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Towards resolution of hierarchy problems in a cosmological context

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Abstract

A cosmological scenario is proposed, which simultaneously solves the mass hierarchy and the small dark energy problem. In the present scenario an effective gravity mass scale (inverse of the Newton's constant) increases during the inflationary period. The small cosmological constant or the dark energy density in the present universe is dynamically realized by introducing two, approximately $O(2)$ symmetric dilatons, taking the fundamental mass scale at TeV.

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1. Introduction

There are many interesting ideas that attempt to solve the hierarchy problem between gravity and particle physics mass scales (the first hierarchy problem), but none seems to have linked this hierarchy problem with another hierarchy in cosmology (the second hierarchy problem); presence of a finite, but very small cosmological constant, or a dark energy, its nature and origin yet to be identified. We attempt to construct models that simultaneously solve these hierarchy problems by radically changing cosmology in the same spirit of ideas as due to Dirac [1], Brans and Dicke [2]. Important new ingredient in the present

work is a choice of the dilaton potential along with a curvature coupling similar to the one given by Brans and Dicke.

Recent observations of WMAP and the large scale structure confirm the basic validity of the inflationary paradigm [3], but at the same time it has left behind a great conundrum of the presence of the dark energy which is close to, but dominant over, the dark matter energy. The implied mass scale of (dark energy density)$^{1/4}$ is very small of order 10^{-3} eV in the microscopic scale. It thus appears that a resolution of great mysteries in cosmology via inflationary scenarios has created another great mystery, which seems even more insurmountable.

We propose a possible scenario towards resolution of two hierarchy problems, while retaining nice

features of the inflationary cosmology. Inflation is achieved by the dilatonic inflaton in our scenario. We avoid fine tuning of parameters, taking a common mass scale of order TeV for the dilatonic inflaton potential. In a broken symmetric model of two dilatons a light scalar boson of mass $\approx \text{TeV}^2/m_{\text{pl}} \approx 1$ meV is predicted, whose coupling to matter is gravitationally suppressed.

The TeV scale model however has a difficulty of generating the baryon asymmetry of the universe, since the reheat temperature is too low. This can be remedied by a further extension of the present model.

2. Theoretical framework

We work in a general framework of four-dimensional Lagrangian field theory, with two parts left unspecified for the time being

$$\mathcal{L} = \sqrt{-g}\left[-f(\varphi_i)R + \frac{1}{2}(\partial\varphi_i)^2 - V(\varphi_i) + \mathcal{L}_m \right]. \tag{1}$$

The dilatonic coupling of the scalar field φ_i to the scalar curvature given by $f(\varphi_i)R$ is taken from [2]. But we depart in the choice of the potential $V(\varphi_i)$ from the Brans–Dicke theory, in which a single dilaton was introduced along with the null potential and $f(\varphi) = \epsilon\varphi^2$.

The Einstein gravity equation is modified to [4]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$
$$= \frac{1}{2f}\left[T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\varphi)}\right] + \frac{1}{f}\left(f_{;\mu;\nu} - g_{\mu\nu}f_{;\lambda}^{;\lambda}\right), \tag{2}$$

where $T_{\mu\nu}^{(\varphi)}$ is contribution to the energy–momentum tensor from the scalar φ, while $T_{\mu\nu}^{(m)}$ is the usual contribution of radiation, matter and other fields. Scalar field evolution is given by

$$\varphi_{i;\lambda}^{;\lambda} = -\frac{\partial V}{\partial\varphi_i} - \frac{\partial f}{\partial\varphi_i}R. \tag{3}$$

One may use

$$-R = \frac{1}{2f(\varphi)}\left[T - (\partial\varphi_i)^2 + 4V(\varphi_i) - 6f_{;\rho}^{;\rho}\right], \tag{4}$$

in the right-hand side of Eq. (3). Here T is the trace of the matter energy–momentum tensor.

An effective gravitational strength is given by $f(\varphi_i) = 1/16\pi G$, and this can be spacetime dependent due to a nontrivial spacetime dependence of φ, thus modifying the Einstein equation in an essential way. The usual Einstein equation with a constant φ_i is however an excellent approximation in the present universe. (We shall discuss its possible variation at the end of this Letter.) Existence of the term, $(f_{;\mu;\nu} - g_{\mu\nu}f_{;\lambda}^{;\lambda})/f$, in the modified Einstein equation (2) is important in our cosmological discussion. From reasons to be clarified later, we introduce two (or more) dilatons and extend the dilatonic coupling to

$$f(\varphi_i) = \epsilon_1\varphi_1^2 + \epsilon_2\varphi_2^2, \tag{5}$$

with ϵ_i positive numbers. We later mention what happens in the case of the exact $O(2)$ symmetry of $\epsilon_1 = \epsilon_2$.

