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Note

# On 1-factors and matching extension

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**Abstract**

We prove the following: (1) Let $G$ be a graph with a 1-factor and let $F$ be an arbitrary 1-factor of $G$. If $G \setminus \{a,b\}$ is $k$-extendable for each $ab \in F$, then $G$ is $k$-extendable. (2) Let $G$ be a graph and let $M$ be an arbitrary maximal matching of $G$. If $G \setminus \{a,b\}$ is $k$-factor-critical for each $ab \in M$, then $G$ is $k$-factor-critical. © 2000 Elsevier Science B.V. All rights reserved.

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We consider only finite simple graphs and follow Chartrand and Lesniak [1] for general terminology and notation. Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. For $A \subset V(G)$, $G[A]$ denotes the subgraph of $G$ induced by $A$ and $G \setminus A$ is the subgraph of $G$ induced by $V(G) \setminus A$. We often identify $G[A]$ with $A$. Further, let $H$ be a subgraph of $G$. For $M \subset E(H)$, $H_e[M]$ denotes the subgraph of $H$ induced by $M$. If $A$ and $B$ are disjoint subsets of $V(G)$, then $E(A,B)$ denotes the set of edges with one end in $A$ and the other in $B$. Further, if $F$ is a subgraph of $G$ and $v$ is a vertex in $G$, we may write simply $G[F]$ instead of $G[V(F)]$, $G \setminus F$ instead of $G \setminus V(F)$, and $E(v,F)$ instead of $E(\{v\},V(F))$. The set of endvertices of an edge $e$ is denoted by $V(e)$ and for a matching $M$, let $V_e(M) = \bigcup_{e \in M} V(e)$.

Let $k \geqslant 0$ and $p > 0$ be integers with $k \leqslant p-1$ and $G$ a graph with $2p$ vertices having a 1-factor. Then $G$ is said to be *$k$-extendable* (*$k$-ext* in brief) if every matching of size $k$ in $G$ can be extended to a 1-factor. A graph $G$ of order $p$ is *$k$-factor-critical* (*$k$-$fc$* in brief), where $k$ is an integer of the same parity as $p$ with $0 \leqslant k \leqslant p$, if $G \setminus X$ has a 1-factor (a perfect matching) for any set $X$ of $k$ vertices of $G$. In particular, $G$ is 0-factor-critical or 0-extendable if and only if $G$ has a 1-factor.

In this note, we will prove the following theorems.

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**Theorem 1.** *Let $G$ be a graph with a 1-factor and let $F$ be an arbitrary (fixed) 1-factor of $G$. If $G \setminus V(e)$ is $k$-extendable for each $e \in F$, then $G$ is $k$-extendable.*

**Theorem 2.** *Let $G$ be a graph and let $M$ be an arbitrary (fixed) maximal matching of $G$. If $G \setminus V(e)$ is $k$-factor-critical for each $e \in M$, then $G$ is $k$-factor-critical.*

Theorems 1 and 2 are extensions of the following theorems, respectively.

**Theorem 3** (Nishimura and Saito [7]). *Let $G$ be a graph with a 1-factor. If $G \setminus V(e)$ is $k$-extendable for each $e \in E(G)$, then $G$ is $k$-extendable.*

**Theorem 4** (Favaron and Shi [5] and Nishimura [6]). *Let $G$ be a graph. If $G \setminus V(e)$ is $k$-factor-critical for each $e \in E(G)$, then $G$ is $k$-factor-critical.*

In actuality, each of the papers [5–7] contains stronger results than Theorems 3 and 4.

We use several lemmas for the proofs of Theorems 1 and 2. In particular, our theorems heavily depends on Lemma 5.

