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Analytical result for the two-loop QCD correction to the decay $H \to 2\gamma$

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Abstract

An analytic formula for the two-loop QCD correction of the decay $H \to 2\gamma$ is presented. To evaluate all master integrals a 'Risch-like' algorithm was exploited.

1. Introduction

The introduction of the Higgs particle into the Standard Model allows to describe the masses of all other particles by means of spontaneous symmetry breaking. The 'hidden' gauge symmetry also makes the model renormalizable. In this sense the Higgs particle is the crucial element in the SM of electroweak interactions. The mass of the Higgs is still unknown and thus the search for the Higgs is a tremendous experimental task. At present one deduces all possible properties of the Higgs boson from perturbation theory even in higher orders to be prepared for the interpretation of coming experiments at LHC. Among these are decay modes of the Higgs boson, which have been investigated in two loop approximation by many authors (for reviews see [1]). Our contribution to this problem is in the present Letter a complete analytic calculation of the decay $H \to 2\gamma$ with the top quark as virtual loop.

2. Results

The partial decay width can be written in the form [1,2]

$$\Gamma[H \to \gamma\gamma] = \left|\sum_f A_f(\tau_f) + A_W(\tau_W)\right|^2 \frac{M_H^3}{64\pi}, \tag{1}$$

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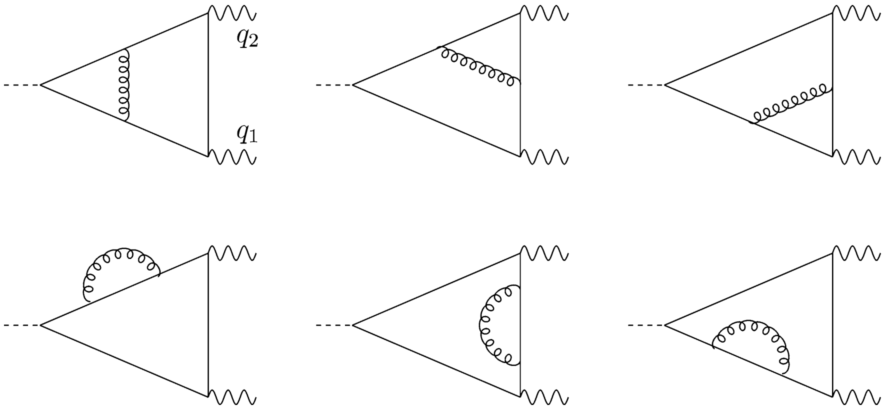

Fig. 1. Two-loop graphs contributing to the QCD corrections to $H\gamma\gamma$ decay.

where $\tau_f = M_H^2/4m_f^2$, $\tau_W = M_H^2/4M_W$. In the one loop order $A_W(\tau_W)$ is the dominating correction. We will focus our attention on $A_t(\tau_t)$ and consider QCD corrections of order $O(\alpha_s)$ to this amplitude. It is convenient to write

$$A_t(\tau_t) = A_t^{(0)} + \frac{\alpha_s}{\pi} A_t^{(1)} + \cdots. \tag{2}$$

The exact one-loop result is well known [3,4] and reads

$$A_t^{(0)} = -\hat{A}_t \left[1 - \frac{(1+\theta_t)^2}{4(1-\theta_t)^2} \ln^2\left(\frac{1}{\theta_t}\right)\right] \frac{6\theta_t}{(1-\theta_t)^2}, \tag{3}$$

where $\hat{A}_t = \frac{2\alpha}{3\pi v} N_C Q_t^2$, $v = 2^{-1/4} G_F^{-1/2}$, N_C is the color factor, Q_t is the electric charge of the t-quark and

$$\theta_t = \frac{\sqrt{1 - \frac{1}{\tau_t}} - 1}{\sqrt{1 - \frac{1}{\tau_t}} + 1}. \tag{4}$$

The two-loop diagrams contributing to $A_t^H(\tau_t)$ are shown in Fig. 1. There are six diagrams contributing to $H\gamma\gamma$ at the two-loop level (see Fig. 1).

