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Modal analysis of Functionally Graded material Plates using Finite Element Method

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Abstract

The present work aims to carry out modal analysis of a functionally graded material (FGM) plate to determine its natural frequencies and mode shapes by using Finite Element Method (FEM). Functionally graded material can be differentiated by varying the composition and structure progressively over its volume, consequentially in corresponding changes in the material constituents. The mechanical properties of a FGM plate change continuously from one surface to another through its thickness direction according to power law. For modal analysis of FGM plate program has been coded in MATLAB software. Some examples are solved, and the results are compared with those available in the literature. The mode shape and natural frequencies of rectangular FGM plate are found at different boundary conditions. It has been observed the effect of volume fraction index, which indicates the percentage of ceramic and metal composition in the FGM. In addition, the effects of power law index on the FGM plate natural frequency and mode shapes with different boundary conditions are studied.

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Key words: *Functionally graded material, classical plate theory, finite element method, natural frequencies, mode shapes*

1. Introduction

Modal analysis is a technique used to determine structure's vibration characteristics: Natural frequencies, and Mode shapes. These are most fundamental of all dynamic analysis types. Whereas technology development at an ever increasing rate, the need for advanced capability materials becomes a main concern in the modal analysis of engineering structures of more intricate and higher performance systems. These requirements can be seen in many

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fields in which engineers are exploring the applications of new engineered materials.

Functionally Graded Materials (FGMs) are a relatively new technology and are being studied for the use in components exposed to high temperature gradients. An FGM material property allows the designer to tailor material response to meet design criteria. An FGM composed of ceramic on the outside surface and metal on the inside surface eliminates the abrupt change between coefficients of thermal expansion, offers thermal protection, and provides load carrying capability. This is possible because the material constituents of an FGM changes gradually through-the-thickness; therefore, stress concentrations from abrupt changes in material properties are eliminated. The FGM plates has numerous applications in advanced engineering fields such as thin-walled structural components in space vehicles, nuclear reactors, and other high thermal application areas. The Functionally graded material (FGM) can be intended for specific function and applications. The material properties such as young's modulus, density and poisons ratio values are varying continuously throughout the thickness direction according to the simple volume constituents defined by power law.

In the recent years functionally graded material plates modelling and analysis are carried out by Birman and Larry (2007). They have presents the principal developments in functionally graded materials (FGMs) with an importance on the recent work published since 2000. Pendhari et al. (2010) have developed the analytical and mixed semi-analytical solutions for a rectangular functionally graded plate. Singha et al. (2011) has investigates the high precision plate bending finite element for nonlinear behaviour of functionally graded plates under transverse load. In their analysis based on the first order shear deformation theory considered the physical/exact neutral surface position. A rectangular functionally graded material plate with simply supported boundary conditions subjected to transverse loading has been investigated by Chi and Yen (2006). Their analysis carried out, bases on the classical plate theory and Fourier series expansion; the series solutions of power-law FGM (simply called P-FGM), sigmoid FGM (S-FGM), and exponential FGM (E-FGM) plates are obtained.

Shahrjerdi et.al.(2008) recently estimates the natural frequency of functionally graded rectangular plate using second-order shear deformation theory (SSDT). The material properties of functionally graded rectangular plates, except the Poisson's ratio, are assumed to vary continuously through the thickness of the plate in accordance with the exponential law distribution. Talha and singh (2010) have studied the static and free vibration analysis of functionally graded material plates by using higher order shear deformation theory with a special modification in the transverse displacement in conjunction with finite element models. Vel and batrab (2004) have developed a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. The exact solution was valid for thick and thin plates, and for arbitrary difference of material properties in the thickness direction. Hashemi et.al. (2011) carried out the new exact closed form method for free vibration analysis of functionally graded rectangular thick plates. Their analysis was based on the Reddy's third-order shear deformation plate theory.

The main objective of this work is to propose a finite element approach for modal analysis of rectangular FGM plates based on the Kirchhoff plate theory or classical plate theory. The material properties are assumed to be graded through the thickness in accordance with a simple power-law distribution. Using the finite element formulation derived for the FGM plate element, a finite element program has been developed. Some examples are solved, and the results are compared with those available in the literature. In addition, the effects of power law index on the FGM plate natural frequency and mode shapes with different boundary conditions are studied.