We do not assume any fine tuning of the potential $V(\varphi_i)$ except that it is a bounded function allowing infinitely many negative values and infinitely many local minima. In this way a large mass hierarchy and dynamical relaxation towards a small cosmological constant may be realized. The simplest choice realizing these is a periodic potential of minimum numbers of parameters

$$V(\varphi_i) = V_0\cos\frac{\varphi_r}{M} + \Lambda, \tag{6}$$

with $\varphi_r = \sqrt{\varphi_i^2}$ and $V_0 > \Lambda > 0$. Here Λ is a collection of all constants in the standard model Lagrangian $\mathcal{L}_m$ such that the potential of the standard model Lagrangian vanishes at its minimum. We assume $O(2)$ rotational symmetry for the potential $V(\varphi_i)$.

Important features of our assertions below are valid irrespective of the precise form of the potential. The essential requirement on the potential for a successful scenario is that (1) boundedness, (2) infinitely many local minima, and (3) infinitely many regions of negative values between minima and maxima. Nevertheless, it would be useful to have a simple realization such as (6) of our idea and to discuss a model explicitly.

For both simplicity and naturalness we assume that all mass parameters are of the same order, thus $V_0 \approx \Lambda = O[M^4]$ for the choice (6). We take the common mass scale M of order TeV.

3. Dynamical equation

Consider the Robertson–Walker metric of flat universe, $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$. Dynamical equation of time evolution is derived straightforwardly. We write it down in terms of the following two field variables $f_\pm$,

$$\varphi_1 = \sqrt{\frac{f_+}{2\epsilon_1}}, \qquad \varphi_2 = \sqrt{\frac{f_-}{2\epsilon_2}}, \qquad f_\pm = f \pm k. \quad (7)$$

The basic dilaton dynamics is given by

$$\ddot{f}_+ + 3H\dot{f}_+$$
$$= \left[1 + 6\frac{\epsilon_1 f_+ + \epsilon_2 f_-}{f}\right]^{-1}$$
$$\times \left[4\epsilon_1 f_+ \left(-\tilde{R} - \frac{1}{2\epsilon_1 \varphi_r} V'\right) + \frac{\dot{f}_+^2}{2f_+}\right], \quad (8)$$

$$\ddot{f}_- + 3H\dot{f}_-$$
$$= \left[1 + 6\frac{\epsilon_1 f_+ + \epsilon_2 f_-}{f}\right]^{-1}$$
$$\times \left[4\epsilon_2 f_- \left(-\tilde{R} - \frac{1}{2\epsilon_2 \varphi_r} V'\right) + \frac{\dot{f}_-^2}{2f_-}\right], \quad (9)$$

$$\tilde{R} = \frac{1}{2f}\left(\frac{\dot{f}_+^2}{8\epsilon_1 f_+} + \frac{\dot{f}_-^2}{8\epsilon_2 f_-} - 4V - T\right). \quad (10)$$

These ought to be solved along with the modified Einstein equation

$$H^2 = \frac{1}{6f}\left(T_{00} + \frac{1}{2}\dot{\varphi}_i^2 + V\right) - H\frac{\dot{f}}{f}. \quad (11)$$

We note interesting features of φ dynamics. First, the usual force term $-\frac{\partial V}{\partial \varphi}$ is modified, as seen from the f dependent terms of (10) $-2f_\pm V/f$ present in $f_\pm$ equations. The second is presence of the induced matter coupling $\propto T$, which may be derived from an effective Lagrangian of the form, $\frac{T}{2}\ln f$. Thus, the dilaton couples to matter via the trace of the energy–momentum tensor T.

One dilaton model fails to solve hierarchy problems from a number of reasons; it requires a fine tuning at the stationary minimum of the potential, namely $V = 0$ at the same time when $-V_{,\varphi} + 4V/\varphi = 0$. It is also difficult to obtain a small dark energy density.