The amplitude for the decay of the Higgs boson into two photons with polarization vectors $\epsilon_\mu(q_1)$ and $\epsilon_\nu(q_2)$ has the following Lorentz structure:

$$A_t^{\mu\nu} = \sum_i A_{t,i}^{\mu\nu} = \sum_i \left(a_{t,i} q_1 q_2 g^{\mu\nu} + b_{t,i} q_1^\nu q_2^\mu + c_{t,i} q_1^\mu q_2^\nu\right), \tag{5}$$

where $c_{t,i}$ has no contribution for on-shell photons. In Eq. (5) the sum runs over all diagrams relevant for the decay $H \to \gamma\gamma$. Due to gauge invariance, we have $\sum_i a_{t,i} = -\sum_i b_{t,i}$. It is easy to find projectors for $a_{t,i}$ and $b_{t,i}$:

$$a_{t,i} = \frac{A_{t,i}^{\mu\nu}}{(d-2)(q_1 q_2)^2} (q_1 q_2\, g_{\mu\nu} - q_{1\nu} q_{2\mu} - q_{1\mu} q_{2\nu}), \tag{6}$$

$$b_{t,i} = \frac{A_{t,i}^{\mu\nu}}{(d-2)(q_1 q_2)^2} \left(-q_1 q_2\, g_{\mu\nu} + q_{1\nu} q_{2\mu} + (d-1) q_{1\mu} q_{2\nu}\right). \tag{7}$$

We calculated both, $a_{t,i}$ and $b_{t,i}$, in order to have an additional check for the correctness of our result.

Tensor integrals and integrals with irreducible numerators were represented in terms of scalar integrals with shifted space–time dimension [5].

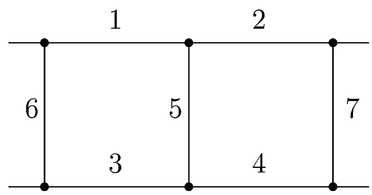

Fig. 2. Generic topology of the two-loop graphs.

We introduced two auxiliary vectors a_1 and a_2 allowing to obtain tensors with k_1 and k_2 by differentiating with respect to $a_{1,2}$, using the differential operator

$$T_{\mu_1\ldots\nu_1\ldots} = \frac{1}{i^{n+k}} \frac{\partial^n}{\partial a_{1\mu_1} \cdots \partial a_{1\mu_n}} \frac{\partial^k}{\partial a_{1\nu_1} \cdots \partial a_{1\nu_k}} \exp(iQ), \tag{8}$$

where

$$\begin{aligned} Q = {} & -q_1 a_1 \big(\alpha_1(\alpha_3+\alpha_4+\alpha_5+\alpha_6)+\alpha_3\alpha_6\big) - q_1 a_2 \big(\alpha_3(\alpha_1+\alpha_2+\alpha_6+\alpha_7)+\alpha_2\alpha_6\big) \\ & - q_2 a_1 \big(\alpha_2(\alpha_3+\alpha_4+\alpha_5+\alpha_6)+\alpha_4\alpha_6\big) - q_2 a_2 \big(\alpha_4(\alpha_1+\alpha_2+\alpha_6+\alpha_7)+\alpha_2\alpha_6\big) \\ & - \frac{1}{4} a_1^2 (\alpha_3+\alpha_4+\alpha_5+\alpha_6) - \frac{1}{2} a_1 a_2 \alpha_6 - \frac{1}{4} a_2^2 (\alpha_1+\alpha_2+\alpha_6+\alpha_7). \end{aligned} \tag{9}$$

Applying the integration by parts method, e.g., to integrals with selfenergy insertions in the fermion line 'scratches' corresponding fermion lines. For the remaining reduction of tensor integrals we apply (8) to the scalar integrals, setting $a_i = 0$ after differentiation. It is also convenient to consider integrals of a more general type with the topology given in Fig. 2. To deal with six-line diagrams (Fig. 1), the corresponding α_i are put to zero.

With the help of recurrence relations one can reduce any integral with six lines to a combination of integrals with at least one line contracted. The two-loop correction in the on-shell renormalization scheme reads