**Lemma 5** (Tutte [10]). (I) *A graph $G$ has a 1-factor iff $o(G \setminus S) \leqslant |S|$ for all $S \subset V(G)$ and*

(II) $o(G \setminus S) - |S| \equiv 0 \pmod 2$ *if $G$ has even order*, *where $o(G)$ denotes the number of odd components of $G$.*

**Lemma 6** (Plummer [8]). *Let $k$ be a positive integer and let $G$ be a $k$-extendable graph. Then $G$ is $(k-1)$-extendable. Further, if $G$ is connected, then $G$ is $(k+1)$-connected.*

**Lemma 7.** *Let $G$ be a graph of order $n \geqslant k+4$ and let $e$ and $f$ be two independent edges of $G$. If $G \setminus V(e)$ and $G \setminus V(f)$ are $k$-connected, then $G$ is $k$-connected.*

**Proof.** Let $G$ be a graph satisfying the conditions of the lemma. Suppose that $G$ is not $k$-connected. Let $S$ be a cutset of $G$ with $|S| \leqslant k-1$ and let $e_1 = a_1 b_1$ and $e_2 = a_2 b_2$ be two independent edges of $G$. Since $G \setminus V(e_i)$ is $k$-connected, clearly $S$ is not a cutset of $G \setminus V(e_i)$ $(i=1,2)$. If $S \subset G \setminus V(e_i)$, then $G \setminus (V(e_i) \cup S)$ and $e_i$ must be components of $G \setminus S$. Therefore, $E(V(e_i),\ G \setminus (V(e_i) \cup S)) = \emptyset$. But since $G \setminus V(e_{3-i})$ is also $k$-connected, we have $|N_{G \setminus (V(e_i) \cup V(e_{3-i}))}(V(e_i))| \geqslant k$, where $N_G(S)$ denotes the neighborhood of $S$ of $V(G)$. Further, since $|S| \leqslant k-1$, $E(V(e_i), G \setminus (V(e_i) \cup S)) \neq \emptyset$, which is a contradiction. Hence $S \not\subset V(G) \setminus V(e_i)$ $(i = 1,2)$. Without loss of generality, we may assume that $\{a_1, a_2\} \subset S$. Now let $D_1$ and $D_2$ be two components of $G \setminus S$. Of course, $E(D_1, D_2) = \emptyset$. If $D_1 \subset G \setminus V(e_1)$, then we have $E(D_1, G \setminus [V(e_1) \cup D_1]) \subset E(D_1, S \setminus \{a_1\})$. Hence $D_1$ is a component of $G \setminus [V(e_1) \cup (S \setminus \{a_1\})]$. If $[G \setminus V(e_1)] \setminus [S \setminus \{a_1\}] \setminus D_1 \neq \emptyset$, then $S \setminus \{a_1\}$ is a cutset in $G \setminus V(e_1)$ of order at most $k-2$, which is a contradiction.

Therefore, $[G \setminus [V(e_1)] \setminus [S \setminus \{a_1\}] \setminus D_1 = \emptyset$, i.e., $G \setminus V(e_1) = G[(S \setminus \{a_1\}) \cup D_1]$. Then $D_2$ must be $b_1(=G[\{b_1\}])$.

On the other hand, since $d_{G\setminus V(e_2)}(b_1) \geqslant K$ and $|E(b_1, S)] \leqslant k-1$, we have $d_{D_1}(b_1) \geqslant 1$. This implies $E(D_1, D_2) \neq \emptyset$, which is a contrdiction. Therefore, $D_1 \not\subset G \setminus V(e_1)$. The same argument gives $D_1 \not\subset G \setminus V(e_2)$. Thus, it must hold that $\{b_1, b_2\} \subset D_1$. However, if $V(D_2) \neq \emptyset$, then $D_2 \subset G \setminus V(e_1)$ or $D_2 \subset G \setminus V(e_2)$. Again similar arguments as in the above lead a contradiction, which completes the proof. $\square$.

**Proof of Theorem 1.** Let $G$ be a graph satisfying the condition of the theorem. If $k=0$, then clearly the theorem holds. We may assume $k > 0$. Suppose that there exists a 1-factor $F$ of $G$ such that $G \setminus V(e)$ is $k$-ext for each $e \in F$ but $G$ is not $k$-ext. Then, for some matching $M$ with size $k, G \setminus R$ has no 1-factor, where $R = G[V_e(M)]$. Further, by Lemma 5, we have $o((G\setminus R)\setminus S) \geqslant |S|+2$ for some vertex subset $S \subset V(G) \setminus R$. Our purpose is to show $G \setminus V(f)$ is not $k$-ext for some $f \in F$. Let $W = (G\setminus R)\setminus S := G\setminus R\setminus S$.