2. FGM plate

A functionally graded material plate of thickness h is made of a mixture of ceramic and metal as shown in Fig. 1(a). The material on the top surface of the plate is ceramic rich whereas the bottom surface material is metal rich. Here, $z_m = h/2$ is the distance measured from mid-surface of the plate as shown in Fig. 1. The volume fraction of ceramic V_c and metal V_m varying through the thickness of the plate, according to a simple power-law, are expressed as

$$V_c(z_m) = \left(\frac{2z_m + h}{2h}\right)^n,\ V_m(z_m) = 1 - V_c(z_m) \tag{1}$$

where n is the volume fraction exponent ($n>0$). The variation of the effective material property P may be written as

$$P(z_m) = P_c(z_m)V_c(z_m) + P_m(z_m)V_m(z_m) \tag{2}$$

For plate made of FGM, the neutral surface may not coincide with its geometric mid-surface. The distance of the neutral surface (d) from the geometric mid-surface may be expressed as.

$$d = \frac{\int_{-h/2}^{h/2} E(z_m) z_m dz_m}{\int_{-h/2}^{h/2} E(z_m) dz_m} \tag{3}$$

In the neutral surface based formulation, the coordinate z_n is with respect to the neutral surface. The volume fraction of ceramic in the new coordinate system can be expressed as

$$V_c(z_n) = \left(\frac{2z_n + h + 2d}{2h}\right)^n \tag{4}$$

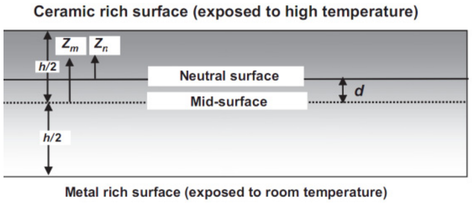


Fig. 1 Neutral plane of FGM plate

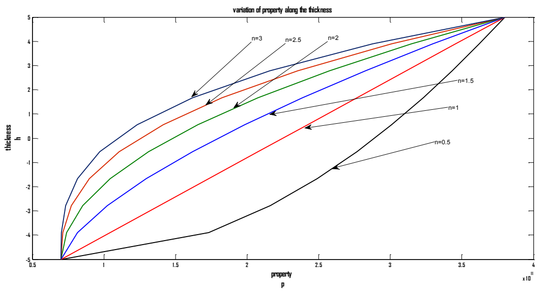


Fig. 2 Variation of young's modulus along the thickness of the FGM plate

Fig.2 shows the effective young's modulus variation with different index values along the thickness direction of the plate.

3. Finite Element formulations

The displacement field of the classical plate theory is:

$$\begin{aligned} U &= u(x,y) - z\frac{\partial w(x,y)}{\partial x} \\ V &= v(x,y) - z\frac{\partial w(x,y)}{\partial y} \\ W &= w(x,y) \end{aligned} \tag{5}$$

The strain-displacement relationship are given as

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}, \quad \gamma_{yz} = 0, \quad \gamma_{xz} = 0 \end{aligned} \tag{6}$$

The stress-strain relationships of the functionally graded plate in the global x-y-z coordinates system can be written as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{Bmatrix} \tag{7}$$

where

$$Q_{11} = \frac{E(z)}{1-\upsilon^2}, Q_{12} = Q_{21} = \frac{\upsilon E(z)}{1-\upsilon^2},$$

$$Q_{66} = \frac{E(z)}{2(1+\upsilon)}$$

The total strain energy can be expressed as

$$U_e = \iiint \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} \right) dxdydz \tag{8}$$

The kinetic energy of the plate can be expressed as follows

$$T_e = \frac{1}{2} \iint_A \rho h \dot{w}^2 \, dA \tag{9}$$

The FGM plate is model using four node rectangular elements. Four node rectangular elements are having four nodes at each corner as shown in Fig.2. There are three degrees of freedom at each node, the displacement component along the thickness (w), and two rotations about x and y directions in terms of the (x ,y) coordinates respectively. The each element consists of four nodes 1, 2, 3 and 4 with w is the transverse displacement and $\theta_x and \theta_y$ represents the rotations about x and y axis respectively.

$$w, \theta_x = \frac{dw}{dy}, \theta_y = \frac{dw}{dx} \tag{10}$$

Therefore the element has twelve degrees of freedom and the displacement function of the element can be represented by a polynomial having twelve terms as shown.

