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Procedia - Social and Behavioral Sciences 197 (2015) 398 – 403
Procedia Social and Behavioral Sciences

7th World Conference on Educational Sciences, (WCES-2015), 05-07 February 2015, Novotel Athens Convention Center, Athens, Greece

A Real Context Problem for Consolidating the Similarity

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Abstract

Constructing a new mathematical structure depends on conceptual understanding and connection with previous constructs. For the purpose of obtaining a new mathematical structure, the process of constructing a concept is defined as abstraction. The weaknesses of new structures that are generated create need their consolidation and so consolidation has been added as a step of the abstraction process. The purpose of this case study is to examine the solving process of a real context problem for consolidating similarity of triangles. The participants are two mathematics education master students. Consequently, it has been determined that participants have similarity knowledge theoretical but in order to consolidate it, they have to encounter more real context problems.

Peer-review under responsibility of Academic World Education and Research Center.

*Keywords:*RBC+C, abstraction, consolidation, real context problem, similarity

1. Introduction

It is frequently stressed on the importance of real context problems in mathematics teaching and learning. The National Council of Teachers of Mathematics [NCTM] in its Standards (NCTM, 2000) articulated and promoted a view of mathematics teaching and learning where students solve real-world problems set in meaningful contexts, communicate their ideas in appropriate mathematical language and symbolism, make conjectures and justify their solutions. Additionally, students should have opportunities to make connections between what they do in the classroom and their daily life, and connect different representations of mathematical concepts so that they view

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Peer-review under responsibility of Academic World Education and Research Center.
doi:10.1016/j.sbspro.2015.07.155

mathematics as an integrated whole rather than a series of seemingly unrelated ideas.

In association with real-world math problems in mathematics education increases the importance of mathematics for students. Secondary School Mathematics Curriculum suggests using real context problems and carrying out modeling activities during education process (MEB, 2013). Mathematics program in our country has been made in reconstruction by adopting constructive approach in recent years. In this context, the importance of research on the transition among the world of mathematics to the real world is increasing.

Mathematizing is a bridge between the real world of the situation and the mathematical world of the model. An example of mathematizing is a student creating a variable expression (a property or part of the mathematical entity) to represent the time required to complete a real-world action (a condition or key piece of the real-world situation) (Zbiek & Conner, 2006). Abstractions can articulate what different situations have in common, they can mediate between the problem solver and new situations to which the abstractions appear to be applicable. Abstraction can be said to enhance the transfer of previously learned knowledge or abilities, on the basis of the perceived analogy among situations (Reeves & Weisberg, 1994).

Hershkowitz, Schwarz, & Dreyfus (2001) characterized abstraction as a process that takes place in a complex context that incorporates tasks, tools, other artifacts, personal histories of the participants and also in social and physical settings. Abstraction in Context is a theoretical model with three observable epistemic actions: Recognizing, Building-with and Constructing–the RBC-model. According to the model constructing incorporates the other two epistemic actions in such a way that building-with actions are nested in constructing actions and recognizing actions are nested in building-with actions and in constructing actions. The authors mention the importance of consolidation of the newly emerged structures and by adding the consolidation process the model called RBC+C model. Consolidation of a construct is likely to occur whenever a construct that emerged in one activity is built-with in further activities. These further activities may lead to new constructs. Hence consolidation connects successive constructing processes and is closely related to the design of sequences of activities (Dreyfus & Kidron, 2014).

According to Dreyfus and Tsamir (2004) the genesis of an abstraction passes through: the need for a new structure; the construction of a new abstract entity; and the consolidation of the abstract entity through repeated recognition of the new structure and building-with it with increasing ease in further activities. They put emphasis on consolidation. Tsamir and Dreyfus (2005) explained the characteristics of consolidation as follows: Immediacy refers to the speed and directness with which a structure is recognized or made use of in order to achieve a goal; self-evidence refers to the obviousness that the use of a structure has for the student; obviousness implies that the student feels no need to justify or explain the use of the structure, though (s)he is able to justify and explain it. Self-evidence is directly related to the confidence or certainty with which a structure is used. Confidence refers to be sure about activity and not to be in doubt. Frequent use of a structure is likely to support the establishment of connections, and thus contribute to the flexibility of its use. A student may be quite proficient in using a structure, even using it flexibly, but without being consciously aware that s(he)is doing so. The awareness of a structure enables the student to reflect on related mathematical and instructional issues, add to the depth of her or his theoretical knowledge and power and ease when using the structure.