**Claim 1** ($F \subset E(R, G \setminus R) \cup E(R)$ and $F \cap M = \emptyset$).

Suppose that an edge $e=ab$ is in $F \cap E(G\setminus R)$. If $e \in E(S)$, then $[G \setminus V(e)] \setminus R \setminus [S \setminus \{a,b\}]$ has all odd components of $W = G\setminus R\setminus S$, i.e., we have $o([G\setminus V(e)]\setminus R\setminus[S\{a,b\}]) = o(W) \geqslant |S|+2$. If $e \in E(W)$, then we have $o([G \setminus V(e)] \setminus R \setminus S) \geqslant o(W) \geqslant |S|+2$. If $e \in E(S, W)$, then for $a \in S$ and $b \in W$, we have $o([G \setminus V(e)] \setminus R \setminus [S \setminus \{a\}]) \geqslant o(W) - 1 \geqslant |S|+1$. Each of them means that $G \setminus V(e)$ is not $k$-ext, a contradition. Therefore, $F \subset E(R, G \setminus R) \cup E(R)$.

Further, if $e \in F \cap M$, then we have $o([G \setminus V(e)] \setminus [R \setminus V(e)] \setminus S) = o(W) \geqslant |S|+2$, which means $G \setminus V(e)$ is not $(k-1)$-ext, i.e., $G \setminus V(e)$ is not $k$-ext by Lemma 6, a contradiction.

By Claim 1, we clearly have $|G \setminus R| \leqslant |R|$.

**Claim 2** (All components of $R_e[F \cup M]$ are alternating paths).

By Claim 1, since $F \cap M = \emptyset$, obviously $R_e[F \cup M]$ induces only even cycles or alternating paths. Note that such an alternating path's endedges are in M.

Suppose that $R_e[F \cup M]$ contains an even cycle $D = a_1 a_{2m} \dots a_{2m-1} a_{2m} a_1$. Let $M_1 = \{a_{2j} a_{2j+1} \mid j=1,2,\dots,m\} \subset M$, where $a_{2m+1} = a_1$, and $M_2 = \{a_{2j-1} a_{2j} \mid j=1,2,\dots,m\} \subset F$. Note that if $G$ has no 1-factor containing $M$, then since $R = G[V_e(M)] = G[V_e((M \setminus M_1) \cup M_2)]$ and $|M| = |(M \setminus M_1) \cup M_2|, G$ also has no 1-factor containing $(M \setminus M_1) \cup M_2$. By the hypothesis and Lemma 6, $G \setminus \{a_1, a_2\}$ is $(k-1)$-ext. But since $[G \setminus \{a_1, a_2\}] \setminus [V_e((M \setminus M_1) \cup (M_2 \setminus \{a_1 a_2\}))] = G \setminus R$ has no 1-factor, $G \setminus \{a_1, a_2\}$ is not $(k-1)$-ext, a contradiction. Thus, Claim 2 holds.

In the rest of proof, $a_1 A a_2$ denotes the component, i.e., alternating path, in $R_e[F \cup M]$ with the endvertices $a_1$ and $a_2$.

**Claim 3** ($S = \emptyset$ and $W = G \setminus R$ has no even component).

Suppose $S \neq \emptyset$. Let $e = ab \in F \cap E(S, R)$, $a \in S$ and $b \in R$. Since $b$ is in $R$, for some alternating path $bAc$ in $R_e[F \cup M]$, there exists the vertex $d \in V(G)\setminus R$ such that $cd \in F$.

Let $M' = M \cup (F \cap bAc) \cap \{cd\} \setminus (M \cap bAc)$. Then note that $R \cup \{d\} \setminus \{b\} = V_e(M')$ and $|M'| = k$. If $d \in S$, then we have $o([G \setminus V(e)] \setminus [R \cup \{d\} \setminus \{b\}] \setminus [S \setminus \{a,d\}]) = o(W) \geqslant |S| + 2$, which means $G \setminus V(e)$ is not $k$-ext, a contradiction. When $d \in W$, even if $d$ is in an odd component of $W$, then we have $o([G \setminus V(e)] \setminus [R \cup \{d\} \setminus \{b\}] \setminus [S \setminus \{a\}]) \geqslant o(W) - 1 \geqslant |S| + 1$. Again we have a contradiction. Thus, $S = \emptyset$. Similarly, we can easily prove that $W$ does not have an even component.