$$w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \tag{11}$$

The expression (12) can be written in the following matrix form

$$w = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2 y & xy^2 & y^3 & x^3 y & xy^3 \end{bmatrix} \{\alpha\}$$

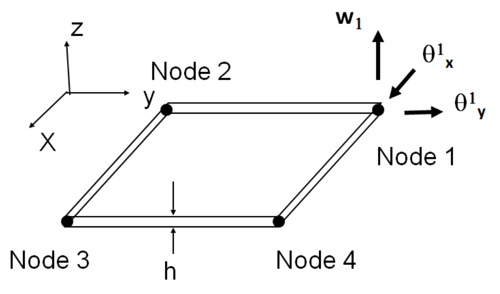
$$w = \left[Poly(x,y) \right] \{\alpha\} \tag{12}$$

$$\{w^e\} = [A]\alpha, \; \alpha = [A]^{-1} \{w^e\} \tag{13}$$

where $\{w^e\}^T = \begin{bmatrix} w_i & \theta_{xi} & \theta_{yi} \end{bmatrix}, i = 1,2,3,4$

Substituting eq. (13) in eq. (12)

$$w = \left[Ploy(x,y) \right] [A]^{-1} \{w^e\}, \; w = [N]\{w^e\} \tag{14}$$



12 degrees of freedom

Fig.3 Geometry of the four node rectangular element

$$N_i^T=\begin{bmatrix} \frac{1}{8}(1+x_ix)(1+y_iy)(2+x_ix+y_iy-x^2-y^2) \\ \frac{1}{8}(1+x_ix)(y+y_i)(y^2-1) \\ \frac{1}{8}(x+x_i)(1+y_iy)(x^2-1) \end{bmatrix} \qquad (15)$$

where i=1, 2, 3 and 4.

The element stiffness matrix and mass matrices are derived on the basis on principle of minimum potential energy and kinetic energy.

The element stiffness matrix is

$$[K_e]=\int_v [B]^T[D][B]dv \qquad (16)$$

where

$$[B]=z\begin{bmatrix} \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial y^2} \\ 2\frac{\partial^2}{\partial x\partial y} \end{bmatrix}[N] \qquad (17)$$

Element mass matrix

$$[M_e]=\int_v [N]^T[\rho][N]dv \qquad (18)$$

The equation of motion for a plate element is obtained by using Hamilton's principle.

$$\delta\int_{t_1}^{t_2}(U_e-T_e+W_e)dt=0 \qquad (19)$$

The element stiffness and mass matrices are assembled in to get global matrices. The equation of motion of the plate can be written as

$$([K]-\omega^2[M])\{w\}=0 \qquad (20)$$

where ω are natural frequencies of the plate

4. Results and discussions

In according to the Talha M. and B.N.Singh, (2010) ϖ is dimensionless frequency parameter is:

$$\varpi=\omega\sqrt{12(1-\upsilon^2)\rho_c L^2W^2/(\Pi^2 E_ch^2)}$$

Validation has been done by considering the values of thickness, length, width, Poisson's ratio, density and young's modulus in the ceramic and metal as:

$Al, \rho=2707kg/m^3, E=70X10^9Pa, \upsilon=0.3$

$Al_2O_3, \rho=2707kg/m^3, E=380X10^9Pa, \upsilon=0.3$

$ZrO_2, \rho=3000kg/m^3, E=151X10^9Pa, \upsilon=0.3$

$SUS304, \rho=8166kg/m^3, E=207X10^9Pa, \upsilon=0.3177$

The table 1 shows the natural frequency parameters obtained from the present study using classical plate theory. Table 1 show the natural frequency parameter obtained from the present study using classical plate theory and Talha & Singh (2010). There is a good matching between the presented results and those from Talha and Singh, (2010), especially for simply supported case.

Table.1 Variation of the frequency parameter (ϖ) with the volume fraction index n for (SSSS) square (Al/ZrO_2) FGM plates (a/h = 20)

		1	2	3	4	5
Ceramic	Talha M.(2010)]	1.9943	5.0388	5.0388	7.9752	10.3318
	Present	1.9953	4.9814	4.9814	7.9262	9.9597
n=0.5	Talha M.(2010)	1.8074	4.5415	4.5415	7.1971	9.3352
	Present	1.7945	4.4801	4.4801	7.1285	8.9575
n=1	Talha M.(2010)	1.7348	4.3322	4.3322	6.8699	8.9029
	Present	1.7213	4.2723	4.2723	6.7982	8.5417
n=5	Talha M.(2010)	1.6215	4.0467	4.0467	6.4057	8.2691
	Present	1.6166	4.0212	4.0112	6.3824	8.0199
n=10	Talha M.(2010)	1.5693	3.9372	3.9372	6.2286	8.0441
	Present	1.5640	3.9047	3.9047	6.2134	7.8070
Metal	Talha M.(2010)	1.4530	3.6695	3.6695	5.8072	7.5198
	Present	1.4329	3.5995	3.5995	5.7293	7.1576