The key for success is introduction of more freedom such as an angular momentum in a higher-dimensional φ-space. We thus introduce 2 dilatonic inflatons φ_1, φ_2, as already stated. This makes it possible to reach a large f value (very weak gravitational interaction) without being trapped in many potential minima of negative cosmological constant.

4. Cosmological evolution

Let us first point out that this model realizes the power-law inflation. Ignoring, for the moment, potential variation and replacing V by its averaged value Λ, we seek solution, with the ansatz valid for large t,

$$f = At^2, \qquad k = Bt^2, \qquad a \propto t^\omega. \quad (12)$$

Leading order solution is found, in which ω is determined in terms of ϵ_i. In the small ϵ_i limit the index of the power ω becomes large.

A large value of ω is favored to approximately mimic the exponential expansion of the cosmological scale factor. The gravity mass scale increases as inflation proceeds like $f \propto a^{2/\omega}$. Thus, there is no difficulty of obtaining a large enough e-folding factor of inflation, at the same time resolving the mass hierarchy problem. This is a feature already visible in the model of extended inflation [5], although it has not been much appreciated.

A naive estimate of the density perturbation gives a magnitude of order $M/m_{\rm pl}$, which is too small if one takes M at TeV. Instead, the best way to realize an acceptable density perturbation is to utilize a version of curvaton idea, which will be discussed separately.

After this inflationary epoch, the inflaton φ_i is expected to settle down to some stationary points. But in our model of $\epsilon_1 \neq \epsilon_2$ there is no stationary point, because the requirement of constant φ_i values implies both $V' = 0$ and $V = 0$, which is nothing but the fine tuning of parameters of the potential. The model without fine tuning however gives a mechanism of dynamical cancellation of the effective cosmological constant, $\Lambda_{\rm eff} = \langle \frac{1}{2}\dot{\varphi}_i^2 + V\rangle$. We shall discuss this mechanism later.

Particle production right after inflation gives rise to the hot big bang. We shall briefly discuss how this comes about. We take an example of the matter energy–momentum tensor, $T = \frac{1}{2}m_\psi^2 \psi^2$, where ψ represents a generic boson. The mode equation for ψ when f deviates around $1/16\pi G$ by a small amount

is

$$\ddot{\psi} + \left(m_\psi^2 + k^2\right)\psi + 3H\dot{\psi} + \tilde{m}_2^2 \xi(t)\psi = 0, \qquad (13)$$

$$\xi(t) = \frac{m_\psi^2}{2\tilde{m}_2^2}\left(\frac{\delta f}{f} - \frac{1}{2}\left(\frac{\delta f}{f}\right)^2 + O\left[\left(\frac{\delta f}{f}\right)^3\right]\right). \quad (14)$$

There exist two mass eigenstates of the dilaton, and we took the lighter dilaton since it gives more important contribution here.

We shall first discuss the fluctuation equation in order to derive these mass values. With the ansatz, $f = f_0 + \delta f$, $k = k_0 + \delta k$ the linearized equations follow

$$\begin{pmatrix} \delta\ddot{f}_+ + 3H\delta\dot{f}_+ \\ \delta\ddot{f}_- + 3H\delta\dot{f}_- \end{pmatrix} = \mathcal{M}^2 \begin{pmatrix} \delta f_+ \\ \delta f_- \end{pmatrix},$$

$$\mathcal{M}_{ij}^2 = O[M^2], \qquad (15)$$

$$\det(\mathcal{M}^2) \approx \frac{V'V''}{\varphi_r^3}\left(\frac{f_+}{\epsilon_1} + \frac{f_-}{\epsilon_2}\right), \qquad (16)$$

where $\det(\mathcal{M}^2)$ appears $O[M^5 m_{\rm pl}^{-1}]$, but actually $O[M^6 m_{\rm pl}^{-2}]$, since $V'(\equiv V_{,\varphi}) \propto 1/m_{\rm pl}$, as is shown later.

The mass diagonalization, taking into account $V' \propto 1/m_{\rm pl}$, yields two eigenmasses of order,

$$\tilde{m}_1 = O[M],$$

$$\tilde{m}_2 = O\left[\frac{M^2}{m_{\rm pl}}\right] = O[1\ {\rm meV}]\left(\frac{M}{1\ {\rm TeV}}\right)^2. \qquad (17)$$

We call the second, light dilaton newdiron, named after Newton and Dirac.