**Claim 4** ($o(G \setminus R) = 2$).

By Claim 3, all components of $G \setminus R$ are odd. Further, by Lemma 5, the number of odd components is even. Let $\{C_1, C_2, \ldots, C_m\}$ be the set of odd components of $G \setminus R$. Suppose $m = o(G \setminus R) \geqslant 4$. Let $e = ab \in F \cap E(C_1, R)$, $a \in C_1$ and $b \in R$. Then, there exists an alternating path $bAc$ in $R$ and $cd \in F$ with $d \in V(G) \setminus R$. Let $M' = M \cup (F \cap bAc) \cup \{cd\} \setminus (M \cap bAc)$. Then note that $R \cup \{d\} \setminus \{b\} = V_e(M')$ and $|M'| = k$. When $d \in W$ (even when $d \in W \setminus V(C_1)$), we have $o([G \setminus V(e)] \setminus [R \cup \{d\} \setminus \{b\}]) \geqslant o(G \setminus R) - 2 \geqslant 2$, which implies $G \setminus V(e)$ is not $k$-ext, a contradiction.

Thus, $\{C_1, C_2\}$ is the set of all components of $G \setminus R$. Notice that the following observation holds:

($\bigstar$) *Let $uA'v$ be a subpath with odd length of $uAw$ in $R_e[F \cup M]$ and let $x$ and $y$ be two distinct vertices of $G \setminus R$ such that $ux \in F$ and $vy \in E(G)$ (clearly if $v = w$, then we can take $y \in V(G) \setminus R$ with $vy \in F$, and if $v \neq w$, then $vy \in E(G) \setminus F$). Then, $M' = M \cup (F \cap uA'v) \cup \{vy\} \setminus (M \cap uA'v)$ is a matching with size $k$. If $x$ and $y$ are in the same component $C_1$ or $C_2$, say $C_1$, then since $C_2$ is also an odd component of $[G \setminus \{u,x\}] \setminus [R \cup \{y\} \setminus \{u\}]$ $(=[G \setminus \{u,x\}] \setminus V_e(M'))$, $[G \setminus \{u,x\}] \setminus [R \cup \{y\} \setminus \{u\}]$ has no 1-factor, i.e., $G \setminus \{u,x\}$ is not $k$-ext.*

Since both components $C_1$ and $C_2$ have odd order, if $|C_1| > |C_2|$, then $|C_1| \geqslant |C_2| + 2$. Therefore, by Claim 1, we can easily find four vertices $x, y \in C_1$ and $u, v \in R$ satisfying the situation of ($\bigstar$), a contradiction. We may assume that $|C_1| = |C_2| = h$ and that $h$ is odd. Then $|V(G)| = |R| + |C_1 \cup C_2| = 2k + 2h$. Since $G \setminus V(e)$ is $k$-ext, $|V(G)| = |G \setminus V(e)| + |V(e)| \geqslant (2k + 2) + 2$. Hence, we may assume $h \geqslant 3$. Further, we may assume that each of alternating paths in $G_e[F \cup M]$ satisfies one endvertex in $V(C_1)$ and the other in $V(C_2)$.