The following numerical results are obtained by considering the steel as the bottom surface and alumina as the top surface in FGM plate according to index value. The geometry of plate and material properties is as follows: *L=1 m* (length), *W=1 m* (width), *h=0.01 m* (thickness).

The variations of natural frequency parameter in FGM *(SUS304/Al_2O_3)* plate for different boundary conditions are shows in fig. 3, 4 and 5.

The effect of power law index *n* on the frequencies can be seen for different boundary conditions. Figs.3-5 shows the first five frequency parameters verses power law index value at different boundary conditions. As expected, the increasing index value leads to reduce the natural frequency parameter. Increasing index value reduces the ceramic constituents, its produce the effective material properties changes which affect the frequency parameter.

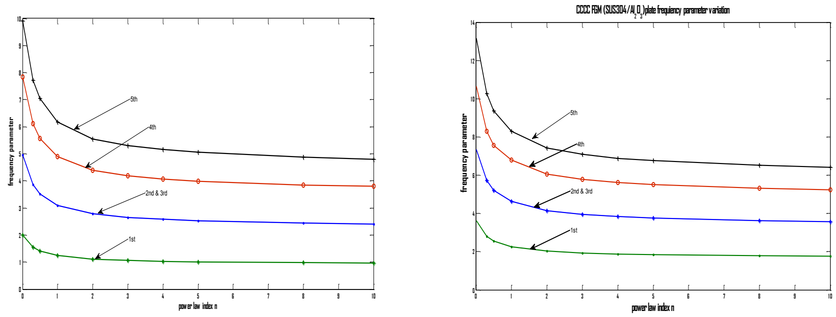


Fig.4 Variation of frequency parameter with index value at SSSS and CCCC boundary conditions

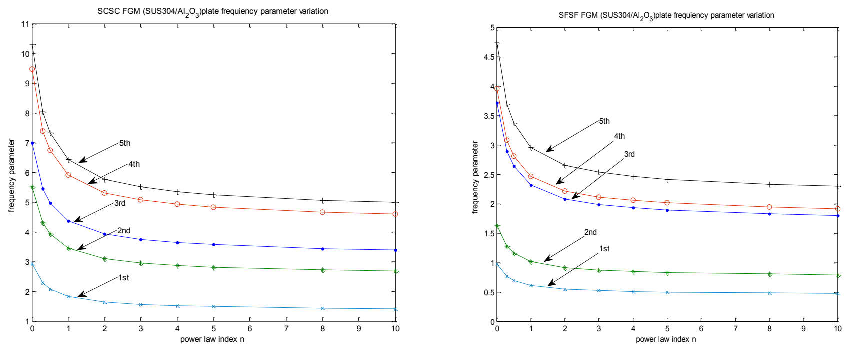
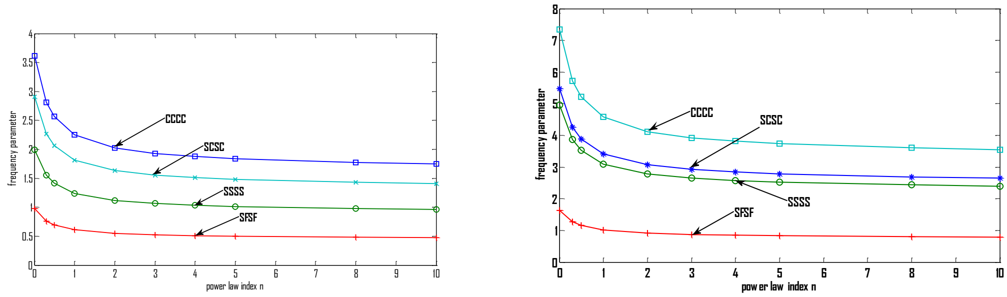


Fig.5 Variation of frequency parameter with index value at SCSC and SFSF boundary conditions



(a) (b)