Most of the researches on abstraction in context analyzed the Recognizing, Building-with and Constructing actions (Hershkowitz, Schwarz, & Dreyfus, 2001; Dreyfus, Hershkowitz, & Schwarz, 2001; Kidron & Dreyfus, 2010) several research effort has been invested in investigating the consolidation process (Dreyfus & Tsamir, 2004; Tabach, Hershkowitz & Schwarz, 2006; Monaghan & Ozmantar, 2006; Dreyfus, Hadas, Hershkowitz & Schwarz, 2006) and in terms of characteristics of consolidation (Dreyfus & Tsamir, 2004; Tsamir & Dreyfus, 2005).

Dreyfus and Tsamir (2004) developed an empirically based, theoretical analysis of consolidation that emerges from a sequence of interviews about the comparison of infinite sets with a talented student. Their analysis showed that consolidation can be identified by means of the psychological and cognitive characteristics of self-evidence, confidence, immediacy, flexibility and awareness. They proposed to take the combination of five characteristics as definition for consolidation. Tsamir and Dreyfus (2005) investigated how stable or fragile consolidated knowledge may be. They showed that under slight variations of context, knowledge structures that have apparently been well – consolidated may become inactive and subordinate to more primitive ones. Dreyfus, Hadas, Hershkowitz and Schwarz (2006), investigated that constructing processes and consolidating processes are often narrowly intertwined. With the aim of identifying mechanisms for consolidating recent knowledge constructs they analyzed processes of abstraction of a group of students working together in a classroom on tasks from a unit on probability. Monhagan and Ozmantar (2006) examined a seventeen years old student working on tasks concerned with absolute value functions. They argued that an abstraction is a consolidated construction that can be used to create new constructions and they gave evidence that an unconsolidated construction cannot be used to create new theoretical knowledge.

In this research a similarity problem in real context was chosen for examining the consolidation. The purpose is to examine the consolidation process of triangle similarity on a real-context problem. Analysis of solving process carried out for students to construct Thales theorem and rules of similarity in terms of characteristics of consolidation.

2. Method

The research is a case study whose data are quantitative. The participants are two mathematics education master students (Bilge and Tuba, denoted by B and T, and researcher by R) who took the courses about constructivism during the undergraduate and master education process. Studying together was asked them to count up the height of a frame's nail on the wall without measuring by ruler. The photograph of environment is as follows:

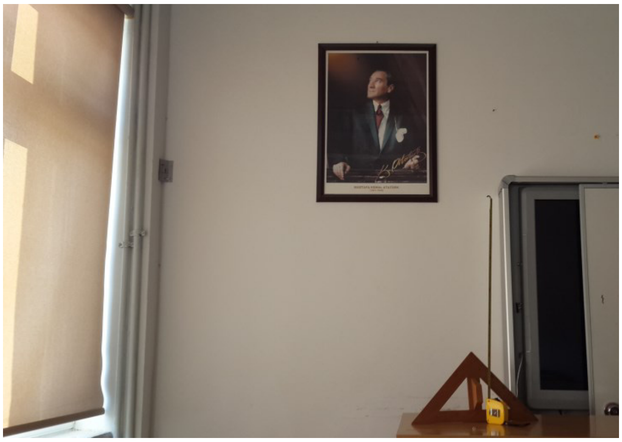

Fig. 1. Environment of activity

Generally, students take theoretical knowledge in undergraduate courses but they aren't able to apply them to real world. So, there is a rupture between theoretical knowledge and its application. In this study we chose two master students who took the courses about constructivism on the purpose of that determine whether we can overcome the aforementioned rupture with master courses. The data were collected with videotape and analyzed with descriptive methods, according to characteristics of consolidation.

3. Findings

First of all, students tried understanding what was wanted from them. Tuba asked whether they could use measuring tools, anywise. When they understood they could use measuring tools for activity except measuring height directly, they thought and began to discuss what they would do to find the height of nail. A dialog which refers for students to focus on estimating and symmetry concept at the beginning of discussion is as follows:

B7: We must find the 1 m on the yellow material (She meant tape measure) then estimate the height but this is very simple. It is estimation, not a precise result.

T8: Imagine that the edge which is on the floor is x-axis and let the symmetry of the edge according to x-axis.