Back to the inflaton decay, large and small amplitude decay of φ is described as follows. First, the large initial amplitude condition is fulfilled, since

$$\xi(t_i) = O\left[\frac{m_\psi^2 M}{\tilde{m}_1^2 m_{\rm pl}}\right] = O\left[\frac{m_\psi^2 m_{\rm pl}}{M^3}\right] \gg 1, \qquad (18)$$

for $\delta\tilde{f}_2$ decay, taking initially $\delta\varphi_i(t_i) = O[M]$. Thus, the inflaton oscillation leads to explosive particle production due to the parametric resonance effect [6]. Thermalization is quickly achieved, giving a reheat temperature of order M.

After this initial phase of preheating and thermalization, the dimensionless amplitude $\xi(t)$ drops to $O[1]$, and the large amplitude oscillation stops. After this takes place, the only process of dilaton decay is two-body perturbative decay. To correctly derive these rates, one has to note correctly normalized fields given by $(2\sqrt{2\epsilon_i})^{-1}\delta f_\pm/\sqrt{f_\pm}$. These decay rates are then $\Gamma_1 = O[M^3/m_{\rm pl}^2]$,

$$\Gamma_2 = O\left[\frac{m_\psi^4}{m_{\rm pl}^2 \tilde{m}_2}\right]$$

$$= O\left[\frac{M^6}{m_{\rm pl}^5}\right] \approx \left(10^{48}\ {\rm s}\right)^{-1}\left(\frac{M}{1\ {\rm TeV}}\right)^6, \qquad (19)$$

where for the newdiron only neutrino-pair and two photon decays are possible due to the small mass. Thus, the newdiron is effectively stable.

For the success of nucleosynthesis, the heavy dilaton must decay prior to nucleosynthesis. This gives a mass scale constraint

$$\Gamma_1 = O\left[\left(10^3\ {\rm s}\right)^{-1}\left(\frac{M}{1\ {\rm TeV}}\right)^3\right] > \left(10^3\ {\rm s}\right)^{-1} \quad \Rightarrow$$

$$M > {\rm a\ few\ TeV}. \qquad (20)$$

More elaborate analysis including the gravitino overproduction favors the mass constraint of order, $M > 100$ TeV, which can be accommodated in the present model.

5. Dynamical relaxation towards vanishing $\Lambda_{\rm eff}$

At some stage of cosmological evolution after inflation, the cosmological constant (or more precisely the vacuum energy density) of order $({\rm TeV})^4$ must relax towards smaller values of order $({\rm meV})^4$. We analyze this problem by assuming the main terms f and k of order f_0 (constant) and $\delta\dot{k} \gg \delta\dot{f}$. The resulting equation for k fluctuation is

$$\delta\ddot{k} + 3H\delta\dot{k}$$
$$= \left(1 + 6\frac{\epsilon_1 f_+ + \epsilon_2 f_-}{f_0}\right)^{-1}$$
$$\times \left[-\frac{2V'}{\varphi_r}(k_0 + \delta k) + \frac{4}{f_0}(\epsilon_1 f_+ - \epsilon_2 f_-)V\right].$$

From the solution of $\delta k = O[m_{\rm pl}] \times$ oscillating function, one obtains the kinetic term of order,

$$\frac{\dot{\varphi}_k^2}{2} = O[1] \times \left(\int^t dt'\, V'(t')\right)^2 = O[M^4]. \qquad (21)$$

Since this quantity is nonvanishing and of order M^4, there is a chance of cancellation against the potential term V. Indeed, for a large f_0 there are a great many trajectories of vanishing $\Lambda_{\rm eff}$.

The requirement of constant f (gravity scale) and the tuning condition $\Lambda_{\rm eff} = 0$ give

$$\frac{\dot{\varphi}_k^2}{2} + V = 0, \qquad 3V - \frac{\epsilon_1 + \epsilon_2}{4\epsilon_1\epsilon_2}\frac{f_0}{\varphi_r}V' = 0. \qquad (22)$$

These two Eqs. (22), to be supplemented by Eq. (21), ought to be solved in favor of (f_0, k_0). Suppose that the Newton's constant is measured to a precision δ (which is at present $\approx 10^{-4}$). Then, there are of order $10^{15}\delta(M/(1\ {\rm TeV}))^{-1}$ possible (f_0, k_0) values that fit measured data.

