Flavor-SU(3) tests from $D^0 \to K^0 K^- \pi^+$ and $D^0 \to \overline{K}^0 K^+ \pi^-$ Dalitz plots

Bhubanjyoti Bhattacharya, Jonathan L. Rosner *

Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, IL 60637, United States

A B S T R A C T

The processes $D^0 \to K^0 K^- \pi^+$ and $D^0 \to \overline{K}^0 K^+ \pi^-$ involve intermediate vector resonances whose amplitudes and phases are related to each other via flavor-SU(3) symmetry. Dalitz plots for these two processes can shed light on the usefulness of this symmetry in studying charm decays. Until this year the only available data on this process came from a conference report in 2002 by the BaBar Collaboration, but now an independent data sample of higher statistics has become available from the CLEO Collaboration. The goal is to predict Dalitz plot amplitudes and phases assuming flavor-SU(3) symmetry and compare them with experiment.

An SU(3) fit can account for the relative magnitudes of the amplitudes for the decays $D^0 \to K^{*-} K^+$ and $D^0 \to K^{*+} K^-$, but neither the current BaBar sample (based on an integrated luminosity of 22 fb^{-1}) nor the CLEO analysis has significant evidence for the decays $D^0 \to K^{*0} \overline{K}^0$ and $D^0 \to \overline{K}^{*0} K^0$. At this level one is unable to compare magnitudes and phases with theoretical predictions. The purpose of this Letter is to advocate an analysis using the full BaBar sample (more than 20 times the 2002 value). It should definitively determine whether predicted magnitudes and phases agree with experiment. A similar analysis should be possible with an even larger sample of events collected by the Belle Collaboration at KEK-B.

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An important contribution to the decay processes $D^0 \to 3P$, where P represents a pseudoscalar meson, involves the intermediate step in which the D meson first decays into a P and a vector meson (V). The vector meson then decays into two pseudoscalars. In general, in a decay with three final P states the combination of any pair of final pseudoscalars may result from the decay of a V as long as charge, isospin, strangeness, etc., are conserved. Evidence of formation of such resonances is seen in Dalitz plots as bands of events corresponding to the invariant mass-squared of the pair of final state P mesons. As such, they provide information about the amplitude and phase for the process $D \to PV$. Overlapping vector resonance bands on Dalitz plots interfere according to their relative phases.

Amplitudes and phases of $D \to PV$ decays were studied in detail using flavor-SU(3) symmetry in Ref. [1]. Relative phase relations based on this symmetry were exploited in Refs. [2–4] to observe its successes in predicting interferences on several $D \to 3P$ Dalitz plots. In the present Letter we consider the Dalitz plots for $D^0 \to K^0 K^- \pi^+$ and $D^0 \to \overline{K}^0 K^+ \pi^-$. We predict amplitudes and phases for the relevant $D \to PV$ intermediate processes using flavor-SU(3) symmetry. Data from the BaBar [5] and CLEO [6] Collaborations do not provide strong enough evidence for the pro-

cesses $D^0 \to K^{*0} \overline{K}^0$ and $D^0 \to \overline{K}^{*0} K^0$ to permit a comparison of phases with predictions, but BaBar's total data, more than twenty times the reported sample, should be able to provide a definitive test. The Belle Collaboration at KEK-B should have at least as many events as the full BaBar sample.

We first review the flavor-SU(3) symmetry technique, and then predict amplitudes and phases for the relevant $D \to PV$ processes, comparing them with data. The flavor symmetry approach used here was discussed in detail in [1]. We denote the relevant Cabibbo-favored (CF) amplitudes, proportional to the product $V_{ud} V_{cs}^*$ of Cabibbo–Kobayashi–Maskawa (CKM) factors, by amplitudes labeled as T ("tree") and E ("exchange"), illustrated in Fig. 1. The singly-Cabibbo-suppressed (SCS) amplitudes, proportional to the product $V_{us} V_{cs}^*$ or $V_{ud} V_{cd}^*$, are then obtained by using the ratio SCS/CF $= \tan \theta_C \equiv \lambda = 0.2305$ [7], with θ_C the Cabibbo angle and signs governed by the relevant CKM factors. The subscript P or V on an amplitude denotes the meson (P or V) containing the spectator quark in the PV final state. The partial width $\Gamma(H \to PV)$ for the decay of a heavy meson H is given in terms of an invariant amplitude $\mathcal{A}$ as:

$$\Gamma(H \to PV) = \frac{p^{*3}}{8\pi M_H^2} |\mathcal{A}|^2 \tag{1}$$

where p^* is the center-of-mass (c.m.) 3-momentum of each final particle, and M_H is the mass of the decaying heavy meson. With this definition the amplitudes $\mathcal{A}$ are dimensionless.

