2} / m_{N}$ as in the canonical seesaw, but $v-S$ mixing may be significant, i.e. $m_{1} m_{2} / m_{S} m_{N}$, whereas $v-N$ mixing is the same as in the canonical seesaw, i.e. $\sqrt{m_{v} / m_{N}}$. In the inverse seesaw, $v-N$ mixing is even smaller, i.e. $m_{v} / m_{2}$, but $v-S$ mixing is much larger, i.e. $m_{2} / m_{1}$, which is only bounded at present by about 0.03 [15]. In the double seesaw, the effective mass of $N$ is $m_{1}^{2} / m_{S}$, so $v-N$ mixing is also $\sqrt{m_{v} / m_{N}}$. Here $m_{S} \gg m_{N}$, so the $v-S$ mixing is further suppressed by $m_{1} / m_{S}$.

In the original scotogenic model [7], neutrino mass is radiatively induced by heavy neutral Majorana singlet fermions $N_{1,2,3}$ as shown in Fig. 2. However, they may be replaced by Dirac fermions. In that case, a $U(1)_{D}$ symmetry may be defined [16], under which $\eta_{1,2}$ transform oppositely. If $Z_{2}$ symmetry is retained, then a radiative inverse seesaw neutrino mass is also possible [17,18]. We discuss here instead the new mechanism of Fig. 1, based on the third one-loop realization of neutrino mass first presented in Ref. [2]. The smallness of $m_{N}$, i.e. the Majorana mass of $N_{L}$, may be naturally connected to the violation of lepton number by two units, as in the original inverse seesaw proposal using Eq. (1). It may also be a two-loop effect as first proposed in Ref. [19], with a number of subsequent papers by other authors, including Refs. [20-22].

In our model, lepton number is carried by $\left(E^{0}, E^{-}\right)_{L, R}$ as well as $N_{L}$. This means that the Yukawa term $\bar{N}_{L}\left(E_{R}^{0} \phi^{0}-E_{R}^{-} \phi^{+}\right)$is allowed, but not $N_{L}\left(E_{L}^{0} \phi^{0}-E_{L}^{-} \phi^{+}\right)$. In the $3 \times 3$ mass matrix span$\operatorname{ning}\left(\bar{E}_{R}^{0}, E_{L}^{0}, N_{L}\right)$, i.e.
$\mathcal{M}_{E, N}=\left(\begin{array}{ccc}0 & m_{E} & m_{D} \\ m_{E} & 0 & 0 \\ m_{D} & 0 & m_{N}\end{array}\right)$,
$m_{E}$ comes from the invariant mass term $\left(\bar{E}_{R}^{0} E_{L}^{0}+E_{R}^{+} E_{L}^{-}\right), m_{D}$ comes from the Yukawa term given above connecting $N_{L}$ with $E_{R}^{0}$ through $\left\langle\phi^{0}\right\rangle=v$, and $m_{N}$ is the soft lepton-number breaking Majorana mass of $N_{L}$. Assuming that $m_{N} \ll m_{D}, m_{E}$, the mass eigenvalues of $\mathcal{M}_{E, N}$ are
$m_{1}=\frac{m_{E}^{2} m_{N}}{m_{E}^{2}+m_{D}^{2}}$,
$m_{2}=\sqrt{m_{E}^{2}+m_{D}^{2}}+\frac{m_{D}^{2} m_{N}}{2\left(m_{E}^{2}+m_{D}^{2}\right)}$,
$m_{3}=-\sqrt{m_{E}^{2}+m_{D}^{2}}+\frac{m_{D}^{2} m_{N}}{2\left(m_{E}^{2}+m_{D}^{2}\right)}$.
In the limit $m_{N} \rightarrow 0, E_{R}^{0}$ pairs up with $E_{L}^{0} \cos \theta+N_{L} \sin \theta$ to form a Dirac fermion of mass $\sqrt{m_{E}^{2}+m_{D}^{2}}$, where $\sin \theta=m_{D} / \sqrt{m_{E}^{2}+m_{D}^{2}}$. This means that the one-loop integral of Fig. 1 is well approximated by
$m_{\nu}=\frac{f^{2} m_{D}^{2} m_{N}}{16 \pi^{2}\left(m_{E}^{2}+m_{D}^{2}-m_{S}^{2}\right)}\left[1-\frac{m_{s}^{2} \ln \left(\left(m_{E}^{2}+m_{D}^{2}\right) / m_{S}^{2}\right)}{\left(m_{E}^{2}+m_{D}^{2}-m_{S}^{2}\right)}\right]$.

This expression is indeed of the form expected of the inverse seesaw.