What happened till this period referred that students had immediacy, flexibility and self- evidence on symmetry and estimating. Because they used prior mathematical structures, tried to solve problem by using different ways and they didn't need to prove prior concepts on their minds. But it was observed that there was lack of awareness and confidence. They didn't sure about their goal and what they were doing. After the abovementioned dialog, they realized that they couldn't find anything by using symmetry. Then researchers asked "Didn't you think anything else except symmetry?". After this question they went to the board for devising and drawing. First of all, they tried to draw a triangle of 45^o-45^o-90^o, but they realized that they couldn't find anything via this design. Then researchers asked what they thought till that moment. Students' of answers are as follows:

R20: You said that you were able to use symmetry but later you renounced that idea. Why?

B21: Because we thought that we couldn't find the certain point by using symmetry.

T22: We will be able to set a point estimate, and then measure the height. But this will be estimation.

B23: We can agree by just looking at 1 m and think how many 1 meters are there. But this will be estimation too.

R24: If there is a problem like this on the paper, which geometric properties will you use for find that height?

B25: Maybe we can use similarity.

It is observed on the abovementioned dialogs that students couldn't use similarity concept for a real context problem, but they could use it for a problem on the paper. After the Bilge's answer researchers asked how they could use similarity. But students had difficulty in explaining the design, so researchers let them to explain it with drawing.

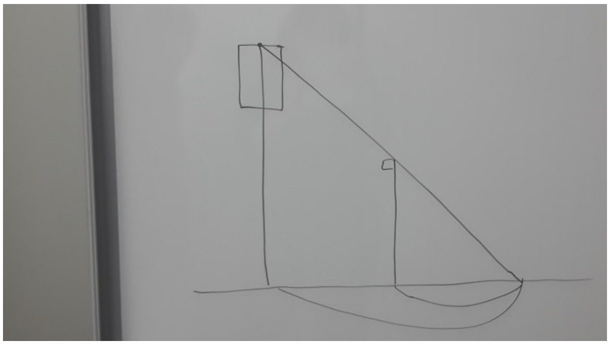

Fig. 2. Students' drawing

B32: Imagine there is a tree (She meant height of nail.) and there is a little pole. If we measure the height of pole, we can use the similarity and find the height of the nail, can't we?

R33: Let's ask your partner, was she persuaded?

T34: I think we can.

R35: At that case, can you try to examine it?

After that period, student began how they could use the similarity for that design.

B36: For example, I thought height of that table as pole. We must determine such a point in here (on the floor). But according to what do we determine it?

For measuring, they tried to devise according to drawing which they had done before. They didn't decide where they had to put the pole. They tried to find a point on the floor, but they weren't sure what they were doing. So researchers asked "It is difficult for you to look by kneeling on the floor, we wonder if you can choose higher place than floor?" Students weren't sure what they were doing still, so researchers asked "if can you choose that table as x-axis?" Then they began to think on that idea.

T49: Let your distance to there and to table be equal.

B50: But how can we know that the height is two times more of my length?

After that they tried to look from the table to the nail. They imagined that Tuba was a pole. Bilge tried to see the nail from Tuba's head level by looking from the table. Then they started to measure the lengths. At that period, they had waited for approval from researchers before they measured them. It is referred to lack of confidence that they waited for approval at all periods.

Firstly, they measured distance of A (238 cm) and then B (270 cm).Secondly, they measured Tuba's length (175cm) and height of table (75 cm). After that, they proportioned and calculated similarity ratio. They were sure that they had to use Thales Theorem, so they didn't need to prove the theorem. They found the result as 373,5 cm and then added 75 cm to it. They found the height as 448,5 cm. But after the operations, they thought that the height could be really such high. The researchers asked "If you need, you can control it. Where is the 175 cm?". In response to this question, Tuba said immediately that we had to subtract 75 cm from my length. Bilge agreed with her friend and calculated again. Finally they found the height as 288 cm. To control whether they had calculated right, they measured the height with tape measure and found the real value as 285 cm.

In conclusion, that students couldn't know what they had to do at the beginning will arise from that they didn't understand the goal accurately. This refers to lack of awareness. There isn't lack of immediacy, flexibility and self-evidence, but the lack of confidence has been observed throughout the study. Because students were in need approval from researchers permanently.