* Corresponding author.
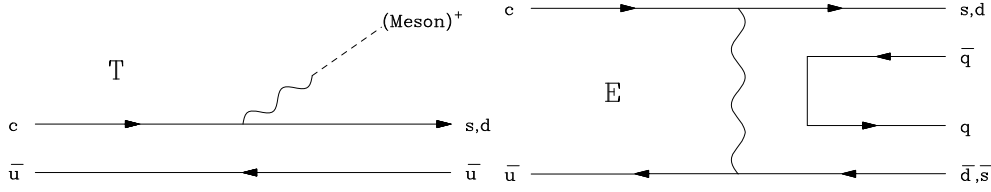 *E-mail address:* rosner@hep.uchicago.edu (J.L. Rosner).

Fig. 1. Graphs describing tree (T) and exchange (E) amplitudes.

Table 1
Solutions for T_V, E_P, T_P and E_V amplitudes in Cabibbo-favored charmed meson decays to PV final states, for η–η' mixing angles of $\theta_\eta = 19.5°$ and $11.7°$.

PV ampl.	$\theta_\eta = 19.5°$		$\theta_\eta = 11.7°$	
	Magnitude (10^{-6})	Relative strong phase	Magnitude (10^{-6})	Relative strong phase
T_V	3.95 ± 0.07	Assumed 0	These results are independent of θ_η	
E_P	2.94 ± 0.09	$\delta_{E_P T_V} = (-93 \pm 3)°$		
T_P	7.46 ± 0.21	Assumed 0	7.69 ± 0.21	Assumed 0
E_V	2.37 ± 0.19	$\delta_{E_V T_V} = (-110 \pm 4)°$	1.11 ± 0.22	$\delta_{E_V T_V} = (-130 \pm 10)°$

Table 2
Amplitudes for $D^0 \to PV$ decays of interest for the present discussion (in units of 10^{-6}). Here we have taken $\theta_\eta = 19.5°$.

Dalitz plot	D^0 final state	Amplitude representation	Amplitude A			
			Re	Im	$|A|$	Phase (°)
$D^0 \to K^0 K^- \pi^+$	$K^{*+} K^-$	$\lambda(T_P + E_V)$	1.533	-0.513	1.616	-18.5
	$\overline{K}^{*0} K^0$	$\lambda(E_V - E_P)$	-0.151	0.163	0.223	132.8
$D^0 \to \overline{K}^0 K^+ \pi^-$	$K^{*-} K^+$	$\lambda(T_V + E_P)$	0.875	-0.677	1.106	-37.7
	$K^{*0} \overline{K}^0$	$\lambda(E_P - E_V)$	0.151	-0.163	0.223	-47.2

Table 3
Amplitudes for $D^0 \to PV$ decays of interest for the present discussion (in units of 10^{-6}). Here we have taken $\theta_\eta = 11.7°$.

Dalitz plot	D^0 final state	Amplitude representation	Amplitude A			
			Re	Im	$|A|$	Phase (°)
$D^0 \to K^0 K^- \pi^+$	$K^{*+} K^-$	$\lambda(T_P + E_V)$	1.608	-0.196	1.620	-6.9
	$\overline{K}^{*0} K^0$	$\lambda(E_V - E_P)$	-0.129	0.481	0.498	105.0
$D^0 \to \overline{K}^0 K^+ \pi^-$	$K^{*-} K^+$	$\lambda(T_V + E_P)$	0.875	-0.677	1.106	-37.7
	$K^{*0} \overline{K}^0$	$\lambda(E_P - E_V)$	0.129	-0.481	0.498	-75.0

The amplitudes T_V and E_P were obtained from fits to rates of CF $D \to PV$ decays not involving η or η' [1]. To specify the amplitudes T_P and E_V, however, one needs information on the η–η' mixing angle (θ_η). Table 1 summarizes these results for two values $\theta_\eta = 19.5°$ and $11.7°$.

In Tables 2 and 3 we list the $D^0 \to PV$ amplitudes relevant in Dalitz plots of interest for $\theta_\eta = 19.5°$ and $\theta_\eta = 11.7°$, respectively. Also included are their representations. We predict the magnitudes and phases for the above amplitudes using flavor SU(3) and compare the magnitudes with data obtained from Dalitz plot fits.