The radiative mechanism of Fig. 1 is also suitable for supporting a discrete flavor symmetry, such as $Z_{3}$. Consider the choice

$$
\begin{array}{ll}
\left(v_{i}, l_{i}\right)_{L} \sim \underline{1}, \underline{1}^{\prime} & \underline{1}^{\prime \prime}, \\
\left(s_{2} \sim i s_{3}\right) / \sqrt{2} \sim \underline{1}^{\prime}, & \left(s_{2}-i s_{3}\right) / \sqrt{2} \sim \underline{1}^{\prime \prime} \tag{8}
\end{array}
$$

with mass terms $m_{s}^{2} s_{1}^{2}+m_{s}^{2}\left(s_{2}^{2}+s_{3}^{2}\right)$, then the induced $3 \times 3$ neutrino mass matrix is of the form

$$
\begin{align*}
& \mathcal{M}_{\nu}=\left(\begin{array}{ccc}
f_{e} & 0 & 0 \\
0 & f_{\mu} & 0 \\
0 & 0 & f_{\tau}
\end{array}\right)\left(\begin{array}{ccc}
I\left(m_{s}^{2}\right) & 0 & 0 \\
0 & 0 & I\left(m_{s}^{\prime 2}\right) \\
0 & I\left(m_{s}^{\prime 2}\right) & 0
\end{array}\right) \\
& \times\left(\begin{array}{ccc}
f_{e} & 0 & 0 \\
0 & f_{\mu} & 0 \\
0 & 0 & f_{\tau}
\end{array}\right) \\
& =\left(\begin{array}{ccc}
f_{e}^{2} I\left(m_{s}^{2}\right) & 0 & 0 \\
0 & 0 & f_{\mu} f_{\tau} I\left(m_{s}^{\prime 2}\right) \\
0 & f_{\mu} f_{\tau} I\left(m_{s}^{\prime 2}\right) & 0
\end{array}\right), \tag{9}
\end{align*}
$$

where $I$ is given by Eq. (7) with $f^{2}$ removed. Let $l_{i R} \sim \underline{1}, \underline{1}^{\prime}, \underline{1}^{\prime \prime}$, then the charged-lepton mass matrix is diagonal using just the one Higgs doublet of the standard model, in keeping with the recent discovery $[23,24]$ of the 125 GeV particle. To obtain a realistic neutrino mass matrix, we break $Z_{3}$ softly, i.e. with an arbitrary $3 \times 3$ mass-squared matrix spanning $s_{1,2,3}$, which leads to

$$
\begin{align*}
& \left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 / \sqrt{2} & i / \sqrt{2} \\
0 & 1 / \sqrt{2} & -i / \sqrt{2}
\end{array}\right) O^{T}\left(\begin{array}{ccc}
I\left(m_{s 1}^{2}\right) & 0 & 0 \\
0 & I\left(m_{s 2}^{2}\right) & 0 \\
0 & 0 & I\left(m_{s 3}^{2}\right)
\end{array}\right) \\
& \times O\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 / \sqrt{2} & 1 / \sqrt{2} \\
0 & i / \sqrt{2} & -i / \sqrt{2}
\end{array}\right), \tag{10}
\end{align*}
$$

where $O$ is an orthogonal matrix but not the identity, and there can be three different mass eigenvalues $m_{s 1, s 2, s 3}$ for the $s_{1,2,3}$ sector. The assumption of Eq. (8) results in Eq. (10) and allows the following interesting pattern for the neutrino mass matrix $\mathcal{M}_{\nu}$. The Yukawa couplings $f_{e, \mu, \tau}$ may be rendered real by absorbing their phases into the arbitrary relative phases between $E_{R}^{0}$ and $v_{e, \mu, \tau}$. If we further assume $f_{\mu}=f_{\tau}$, then $\mathcal{M}_{\nu}$ is of the form [25]
$\mathcal{M}_{\nu}=\left(\begin{array}{ccc}A & C & C^{*} \\ C & D^{*} & B \\ C^{*} & B & D\end{array}\right)$,
where $A$ and $B$ are real. Note that this pattern is protected by a symmetry first pointed out in Ref. [26], i.e. $e \rightarrow e$ and $\mu-\tau$ exchange with $C P$ conjugation, and appeared previously in Refs. [27,28]. As such, it is also guaranteed to yield maximal $v_{\mu}-v_{\tau}$ mixing $\left(\theta_{23}=\pi / 4\right)$ and maximal $C P$ violation, i.e. $\exp (-i \delta)= \pm i$, whereas $\theta_{13}$ may be nonzero and arbitrary. Our scheme is thus a natural framework for this possibility. Further, from Eq. (7), it is clear that it is also a natural framework for quasidegenerate neutrino masses as well. Let
$F(x)=\frac{1}{1-x}\left[1+\frac{x \ln x}{1-x}\right]$,
where $x=m_{s}^{2} /\left(m_{E}^{2}+m_{D}^{2}\right)$, then Eq. (7) becomes
$m_{\nu}=\frac{f^{2} m_{D}^{2} m_{N}}{\left(m_{E}^{2}+m_{D}^{2}\right)} F(x)$.

