Physics Letters B 674 (2009) 137–138

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Constraints on an asymptotic safety scenario for the Wess–Zumino model

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ARTICLE INFO

Article history:
Received 5 August 2008
Received in revised form 18 January 2009
Accepted 8 March 2009
Available online 14 March 2009
Editor: L. Alvarez-Gaumé

PACS:
11.10.Gh
11.10.Hi
11.30.Pb

ABSTRACT

Using the nonrenormalization theorem and Pohlmeyer's theorem, it is proven that there cannot be an asymptotic safety scenario for the Wess–Zumino model unless there exists a non-trivial fixed point with (i) a negative anomalous dimension (ii) a relevant direction belonging to the Kähler potential.

In this Letter, we will consider the existence of certain renormalization group fixed points in theories of a chiral superfield. Suppose that a non-trivial fixed point exists and, moreover, that there is a renormalized trajectory [1] emanating from it, such that the low energy effective theory is well described by the Wess–Zumino model. It will be proven that, for such an asymptotic safety scenario [2] to occur, the putative fixed point must have both a negative anomalous dimension[1] and at least one relevant operator belonging to the Kähler potential. This generalizes earlier work [4] on zeros of the β-function of the Wess–Zumino model in a way that will be precisely spelt out below.

To formulate our argument, we introduce the Wilsonian effective action, S_Λ, constructed by integrating out degrees of freedom between the bare scale and a lower, effective scale, Λ (this implies that we have transferred to Euclidean space, so that momenta can be readily separated into large and small).[2] The Wilsonian effective action, being infrared safe, does not suffer from the holomorphic anomaly in the massless case. Therefore, the nonrenormalization theorem always holds and the superpotential does not renormalize, even nonperturbatively [5].

To conveniently uncover fixed point behaviour, we rescale to dimensionless variables by dividing all quantities (coordinates and fields) by Λ raised to the appropriate scaling dimension. In the case of the chiral superfield, Φ (and its conjugate) we must take account of the anomalous scaling according to

$$\Phi \to \Phi\sqrt{Z}\Lambda, \tag{1}$$

where Z is the field strength renormalization and the anomalous dimension is defined by

$$\gamma(\Lambda) \equiv \Lambda \frac{d\ln Z}{d\Lambda}. \tag{2}$$

As a consequence of the rescalings, the superpotential does now renormalize, but just according to the (anomalous) mass dimension of the various couplings. In particular, denoting the rescaled three-point superpotential coupling by $\lambda(\Lambda)$, we have that

$$\beta_\lambda \equiv \Lambda \frac{d\lambda}{d\Lambda} = \frac{3\lambda\gamma}{2}. \tag{3}$$

In the rescaled variables, a fixed point is defined by

$$\Lambda\partial_\Lambda S_\star[\bar\Phi, \Phi] = 0, \tag{4}$$

where $\Lambda\partial_\Lambda$ is performed at constant $\bar\Phi, \Phi$ and a star is used to denote a fixed point quantity (it is emphasized that a fixed-point action is something which is *solved* for, using an exact renormalization group equation, not something which is chosen by hand). Immediately, it is apparent from (3) and (4) that if $\lambda_\star \neq 0$, then it must be that $\gamma_\star = 0$.

However, there is a theorem due to Pohlmeyer [6] which tells us that, if the two-point function in a scale invariant theory is canonical—i.e. the anomalous dimension is zero—then the field is a massless free field. Therefore, in the current scenario, the only fixed point (i.e. scale invariant) theory with $\gamma_\star = 0$ must correspond to the Gaussian fixed point. This was the reasoning used in

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[1] It is worth pointing out that in the vicinity of a nonperturbative fixed point, we cannot rule out a negative anomalous dimension, γ, by the usual unitarity arguments. These relate the unitarity constraint $0 \leqslant Z \leqslant 1$ to a non-negative γ via a perturbative calculation; but there is no reason to believe such a calculation at a nonperturbative fixed point (see [3] for an interesting discussion on negative anomalous dimensions).

[2] To explicitly compute S_Λ would require that we write down an exact renormalization group [1] equation. However, the following arguments are sufficiently general that all we need to do is suppose that this can be done.

0370-2693
doi:10.1016/j.physletb.2009.03.009

[4] to rule out non-trivial zeros of the β-function in the Wess–Zumino model; the same logic has also been applied to the O(N) symmetric Wess–Zumino model [7]. Here, though, we deal with general fixed point actions.

