Space-grid Method in Missile Guidance and Control

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Abstract

Taking guidance and control as the object, a new method named space-grid method was applied in the missile guidance in this paper. Assumed condition and basic theory were introduced firstly. The process and characteristic of the method was showed subsequently. Based on these work, the guidance and control way of space-grid method was analyzed in detail. Through simulation, it proved that the guidance and control way was effective and feasible. It could satisfy precision demand. The space-grid method has advantages of calculation quickly on missile computer, strong ability of antijamming and control simply. For these reasons, the new method has broad application foreground and reference value in the aspects of guidance precision and efficiency.

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1. Introduction

Perturbation guidance is widely used in missile guidance. Compared with other guidance methods, it has advantages of lower demand of guidance computer and lots of data can be calculated before launch. But its theory background is small deviation. When large deviation occurred, guidance error is produced. Along with longer flight time and farther range, the error becomes larger. For adjusting big disturbance to reduce its influence on missile, a new method called space-grid method was applied in missile guidance. This method can modify big disturbance in real time. And the advantages of calculation simply and data small in missile storage makes it has broad research and application foreground. Space-grid method has

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no scheduled flight program. Then guidance and control way is different from perturbation guidance. So the main work is analyzing guidance and control way of the new method. Before this, we should know the assumption and basic theory and the partition rule of this method to set the stage for analyzing the guidance and control way.

2. Description of space-grid method

2.1. Basic principle

Velocity of the space that missile passing by is saved in a storage. When the missile is in this area, the current velocity could be modified by the velocity in the storage. This velocity in the storage is called required velocity. The missile powered off if the current velocity at current position is same as the required velocity at the same position. This is basic principle of the space-grid method. But not all required velocity at any position could be saved in the area missile passing by. So we must set partition principle to carve the area and save state parameters at grid nod. Then the saved state parameters are considered as benchmark to modify velocity deviation. For the demand of structural design and small normal overload, the space-grid method could be used only out space. This restrict makes the method work only at the second stage in powered phase.

2.2. Partition principle

Space-grid boundary, node distance, fiducial coordinates and node parameters are the four elements to build one space grid. Node parameters are the essential one.

- The boundary of space grid
  The change of position along transverse and normal are much shorter than it along longitudinal direction. So the area of space grid is restricted in cuboid region that the longest side is longitudinal direction, the middle side is normal and the shortest side is transverse. For ensuring the precision of space-grid method and avoiding missile track overstepping the boundary of space grid brought by disturbance outside, we should use simulation to choose proper length of the boundary.

- The interval of grid node
  If the interval of grid node is long, it makes the relation among node parameters is weak. Contrarily, if the interval is short, it makes a mass of data and overtime of calculation. Through simulation, we can find the proper interval based on the change of velocity at different nodes and the sensitive degree of position.

- The fiducial coordinate
  The benchmark of computation on missile is inertia space. Fight parameters are under inertia fixed coordinates. If the velocity of space grid is in inertia coordinate, we could compare the current velocity with the space-grid velocity to improve the computation speed. If the node position is in inertia fixed coordinates, it may lead the trend of space grid and trajectory has great difference. Then a lot of data are invalidated, and much calculation is unwanted. So choosing node position in locus coordinate system, it can exert the function of space grid.

- The node parameters
  The node parameters are the key of space-grid method. They contain time, position and velocity. Velocity is the core of the node parameters. Because of rotation of globe, time is a necessary factor to be considered. Different time and space grid corresponded with it make of the space grid group. Compared with the current flight time and the grid time, we can obtain needed space grid.
  Actual flight track can’t as same as standard trajectory well. But the difference doesn’t affect precision. We should save Euler’s angles which connect fiducial locus coordinate system and launch coordinate
system. Then turning current position in inertia fixed coordinates to that in fiducial locus coordinate system.

Based on elliptical orbit theory[1], ellipses through any two space node are incalculability. So restrict of node velocity is required to ensure the track from current position to target is the only one. At the same time, search order and principle of velocity must be set to make sure the change of velocity has some rules to follow. Otherwise no connection among nodes can lead the invalidation of space-grid method. Only changing one direction of velocity to adjust longitudinal and lateral deviation is impossible. So one direction of velocity which defined by $v_{\text{const}}$ is fixed. The other two directions of velocity which defined by $v_{\text{alter}}$ can be changed. Modifying longitudinal deviation firstly and lateral deviation secondly until we get the node velocity which satisfied impact point precision.

By the analysis above all, we can find the space-grid method is a discretization method using several groups of space grids laid on time, position and velocity. From simulation, velocity and position on any two directions show the relationship of double bilinearity. Space grids in different time are parallel. So the required velocity could be obtained by linear interpolation. Then the difference between required velocity and current velocity is considered as the sign of guidance and control to modify the disturbance. But how to work out the directions of attitude angles and the value of control force is a question to be solved. Next part is the analysis of detailed guidance and control way of space-grid method.