Since $F(0)=1$ and goes to zero only as $x \rightarrow \infty$, this scenario does not favor a massless neutrino. If $f_{e, \mu, \tau}$ are all comparable in magnitude, the most likely outcome is three massive neutrinos with comparable masses.

Since the charged leptons also couple to $s_{1,2,3}$ through $E^{-}$, there is an unavoidable contribution to the muon anomalous magnetic moment given by [29]
$\Delta a_{\mu}=\frac{(g-2)_{\mu}}{2}=\frac{f_{\mu}^{2} m_{\mu}^{2}}{16 \pi^{2} m_{E}^{2}} \sum_{i}\left|U_{\mu i}\right|^{2} G\left(x_{i}\right)$,
where
$G(x)=\frac{1-6 x+3 x^{2}+2 x^{3}-6 x^{2} \ln x}{6(1-x)^{4}}$,
with $x_{i}=m_{s i}^{2} / m_{E}^{2}$ and
$U=O\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & 1 / \sqrt{2} & 1 / \sqrt{2} \\ 0 & i / \sqrt{2} & -i / \sqrt{2}\end{array}\right)$.
To get an estimate of this contribution, let $x_{i} \ll 1$, then $\Delta a_{\mu}=$ $f_{\mu}^{2} m_{\mu}^{2} / 96 \pi^{2} m_{E}^{2}$. For $m_{E} \sim 1 \mathrm{TeV}$, this is of order $10^{-11} f_{\mu}^{2}$, which is far below the present experimental sensitivity of $10^{-9}$ and can be safely ignored. The related amplitude for $\mu \rightarrow e \gamma$ is given by
$A_{\mu e}=\frac{e f_{\mu} f_{e} m_{\mu}}{32 \pi^{2} m_{E}^{2}} \sum_{i} U_{e i}^{*} U_{\mu i} G\left(x_{i}\right)$.
Using the most recent $\mu \rightarrow e \gamma$ bound [30]
$B=\frac{12 \pi^{2}\left|A_{\mu e}\right|^{2}}{m_{\mu}^{2} G_{F}^{2}}<5.7 \times 10^{-13}$,
and the approximation $\sum_{i} U_{e i}^{*} U_{\mu i} G\left(x_{i}\right) \sim 1 / 36$ (based on tribimaximal mixing with $x_{1} \sim 0$ and $x_{2} \sim 1$ ) and $m_{E} \sim 1 \mathrm{TeV}$, we find
$f_{\mu} f_{e}<0.03$.
Let $f_{e, \mu, \tau} \sim 0.1, m_{N} \sim 10 \mathrm{MeV}, m_{D} \sim 10 \mathrm{GeV}, m_{E} \sim 1 \mathrm{TeV}$, then the very reasonable scale of $m_{v} \sim 0.1 \mathrm{eV}$ in Eq. (7) is obtained, justifying its inverse seesaw origin. Since $N_{L}$ is the lightest particle with odd $Z_{2}$, it is a would-be dark matter candidate. However, suppose we add $N_{R}$ so that the two pair up to have a large invariant Dirac mass, then the lightest scalar (call it $S$ ) among $s_{1,2,3}$ is a dark-matter candidate. It interacts with the standard-model Higgs boson $h$ according to
$-\mathcal{L}_{\text {int }}=\frac{\lambda_{h S}}{2} v h S^{2}+\frac{\lambda_{h S}}{4} h^{2} S^{2}$.
If we assume that all its other interactions are suppressed, then the annihilations $S S \rightarrow h \rightarrow$ SM particles and $S S \rightarrow h h$ determine its relic abundance, whereas its elastic scattering off nuclei via $h$ exchange determines its possible direct detection in underground experiments. A detailed analysis [31] shows that the present limit of the invisible width of the observed 125 GeV particle (identified as $h$ ) allows $m_{S}$ to be only within several GeV below $m_{h} / 2$ or
greater than about 150 GeV using the recent LUX data [32]. Note that the vector fermion doublet ( $E^{0}, E^{-}$) is not the usually considered vector lepton doublet because it is odd under $Z_{2}$ and cannot mix with the known leptons.

In conclusion, we have shown how neutrino mass and dark matter may be connected using a one-loop mechanism proposed in 1998. This scotogenic model is naturally suited to implement the notion of inverse seesaw for neutrino mass, allowing the scale of new physics to be 1 TeV or less. The imposition of a softly broken $Z_{3}$ flavor symmetry yields an interesting pattern of radiative neutrino mass, allowing for maximal $\theta_{23}$ and maximal $C P$ violation. The real singlet scalars in the dark sector carry lepton flavor, the lightest of which is absolutely stable. Our proposal provides thus a natural theoretical framework for this well-studied phenomenological possibility.

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