However, the condition that $\lambda_\star = 0$ is not sufficient to rule out an asymptotic safety scenario for the Wess–Zumino model. This is because, although a putative non-trivial fixed point cannot possess a three-point superpotential term, it could be that (i) λ constitutes a relevant direction at the fixed point (ii) trajectories initiated along the λ direction happen to flow towards the Gaussian fixed point. Note that a marginally relevant λ will not do, because this requires $\gamma_\star = 0$ and we again fall foul of Pohlmeyer's theorem.

Let us suppose that such a scenario is realized: the non-trivial fixed point action is perturbed in the λ direction, inducing a flow towards the Gaussian fixed point. Now, in the vicinity of the Gaussian fixed point the low energy effective theory is described arbitrarily well by the Wess–Zumino model. This follows simply because, although λ is irrelevant with respect to the Gaussian fixed point, it is only marginally so, and so all other couplings (besides the mass, which can be ignored in this discussion) die off much faster.

Trajectories which emanate from fixed points are called renormalized trajectories [1]. As straightforwardly shown in [8], a renormalized trajectory is such that all scale dependence of the action appears through (i) the relevant couplings with which the fixed point action has been perturbed (ii) the anomalous dimension of the field. This is referred to a 'self-similarity' [8,9]; it is worth noting that self-similarity is a nonperturbative statement of renormalizability [8]. In the current context, we have supposed that the fixed point action has been perturbed in the λ-direction. Were it not for the nonrenormalization theorem, we would expect the action along the resulting renormalized trajectory to depend on both $\lambda(\Lambda)$ and $\gamma(\Lambda)$. However, the two quantities are related by (3) and so we can write simply

$$S_\Lambda[\bar{\Phi}, \Phi] = S[\bar{\Phi}, \Phi]\big(\gamma(\Lambda)\big). \tag{5}$$

As just stated, in order to construct this renormalized trajectory, it must be that $\lambda(\Lambda)$ is relevant with respect to the non-trivial fixed point. This requires that $\gamma_\star < 0$, as follows from (3). Crucially, however, sufficiently close to the Gaussian fixed point—where we can rely on perturbation theory done with the Wess–Zumino model—we know that the anomalous dimension is positive.

Therefore, in going from the UV fixed point down to the vicinity of the Gaussian fixed point, $\gamma(\Lambda)$ must pass through zero (at least once). Consider the first time that this happens. Since all scale dependence along our renormalized trajectory is carried by $\gamma(\Lambda)$ then, if $\gamma(\Lambda)$ ever vanishes, we must be at a fixed point. Now, on the one hand, this fixed point cannot be the Gaussian one: the action in the vicinity of the Gaussian fixed point is (essentially) the Wess–Zumino action, but $\gamma(\Lambda)$ has not yet increased above zero, by assumption. On the other hand, Pohlmeyer's theorem tells us that this fixed point cannot be anything else! Therefore, our original assumption that there exists a non-trivial fixed point with a trajectory, spawned along the λ direction, emanating from it such that the low energy effective theory is well described by the Wess–Zumino model, must be incorrect.

However, suppose that the fixed point also possesses a relevant operator coming from the Kähler potential, $\mathcal{O}[\bar{\Phi}, \Phi]$, with coupling $g(\Lambda)$ (obviously, we can generalize this to several such operators). Perturbing the fixed point action in both the λ and g directions, the action along the resulting renormalized trajectory now reads

$$S_\Lambda[\bar{\Phi}, \Phi] = S[\bar{\Phi}, \Phi]\big(g(\Lambda), \gamma(\Lambda)\big). \tag{6}$$

Whilst it is still true that, in order for an asymptotic safety scenario to be realized for the Wess–Zumino model, the anomalous dimension must pass through zero, it is no longer true that the vanishing of $\gamma(\Lambda)$ at some scale necessarily corresponds to fixed point, since $g(\Lambda)$ could still be flowing.

Assuming such an asymptotic safety scenario to exist, we now have the following picture of the renormalization group flows. If we perturb away from the non-trivial fixed point in just the λ direction, then we must shoot off away from the Gaussian fixed point. (A finite distance along the resulting trajectory, it may be that $\mathcal{O}[\bar{\Phi}, \Phi]$ is generated, but now we have $g(\Lambda) = g(\gamma(\Lambda))$.) However, by appropriately perturbing the fixed point in both the λ and g directions, we flow towards the Gaussian fixed point, with the low energy effective action being well described by the Wess–Zumino action. The question as to whether such non-trivial fixed points actually exist will be addressed in a companion paper [10].

Acknowledgements

I acknowledge IRCSET for financial support. I would like to thank Denjoe O'Connor for comments on the manuscript and, particularly, Werner Nahm for a very useful discussion.

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