$$\begin{aligned} A_t^{(1)} = -\hat{A}_t \Bigg\{ & \frac{\theta_t(1+\theta_t)(1+\theta_t^2)}{(1-\theta_t)^5} \Bigg[\frac{1}{48} \ln^4\theta_t + \left(\frac{7}{2} \mathrm{Li}_2(\theta_t) + 2\,\mathrm{Li}_2(-\theta_t) + \zeta_2 \right) \ln^2\theta_t + \frac{9}{2}\zeta_4 \\ & \qquad - 4\big(\mathrm{Li}_3\big(\theta_t^2\big) - \zeta_3\big) \ln\theta_t + 36\,\mathrm{Li}_4(-\theta_t) + 27\,\mathrm{Li}_4(\theta_t) \Bigg] \\ & + \frac{\theta_t(3\theta_t^3 - 7\theta_t^2 + 25\theta_t + 3)}{12(1-\theta_t)^5} \ln^3\theta_t + \frac{\theta_t(5\theta_t^2 - 6\theta_t + 5)}{2(1-\theta_t)^4} \ln^2\theta_t \ln(1-\theta_t) \\ & + \frac{2\theta_t(3\theta_t^2 - 2\theta_t + 3)}{(1-\theta_t)^4} \ln\theta_t\,\mathrm{Li}_2(\theta_t) + \frac{\theta_t(1+\theta_t)^2}{(1-\theta_t)^4} \big[4\ln\theta_t\,\mathrm{Li}_2(-\theta_t) - \zeta_2 \ln\theta_t - 8\,\mathrm{Li}_3(-\theta_t) \big] \\ & - \frac{\theta_t(7\theta_t^2 - 2\theta_t + 7)}{(1-\theta_t)^4} \mathrm{Li}_3(\theta_t) + \frac{\theta_t(\theta_t^2 - 14\theta_t + 1)}{(1-\theta_t)^4} \zeta_3 + \frac{3\theta_t^2}{(1-\theta_t)^4} \ln^2\theta_t \\ & - \frac{3\theta_t(1+\theta_t)}{(1-\theta_t)^3} \ln\theta_t - \frac{5\theta_t}{(1-\theta_t)^2} \Bigg\}. \end{aligned} \tag{10}$$

The expansion at $\tau_t = 0$ agrees with the known result [6,7]:

$$\begin{aligned} A_t^{(1)} = -\hat{A}_t \Bigg(& 1 - \frac{122}{135}\tau_t - \frac{8864}{14175}\tau_t^2 - \frac{209186}{496125}\tau_t^3 - \frac{696616}{2338875}\tau_t^4 \\ & - \frac{54072928796}{245827456875}\tau_t^5 - \frac{21536780128}{127830277575}\tau_t^6 - \cdots \Bigg) \end{aligned} \tag{11}$$

and the asymptotic behavior near threshold $\tau_t \simeq 1$ is:

$$A_t^{(1)} = -\hat{A}_t\left[\frac{5}{4} - \frac{3}{16}\pi^2 + \frac{1}{2}\pi^2\ln 2 - \frac{7}{4}\zeta_3 + \left(2 - \frac{9}{8}\pi^2 + \frac{7}{4}\pi^2\ln 2 + \frac{\pi^2}{4}\ln(1-\tau_t)\right)(1-\tau_t) + O\big((1-\tau_t)^{3/2}\ln(1-\tau_t)\big)\right]. \tag{12}$$

At $m_H^2 > 4m_t^2$ the imaginary part of (10) is in agreement with the result of [8].

The only master integral of the transcendentality four contributing to the result is:

$$\begin{aligned} H_4 &= \frac{m_t^4}{(i\pi^2)^2}\iint d^4k_1\, d^4k_2\, P_{k_1+q_1,m_t} P_{k_1+q_2,m_t} P_{k_2+q_1,m_t} P_{k_2+q_1,m_t} P_{k_1-k_2,m_t} P_{k_2,m_t} \\ &= \frac{2\theta_t^2}{(1-\theta_t)^3(1+\theta_t)}\left[\left(\frac{7}{2}\mathrm{Li}_2(\theta_t) + 2\,\mathrm{Li}_2(-\theta_t) + \zeta_2\right)\ln^2\theta_t - 4\big(\mathrm{Li}_3(\theta_t^2) - \zeta_3\big)\ln\theta_t + \frac{1}{48}\ln^4\theta_t \right. \\ &\qquad \left. + \frac{9}{2}\zeta_4 + 27\,\mathrm{Li}_4(\theta_t) + 36\,\mathrm{Li}_4(-\theta_t)\right], \end{aligned} \tag{13}$$

where $P_{k,m}^{-1} = k^2 - m^2 + i\epsilon$ and $q_1^2 = q_2^2 = 0$, $(q_1 - q_2)^2 = M_H^2$. For the calculation of this result a method to some extent resembling the 'Risch algorithm' [9] was developed. To conclude, we stress that the availability of analytic results is of great value since they allow to cover the whole kinematical domain with one expression, serve as checks for approximation, allow to represent the analytic properties of the amplitudes and can also be used for numerical evaluation in all domains of interest.

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