Let $V(C_1) = \{x_1, x_2, \ldots, x_h\}$, $V(C_2) = \{y_1, y_2, \ldots, y_h\}$. We may assume that $u_i$ and $v_i$ are endvertices of an alternating path $P_i$ in $R_e[F \cup M]$, i.e., $R_e[F \cup M] = P_1 \cup P_2 \cup \cdots \cup P_h$, where $P_i = u_i A v_i$. And let $x_i u_i \in F$ and $y_i v_i \in F (i = 1, 2, \ldots, h)$. Furthermore, let $U = \{z \,||E(u_i A' z)| \equiv 0 \pmod 2, i = 1, 2, \ldots, h\}$ and $V = \{z \,||E(u_i A' z)| \equiv 1 \pmod 2, i = 1, 2, \ldots, h\}$, where $u_i A' z$ denotes the subpath of $u_i A v_i$ with endvertices $u_i$ and $z$. Of course, $|U| = |V| = k$, $\{u_1, u_2, \ldots, u_h\} \subset U, \{v_1, v_2, \ldots, v_h\} \subset V$, and $G_e[F \cup M] = \cup_{i=1}^{h} x_i u_i A v_i y_i$.

Now, we have that $E(x_i, P_j \cap V) = \emptyset$ for $i \neq j$. Because if $v \in P_j \cap V$ is a vertex satisfying $x_i v \in E(G)$, then four vertices $x_j, x_i, u_j$, and $v$ are playing the roles $x, y, u$, and $v$ in ($\bigstar$), respectively. Thus, $E(C_1 \setminus \{x_i\}, P_i \cap V) = \emptyset$ for each $i$. Similarly, $E(y_i, P_j \cap U) = \emptyset$ for $i \neq j$.

Let

$$M' = M \cup (F \cap u_1Av_1) \cup (F \cap u_2Av_2)$$
$$\cup \{x_1u_1, x_2u_2\} \setminus ((M \cap u_1Av_1) \cup (M \cap u_2Av_2)).$$

Then $M'$ is a matching with $|M'| = k$ in $G \setminus \{v_1, y_1\}$. And note that $v_1y_1 \in F$. By the previous paragraph, $E(v_2, C_1 \setminus \{x_2\}) = \emptyset$ and hence $E(C_1 \setminus \{x_1, x_2\}, (C_2 \cup \{v_2\}) \setminus \{y_1\}) = \emptyset$. Since $|C_1 \setminus \{x_1, x_2\}| \equiv |(C_2 \cup \{v_2\}) \setminus \{y_1\}| \equiv 1 \pmod 2$, $(G \setminus \{v_1, y_1\}) \setminus V_e(M')$ has at least two odd components, and hence $M'$ cannot extend to a 1-factor in $G \setminus \{v_1, y_1\}$. This contradicts the assumption, and the theorem follows. □

Next, we will give a proof of Theorem 2. The proof technique is very similar to the one of Theorem 1. But this proof is much easier than that of Theorem 1.

**Lemma 8** (Favaron [4]). (I) *If $G$ is $k$-factor-critical of order $p > k$, then $G$ is $k$-connected, and*

(II) *for $k \geqslant 2$, any $k$-factor-critical graph of order $p > k$ is $(k-2)$-factor-critical.*

**Proof of Theorem 2.** Let $G$ be a graph satisfying the condition of the theorem. If $k = 0$, then clearly the theorem holds. We may assume $k > 0$. By the hypothesis and Lemma 8(I), since $|G \setminus V(e)| \geqslant k + 2$, we have $|V(G)| = |G \setminus V(e)| + |V(e)| \geqslant k + 4$. Further, we may assume the size of maximal matching is at least 2. By Lemma 7, $G$ is connected.

Suppose that there exists a maximal matching $M$ of $G$ such that $G \setminus V(e)$ is $k$-fc for each $e \in M$ but $G$ is not $k$-fc. Then, for some vertex subset $R$ of order $k, G \setminus R$ has no 1-factor. Further, by Lemma 5, we have $o(G \setminus R \setminus S) \geqslant |S| + 2$ for some vertex subset $S \subset V(G) \setminus R$. Our purpose is to show $G \setminus V(f)$ is not $k$-fc for some $f \in M$. Let $W = G \setminus R \setminus S$.

**Claim 1** ($M \subset E(R, G \setminus R)$).

If $e \in M \cap E(R)$, then we have $o([G \setminus V(e)] \setminus [R \setminus V(e)] \setminus S) = o(W) \geqslant |S| + 2$, which means $G \setminus V(e)$ is not $(k-2)$-*fc*, i.e., $G \setminus V(e)$ is not $k$-*fc* by Lemma 9(II), a contradiction.