The ratio of the amplitude $|\mathcal{A}(D^0 \to K^{*-} K^+)|$ relative to $|\mathcal{A}(D^0 \to K^{*+} K^-)|$ is predicted to be equal to a corresponding ratio of Cabibbo-favored amplitudes (taken from Ref. [1]):

$$\frac{|\mathcal{A}(D^0 \to K^{*-} K^+)|}{|\mathcal{A}(D^0 \to K^{*+} K^-)|} = \frac{|\mathcal{A}(D^0 \to K^{*-} \pi^+)|}{|\mathcal{A}(D^0 \to \rho^{*+} K^-)|}$$
$$= 0.685 \pm 0.032. \quad (2)$$

These ratios are less than one because the T amplitudes in the numerators involve the coupling of the weak current to a pseudoscalar meson, whose decay constant is less than that for the vector meson involved in the denominators: $|T_V| < |T_P|$ (see Table 1).

Flavor SU(3) predicts equal magnitudes for the much smaller amplitudes $\mathcal{A}(D^0 \to \overline{K}^{*0} K^0)$ and $\mathcal{A}(D^0 \to K^{*0} \overline{K}^0)$:

$$\frac{|\mathcal{A}(D^0 \to \overline{K}^{*0} K^0)|}{|\mathcal{A}(D^0 \to K^{*+} K^-)|} = \frac{|\mathcal{A}(D^0 \to \overline{K}^0 K^{*0})|}{|\mathcal{A}(D^0 \to K^{*+} K^-)|}$$
$$= \begin{cases} 0.138 \pm 0.033 & (\theta_\eta = 19.5°), \\ 0.307 \pm 0.035 & (\theta_\eta = 11.7°). \end{cases} \quad (3)$$

The predicted magnitude of these amplitudes is very sensitive to the mixing angle θ_η, as a result of cancellation between the amplitudes E_V and E_P (see Table 1).

In order to obtain amplitudes from Dalitz plot fit fractions to compare with predictions, one must recognize that the $D \to PV$ process is an intermediate to the complete 3 body decay $D \to 3P$. The Dalitz plot fit fractions also contain information about the vector meson decay and this must be factored out for comparison with flavor-SU(3) predictions. The fraction of a vector meson's decay amplitude to a pair of P mesons is given by the relevant isospin Clebsch–Gordan factor.

To obtain the correct Clebsch–Gordan factor including its sign, one notes that the spin part of the amplitude for the process $D \to RC \to ABC$ (R represents the intermediate resonance while A, B and C are the final pseudoscalar mesons) is proportional to the product $\vec{p}_A \cdot \vec{p}_C$ ($\vec{p}_i$ is the 3-momentum of the final state particle i in the rest frame of R). Since the particles A and B have equal and opposite 3-momenta in the resonance rest frame, this implies that swapping A and B while calculating the amplitude would result in an additional phase difference of π. It is thus important

Table 4

Conventions for the order of two pseudoscalar mesons in vector meson decay and associated Clebsch–Gordan factors assuming the cyclic convention of Ref. [6].

Dalitz plot	Bachelor particle		Vector meson decay			p^* (in MeV)
	Meson	Index	Process	Indices	Clebsch factor	
$D^0 \to K^0 K^- \pi^+$	K^0	1	$\overline{K}^{*0} \to K^- \pi^+$	23	$-\sqrt{2/3}$	605
	K^-	2	$K^{*+} \to K^0 \pi^+$	13	$-\sqrt{2/3}$	610
	π^+	3	–	–	–	–
$D^0 \to \overline{K}^0 K^+ \pi^-$	$\overline{K}^0$	1	$K^{*0} \to K^+ \pi^-$	23	$\sqrt{2/3}$	605
	K^+	2	$K^{*-} \to \overline{K}^0 \pi^-$	13	$\sqrt{2/3}$	610
	π^-	3	–	–	–	–

Table 5

Dalitz plot fits to data from the BaBar [5] and CLEO [6] Collaborations.

Dalitz plot	D^0 final state	Fit fraction (%)	
		BaBar	CLEO
$D^0 \to K^0 K^- \pi^+$	$K^{*+} K^-$	$63.6 \pm 5.1 \pm 2.6$	$67.6 \pm 6.4 \pm 3.8$
	$\overline{K}^{*0} K^0$	$0.8 \pm 0.5 \pm 0.1$	$1.8 \pm 1.7 \pm 0.8$
$D^0 \to \overline{K}^0 K^+ \pi^-$	$K^{*-} K^+$	$35.6 \pm 7.7 \pm 2.3$	$20.4 \pm 2.1 \pm 0.8$
	$K^{*0} \overline{K}^0$	$2.8 \pm 1.4 \pm 0.5$	$3.9 \pm 1.5 \pm 0.4$

Table 6

Comparison of ratios $\mathcal{B}(D^0 \to K_S K^+ \pi^-)/\mathcal{B}(D^0 \to K_S K^- \pi^+)$.