3. Guidance and control way

3.1. Calculation of required velocity

The guidance rule of space-grid method is the direction of acceleration $\mathbf{a}$ and the needed increased velocity $\mathbf{v}_g$ are same[2]-[3]. $\mathbf{v}_g$ is the difference of required velocity $\mathbf{v}_r$ and current velocity $\mathbf{v}$. For the ceaseless change of missile position, it makes the change of required velocity. So the guidance way is not the optimization one. But the guidance way can satisfy the precision demand.

Suppose $v_{\text{const}}=v_x$, using $t, x, y, z$ and $v_x$ to compute $v_y$ and $v_z$. This is six dimension interpolations. As the linear relationship of node velocity and position, we can get linear match function to calculate $v_y$ and $v_z$. For the particularity of the six dimension arrays, the characteristic of linearity layer upon layer makes five linear functions instead of the only match function. Then the compute process is predigested. Taking calculating $v_y$ as an example, supposing the number of time $t$ is $r$, the number of $x$ direction of velocity $v_x$ is $s$, the number of $x$ direction of position is $m$, the number of $y$ direction of position is $n$ and the number of $z$ direction of position is $p$. Then calculation processes of linear functions are as follows:

- Using $y_1, \ldots, y_n$ and $v_y$ corresponding to them, linear function of $y$ and $v_y$ is obtained.

\[ v_y = a_y + b_y \quad (1) \]

- Using $x_1, \ldots, x_m$ and equation (1), linear functions of $x$ and $a, x$ and $b$ are obtained.

\[ a = c_{1j}x + d_{1j} \quad b = c_{2j}x + d_{2j} \quad (2) \]

where $j=1, \ldots, p$. $c_{1j}, c_{2j}, d_{1j}$ and $d_{2j}$ are equation coefficients, their number are equal to dimensions of $z$.

- Using $z_1, \ldots, z_p$ and equation (2), linear functions of $z$ and $c_1, \ldots, z$ and $d_2$ are obtained.

\[ c_1 = e_{1k}z + f_{1k} \quad c_2 = e_{2k}z + f_{2k} \quad (3) \]
\[ d_1 = e_{3k}z + f_{3k} \quad d_2 = e_{4k}z + f_{4k} \quad (4) \]
3.4. How to calculate $v_{const}$

**Using $vx_1, ... , vx_s$ and equation (3)-(4), linear functions of $vx$ and $e_1, ..., vx$ and $f_1$ are obtained.**

\[
e_i = g_{i,q}v_x + h_{i,q}, \ldots, e_4 = g_{i,q}v_x + h_{i,q}
\]

\[
f_i = g_{i,q}v_x + h_{i,q}, \ldots, f_4 = g_{i,q}v_x + h_{i,q}
\]

where $q = 1, \ldots, r$, $g_{i,q}$, $h_{i,q}$ are equation coefficients, their number are equal to dimensions of $t$.

**Using $t_1, ..., t_s$ and equation (5)-(6), linear functions of $t$ and $g_1, ..., t_s$ and $h_8$ are obtained by functions of**

\[
g_1 = u_1t + w_5 \ldots g_8 = u_8t + w_16, h_1 = g_9t + w_9 \ldots h_8 = g_{16}t + w_{16}.
\]

Where $u_1, ..., u_16, w_1, ..., w_16$ are equation coefficients.

### 3.2. Process of guidance and control

Guidance rule of space-grid method requires the angle of $a$ and $vg$ must be known. Suppose direction of $vg$ is denoted by pitch angle $\phi g$ and yaw angle $\psi g$. Roll angle is not considered. So $\phi g$ and $\psi g$ are given by

\[
tan(\phi g) = \frac{v_{gx}}{v_{gy}}
\]

\[
tan(\psi g) = \frac{-v_{gy}}{v_{gx}}
\]

where $v'_{gx}$ is square root of $vgx$ and $vgy$.

Suppose direction of $a$ is denoted by pitch angle $\phi a$ and yaw angle $\psi a$. So the angles are obtained as

\[
tan(\phi a) = \frac{-\Delta v_{gx}}{v'_{gy}}
\]

\[
tan(\psi a) = \frac{\Delta v_{gy}}{v'_{gx}}
\]

where $\Delta v_{gx}$ is square root of $\Delta vx$ and $\Delta vy$.

Substituting Eqs. (9)-(10) into triangular function, when $\Delta \phi = \phi g - \phi a$ and $\Delta \psi = \psi g - \psi a$ are very small, the equation reduces to $\Delta \phi_a = (v_{gx}\Delta v_{gx} - v_{gy}\Delta v_{gy})/(v_{gx}\Delta v_{gx} + v_{gy}\Delta v_{gy})$ and $\Delta \psi_a = (v'_{gx}\Delta v_{gy} + v'_{gy}\Delta v_{gx})/(v'_{gx}\Delta v_{gy} + v'_{gy}\Delta v_{gx})$. When $\Delta \phi = \Delta \psi = 0$, directions of $a$ and $vg$ are equal. Then the missile powers off.