Fig.6 (a), (b) First and second mode frequency parameter variation by power law index with various boundary conditions

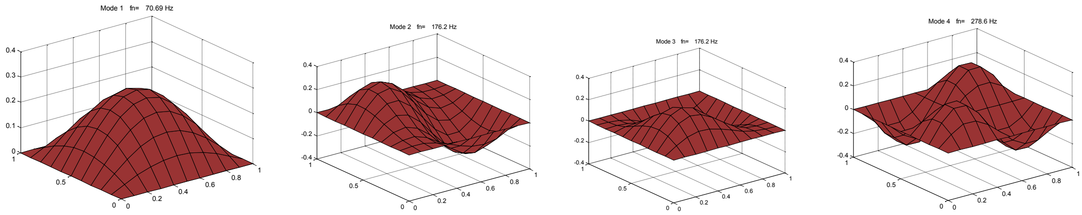


Fig.7 SSSS square FGM plate mode shapes 1, 2, 3 and 4 with index value *n=1*

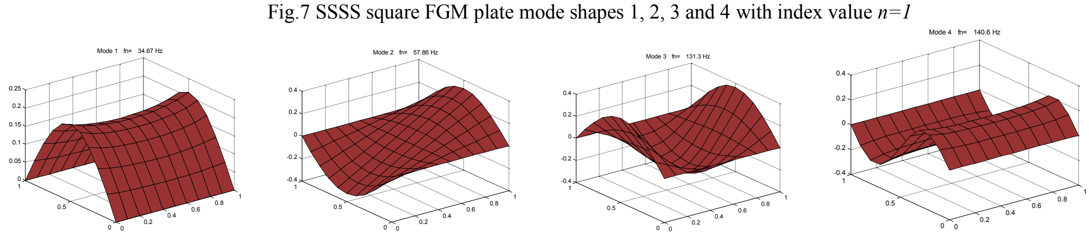


Fig.8 SFSF square FGM plate mode shapes 1, 2, 3 and 4 with index value *n=1*

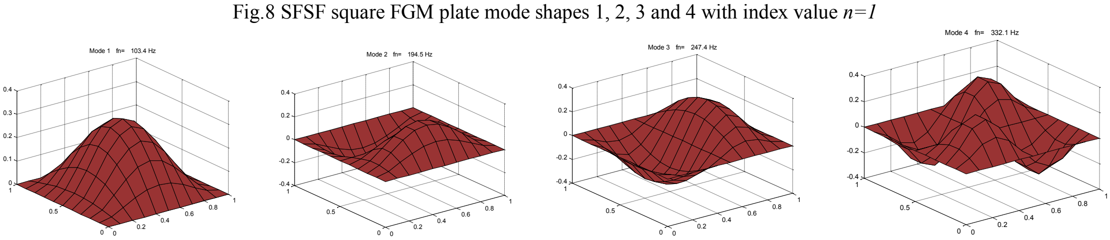


Fig.9 CCCC square FGM plate mode shapes 1, 2, 3 and 4 with index value *n=1*

A vibrating system with infinite number degrees of freedom (DOF) has infinite number natural frequencies, and for each natural frequency there is a relationship between the amplitudes of the infinite number independent motions, known as the mode shape. There is one mode shape for each natural frequency and it depends on the value of that natural frequency. For a FGM plate the mode shapes are the shapes of the structure at its maximum deformation during a cycle of vibration. Fig. 5, 6 and 7 shows the first four modes shapes of an FGM plate with different boundary conditions.

5. Conclusions

A finite element formulation for modal analysis of FGM plates has been derived. The material composition of the FGM structure is assumed to vary according to a simple power-law distribution through the thickness. The developed computer code is validated with the published results for natural frequencies of FGM plates. Results obtained from present finite element analysis agree well with those reported. Developed finite element code is employed to compute the natural frequencies and mode shapes of the FGM plate with different boundary conditions and with different power law index values. Convergence study has also been performed and it is observed that results are very good with mesh size 10X10. Effects of power index ‘n’ and the four different boundary conditions on the natural frequencies are investigated. Natural frequencies for CCCC boundary condition are the largest than the corresponding frequencies obtained for SCSC, SSSS and SFSF boundary conditions. Increasing the power law index value *n* reduces the first five natural frequencies for all the different boundary conditions. The effect of power law index is more prominent within the range up to 4, after which the effect is not that much significant.

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