BaBar [5]	CLEO [6]
0.683 ± 0.078	0.592 ± 0.048

Table 7

Comparison of ratios of D^0 decay amplitudes extracted from Dalitz plot fits with theoretical predictions of flavor SU(3).

Ratio	Experiment		Theory	
	BaBar	CLEO	$\theta_\eta = 19.5°$	$\theta_\eta = 11.7°$
$\frac{|\mathcal{A}(K^{*-}K^+)|}{|\mathcal{A}(K^{*+}K^-)|}$	0.618 ± 0.083	0.423 ± 0.037	0.685 ± 0.032	0.685 ± 0.032
$\frac{|\mathcal{A}(\overline{K}^{*0}K^0)|}{|\mathcal{A}(K^{*+}K^-)|}$	$0.159^{+0.045}_{-0.064}$	$0.231^{+0.121}_{-0.231}$	0.138 ± 0.033	0.307 ± 0.035
$\frac{|\mathcal{A}(K^{*0}\overline{K}^0)|}{|\mathcal{A}(K^{*+}K^-)|}$	$0.245^{+0.061}_{-0.079}$	$0.261^{+0.051}_{-0.061}$	0.138 ± 0.033	0.307 ± 0.035

to know the phase convention used to obtain the amplitudes. In the present case, we assume a convention employed by the CLEO Collaboration [6]. This convention is presented in Table 4. Using this convention one may then calculate the appropriate isospin Clebsch–Gordan coefficients, also noted in Table 4.

The phase space factors for the two $D \to PV$ processes from each Dalitz plot are not the same since the mesons involved have slightly different masses. This very small difference, noted in Table 4, has been neglected.

The fit fractions obtained by the BaBar and CLEO analyses for relevant intermediate $D^0 \to PV$ decays corresponding to each Dalitz plot are quoted in Table 5. We use the best CLEO fits which include the channels $\overline{K}^{*0} K^0$ and $K^{*0} \overline{K}^0$. Fits not including these channels actually are superior in quality; the fit fractions for $K^{*-} K^+$ and $K^{*+} K^-$ do not differ much from those quoted.

Fit fractions quoted in Table 5 are normalized so as to represent percentage of each decay mode in the specific Dalitz plots. This normalization is different for the two different Dalitz plots. In order to compare amplitudes for $D \to PV$ processes from two different Dalitz plots it is useful to choose a universal normalization. To achieve this we make use of the branching fractions for the $D \to 3P$ processes for each Dalitz plot, so as to calculate the fraction of each $D \to PV$ process relative to a common rate or amplitude. We thus utilize ratios of branching fractions of $D^0 \to K_S K^+ \pi^-$ and $D^0 \to K_S K^- \pi^+$ given in Table 6. The BaBar value has been extracted by us from the ratios [5]

$$\frac{\mathcal{B}(D^0 \to \overline{K}^0 K^+ \pi^-)}{\mathcal{B}(D^0 \to \overline{K}^0 \pi^+ \pi^-)} = (5.68 \pm 0.25 \pm 0.41)\%,$$

$$\frac{\mathcal{B}(D^0 \to K^0 K^- \pi^+)}{\mathcal{B}(D^0 \to \overline{K}^0 \pi^+ \pi^-)} = (8.32 \pm 0.29 \pm 0.56)\%, \qquad (4)$$

while the CLEO value is quoted directly by them.

We make use of the data quoted in Table 5 and the ratios in Table 6 to calculate the relative amplitudes of the relevant $D \to PV$ decays. The magnitudes of the amplitudes are obtained relative to that of the process $D^0 \to K^{*+} K^-$ with maximum amplitude. These results are listed in Table 7. In Table 7 we also list the predictions of magnitudes of corresponding amplitudes obtained using flavor-SU(3) symmetry.

The success of the theoretical predictions is mixed. While the observed ratios quoted in Table 6 and the first line of Table 7 are less than one as predicted, the CLEO value is significantly below that of BaBar and the predicted value (2). The second and third ratios in Table 7 are indeed seen to be small, but the evidence for them is scant, with CLEO favoring fits without such amplitudes.

Until significant evidence for the decays $D^0 \to \overline{K}^{*0} K^0$ and $D^0 \to K^{*0} \overline{K}^0$ is found, it is premature to compare the phases predicted in Tables 2 and 3 with experiment. BaBar's total sample is more than 20 times as large as reported in Ref. [5], and an updated analysis would provide much more convincing statistics. The Belle Collaboration should have at its disposal at least as many events as the full BaBar sample.

Acknowledgements

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