### 3.3. Shutdown control of engine

When $\text{vg} = 0$, that is the current velocity $v$ is equal to $vR$, missile powers off. Then through free-flight phase and reentry phase, it can hit the target. So shutdown condition is $\text{vg} = 0$. During the guidance process, $\text{vgx} = \text{vgy} = \text{vgz} = 0$ in inertia fixed coordinates at shutdown time. But the three variables reach zero is difficult to achieve in engineering practice. So choosing variable has largest change rate as the control object to obtain the shutdown condition. This means is prone to be come true and satisfies the control precision. Generally speaking, $\text{vgx} > \text{vgy} > \text{vgz}$. $\text{vgx} = 0$ is looked as shutdown condition.

### 3.4. How to calculate $v_{const}$

As section 3.1-3.3 described in this part, we can use the required velocity from space grid to guidance and control missile. But $v_{const}$ has different value in space grid. By linear interpolation, other $v_{const}$ can be obtained outside grid node. How to choose $v_{const}$ affects track and energy consumption. Taking energy consumption as performance index, we select proper $v_{const}$. Based on $v_{const}$ and $t$, $x$, $y$, $z$, $v_{alter}$ can be obtained. When $t$, $x$, $y$, $z$ are given, $v_{alter}$ lies on $v_{const}$. Energy consumption has close relation with $\Delta v_x$, $\Delta v_y$, $\Delta v_z$. Suppose $v_{const} = v_x$, the question can be described as $J = \min J(\Delta v_x, \Delta v_y, \Delta v_z)$. And the function subject to $v_{rx}e(v_{dx} - c, v_{dx} + c)$, $v_{ry} = f(v_{rx})$, $v_{rz} = f(v_{rx})$. Where $J$ is energy consumption, $v_{rx}$ is $x$ direction of current velocity, $c$ is change area of $v_{rx}$, $\Delta v_x = v_{rer} - v_{d}(i = x, y, z)$. The key part is expression of $J$.

Components of control force are usually under missile body coordinate system. Euler’s angles of this coordinate system and inertia fixed coordinates are $\phi$, $\psi$, $\psi$. Only gravitation $g$ and control force $Pr$ act on missile upon atmosphere. $Pr$ is function of thrust $P$ and $\phi$, $\psi$. For $g$ can be calculated at any position by gravity models easily. Suppose $P$ is constant and ignoring $\psi$, so $J$ is function of $P$. Then $P_r$ is function of
φ, ψ, \( J \) changes into \( J(P_r, \varphi, \psi) \). So function of \( J \) translates into \( J = \min P_r(\varphi, \psi) \). Using searching method finds proper \( v_{cr} \). Then suppose \( v_{conr} = vy, vz \), we can find their minimize control force and required velocity. Comparing the three minimize control force, the minimum one is the proper one.

4. Simulation results

Consider partition principle in the second section and the guidance and control way in the third section. Simulation of guidance and control way by space-grid method is given in the following text. Suppose flight time is 80 seconds in powered phase. When flight time reached 70 seconds, velocity disturbance or position disturbance add to join in trajectory calculation respectively. Perturbation method and space-grid method are used in simulation respectively. Not considering reentry error, the results are as follows:

Table 1. Velocity disturbance simulation results

<table>
<thead>
<tr>
<th>Impact point precision of different method</th>
<th>( \Delta v_x=10m )</th>
<th>( \Delta v_x=5m )</th>
<th>( \Delta v_x=5m )</th>
<th>( \Delta v_x=10m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal error of perturbation method</td>
<td>-1476.2</td>
<td>-621.4</td>
<td>584.9</td>
<td>1481.7</td>
</tr>
<tr>
<td>Longitudinal error of space-grid method</td>
<td>0.48</td>
<td>0.79</td>
<td>0.63</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 2. Position disturbance simulation results

<table>
<thead>
<tr>
<th>Impact point precision of different method</th>
<th>( \Delta v_x=1000m )</th>
<th>( \Delta v_x=1000m )</th>
<th>( \Delta v_y=1000m )</th>
<th>( \Delta v_y=1000m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal error of perturbation method</td>
<td>-358.5</td>
<td>361.9</td>
<td>-402.4</td>
<td>398.1</td>
</tr>
<tr>
<td>Longitudinal error of space-grid method</td>
<td>0.25</td>
<td>0.82</td>
<td>0.51</td>
<td>0.39</td>
</tr>
</tbody>
</table>

From Table 1, we can see the results of the two methods have visible distinction on longitudinal error. Lateral error is the same as longitudinal error. So precision of space-grid method is much better. Table 2 shows the position disturbance simulation results. The results show that the impact point error is shorter than one meter. The results satisfy the precision demand greatly. Contrasting results of the two methods, space-grid method has broader application than perturbation method.

5. Conclusion

Compared with perturbation method, space-grid method has advantage of anti-jamming ability. It has great applied significance when getting in large disturbing condition. This method has great adaptability in complicated operational environment, broad application foreground in the aspects of guidance precision and reference value in computational efficiency on onboard computer. So space-grid method is worthy to have detailed study in the following research.

References