If there exists an edge $e \in M \cap E(G \setminus R)$, then we can obtain $G \setminus V(e)$ is not $k$-*fc* by the same argument as in the proof of Claim 1 of Theorem 1, a contradiction.

**Claim 2** ($S = \emptyset$ and $W = G \setminus R$ has no even component).

Suppose $S \neq \emptyset$. Since $M \neq \emptyset$, some edge $e = ab \in M$ satisfies $e \in E(S, R)$ or $e \in E(W, R)$. Let $a \in S \cup W$ and $b \in R$. If $e \in E(S, R)$, then for a vertex $c \in W$, we have $o([G \setminus V(e)] \setminus (R \cup \{c\} \setminus \{b\}) \setminus (S \setminus \{a\})) \geqslant o(W) - 1 \geqslant |S| + 1$, which implies $G \setminus V(e)$ is not $k$-*fc*, a contradiction. If $e \in E(W, R)$, then for a vertex $d \in S$, we have $o([G \setminus V(e)] \setminus (R \cup \{d\} \setminus \{b\}) \setminus (S \setminus \{d\})) \geqslant o(W) - 1 \geqslant |S| + 1$. Again, we have a contradiction. Thus, we have $S = \emptyset$.

Suppose that $W$ has an even component $D$. By the connectedness of $G$ and the maximality of $M$, there exists an edge $e = ab \in M \cap E(D, R)$. Let $a \in D$ and $b \in R$. Then, since $|D|\geqslant 2$, for a vertex $c\in D\setminus\{a\}, [G\setminus V(e)]\setminus(R\cup\{c\}\setminus\{b\})$ has all odd components of $G\setminus R$, i.e., we have $o([G\setminus V(e)]\setminus(R\cup\{c\}\setminus\{b\}))\geqslant o(G\setminus R)\geqslant 2$, a contradiction.

**Claim 3** ($o(G\setminus R)=2$).

Let $\{C_1,C_2,\dots,C_m\}$ be the set of odd components of $G\setminus R$, where $m=o(G\setminus R)$. Suppose $m\geqslant 4$ since $m$ is even. Without loss of generality, we may assume $e=ab\in E(C_1,R)\cap M$. Let $a\in C_1$, $b\in R$ and $c\in V(C_2)$. Then $[G\setminus V(e)]\setminus[R\cup\{c\}\setminus\{b\}]$ has odd components $C_3,\dots,C_m$ of $G\setminus R$, i.e., $o([G\setminus V(e)]\setminus[R\cup\{c\}\setminus\{b\}])\geqslant m-2\geqslant 2$, which implies $G\setminus V(e)$ is not $k$-*fc*, a contradiction.

Since $|V(G)|\geqslant k+4$ and $|R|=k$, we have $|C_1\cup C_2|\geqslant 4$. Further, since $C_1$ and $C_2$ are odd components, $|C_1|\geqslant 3$ or $|C_2|\geqslant 3$. We may assume $|C_1|\geqslant 3$. By Claim 1, $M\subset E(R,G\setminus R)$, the maximality of $M$, and the connectedness of $G$, there exists an edge $e\in M\cap E(C_1,R)$. Let $e=ab$, $a\in C_1$, $b\in R$. Then, we can take a vertex $c\in C_1\setminus\{a\}$ so that $o([G\setminus V(e)]\setminus(R\cup\{c\}\setminus\{b\}))\geqslant o(G\setminus R)=2$. This shows $G\setminus V(e)$ is not $k$-fc, which completes the proof of Theorem 2. □

**Remark.** In [9], Saito has proved the following 'similar type' result for the existence of a $k$-(regular) factor. This result gives an extension of a result in [2] which is similar to Theorem 3 or 4. (Recently, Enomoto and Tokuda [3] gave a further extension of Saito's result.)

**Theorem 9.** *Let $G$ be a graph with a 1-factor and let $F$ be an arbitrary (fixed) 1-facror of $G$. If $G\setminus V(e)$ has a $k$-factor for each $e\in F$, then $G$ has a $k$-factor.*

Our results are along this line of study for 'extendability' and 'factor-criticality'.

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