4. Conclusion and Discussion

In this research, it were emphasized that five characteristics of consolidation which were defined by Dreyfus and Tsamir (2004) and it were seen that these characteristics could be used for whether a construct was consolidated or not. It was determined that students didn't have difficult on immediacy, flexibility and self-evidence characteristics; but they had difficult awareness and confidence characteristics. For students to wait for researchers' approval about the exercise was the clearest sign that they had lack of awareness and confidence.

Constructing does not refer to the construct becoming freely and flexibly available to the learner: becoming freely and flexibly available pertains to consolidation (Dreyfus & Kidron, 2014). When considered this point of view, it can be said that the students built and constructed the similarity concept but they couldn't consolidate it enough.

Monhagan and Ozmantar (2006) argued that an abstraction is a consolidated construction that can be used to create new constructions and they gave evidence that an unconsolidated construction cannot be used to create new theoretical knowledge. Our research has showed that consolidation has enabled to not only create new constructs but also be used current constructs in real context problems. A mathematical construct can be consolidated by using in a real context problem.

Tsamir and Dreyfus (2005) showed that under slight variations of context, knowledge structures that have apparently been well –consolidated may become inactive and subordinate to more primitive ones. For example at schools in Turkey, many exercises are done for consolidating the constructs. For this purpose, when teaching similarity, teachers solve many routine problems at middle and high schools. When solving these kinds of problems, it is apparently seen that students are successful and the construct is consolidated well. But changing variations, like solving a non-routine or real context problem, it is seen that the construct is not consolidated adequately. Also in this research, it has been faced the fact that the similarity concept didn't consolidate actually because of lack of using real context problems.

Consequently, it was clearly seen that student had similarity knowledge theoretical but there was lack of using it in daily life. Considering the five characteristics of consolidation, the lack of two characteristics indicated that the structure wasn't able to consolidating adequately. To consolidate the structures, it is recommended to include more daily life examples and exercises in mathematics courses.

References

Dreyfus, T., Hershkowitz, R., & Schwarz, B. B. (2001). Abstraction in context II: the case of peer interaction. *Cognitive Science Quarterly*, 1(3/4), 307–368.

Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *Journal of Mathematical Behavior*, 23, 271–300.

Dreyfus, T., Hadas, N., Hershkowitz, R., & Schwarz B.B.(2006). Mechanisms for consolidating knowledge constructs. In J Novotna, H.Moraova, M. Kratka, & N. Stelinkova (Eds.). *Proceedings of the 30th International Conference for the Psychology of Mathematics Education*, 2, 465-472, Prague.

Dreyfus, T. & Kidron, I. (2014). Introduction to Abstraction in Context. In Bikner-Ahsbahs, A. & Prediger, S. (eds.), *Networking of Theories as a Research* 85 *Practice in Mathematics Education*, Advances in Mathematics Education, Springer International Publishing Switzerland..

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education*, 32, 195–222.

Kidron, I. (2008). Abstraction and consolidation of the limit precept by means of instrumented schemes: The complementary role of three different frameworks. *Educational Studies in Mathematics*,69, 197–216.

Kidron, I. & Dreyfus, T. (2010). Justification enlightenment and combining constructions of knowledge. *Educational Studies in Mathematics*. 74, 75–93.

MEB (2013). *Secondary School Mathematics Curriculum*. Ankara: MEB Press.

Monaghan, J., & Ozmantar, M. F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics*, 62, 233–258.

National Council of Teachers of Mathematics, (2000). *Principles and standards for school mathematics.* Edited by NCTM, Reston, VA: National Council of Teachers of Mathematics.

Schwarz, B., Hershkowitz, R. & Dreyfus, T. (2002). Abstraction in Context: Construction and Consolidation of Knowledge Structures. In Cockburn, A. D. & Nardi, E. (Eds.), *Proceedings of PME26*, Norwich, UEA, 1, 120–125.

Tabach, M., Hershkowitz, R. & Schwarz, B. B. (2001). The Struggle Towards Algebraic Generalization and its Consolidation. In van den Heuvel, M. (Ed.), *Proceedings of PME25*. Utrecht, The Netherlands, 4, 241–248.

Tsamir, P. & Dreyfus, T., (2005). How fragile is consolidated knowledge? Ben's comparisons of infinite sets. *Journal of Mathematical Behavior*, 24, 15-38.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics*, 63(1), 98–112.