Note that the coefficient of these Eqs. (22), f_0/φ_r is of order $m_{\rm pl}$. Hence the factor V' must be small of order $M^4/m_{\rm pl}$ to cancel against other terms of order M^4. This explains the already mentioned mass relation, $\tilde{m}_2 \approx M^2/m_{\rm pl}$ due to the presence of V'.

There are a great many (f, k) values of solutions to both of Eqs. (22), close to trajectories of $V' = 0$, or more precisely $\varphi_r = MN\pi + O[M^2/m_{\rm pl}]$ for a large integer N. Thus, a very small $\Lambda_{\rm eff}$ may be dynamically obtained near these points which however cannot be stationary anchor points, hence one expects that neverending shifts towards these points occur in the present version of model of $\epsilon_1 \neq \epsilon_2$.

We however point out with the exact $O(2)$ symmetry of $\epsilon_1 = \epsilon_2 = \epsilon$ there exist stationary anchor points of very small effective cosmological constant. This comes about, because the angular momentum $L = \varphi_1\dot{\varphi}_2 - \varphi_2\dot{\varphi}_1$ is conserved with the $O(2)$ symmetry, and the stationary condition reduces to

$$-V' + \frac{4V}{\varphi_r} + \frac{L^2}{8\epsilon\varphi_r^3} = 0, \qquad \frac{L^2}{2\varphi_r^2} + V = 0, \qquad (23)$$

which should be solved for (f_0, L). There are again such candidate values of order $10^{15}\delta(M/(1\ {\rm TeV}))^{-1}$ that fit observation. The critical question for realization of this result concerns a natural initial setting for the conserved quantity L^2 which should be of order $M^4m_{\rm pl}^2$. We shall address this question elsewhere.

The $O(2)$ symmetric model has one heavy dilaton of mass $O[M]$, whose decay rate is of order $M^3/m_{\rm pl}^2$. What happens to the light dilaton is that it becomes massless, which however completely decouples from the rest of the world.

In both scenarios of dynamical relaxation towards the vanishing cosmological constant, the exact tuning is not necessary and moreover is unlikely to occur. Under this circumstance one expects a residual dilaton energy as the dark matter candidate. The tuning to the amplitude precision of the leading order $M^2/m_{\rm pl}$ yields the dark energy density of order,

$$\rho_{\rm DM} = O\left[\left(\frac{M^2}{m_{\rm pl}}\right)^4\right]\left(\frac{T_0}{T_d}\right)^3, \qquad (24)$$

with T_d/T_0 the expansion factor after the relaxation. If the relaxation epoch is close to the present age, the dilaton oscillation energy is of the right order of magnitude to explain the present amount of dark matter.

There is another possibility. Suppose that the approach to anchor points had occurred around nucleosynthesis or at the heavy dilaton decay. Then, one gets a right order of magnitude of the present $\Lambda_{\rm eff}$, since its present value

$$(\delta\Lambda_{\rm eff})_0 = O\left[\frac{M^6}{m_{\rm pl}^2}\right]\left(\frac{T_0}{\rm MeV}\right)^3$$

$$\approx (1\ {\rm meV})^4\left(\frac{M}{1\ {\rm TeV}}\right)^6. \qquad (25)$$

Finally, let us discuss what happens if the scenario works, as expected. The result differs, depending whether the dark matter is provided by the dilaton oscillation or another form of stable particles such as lightest supersymmetric particles. If the dark matter is made of newdiron, one obtains, from the consistency with the modified Einstein equation, for the w value defined by $w \equiv p/\rho$,

$$w = -1 - \frac{\rho_{\rm DM}}{\rho_{\rm DE}}. \qquad (26)$$

If the dark matter is attributed to another source, one has the usual $w = -1$. Clearly, a better understanding of the relaxation process is welcome. The time dependence of w differs, depending on how the relaxation occurs, in particular, when this occurs. Observation of future deep sky surveys is crucial to test the model of dynamical relaxation.

6. Variation of Newton's constant

In the present model variation of the gravitational constant is inevitable, although its magnitude is model

dependent. We shall discuss the simplest case of how much it varies due to nonrelativistic (NR) matter of mass density ρ_m excluding the dilaton oscillation. For simplicity, we take the $O(2)$ symmetric model. The quantity f varies, with m_d the heavy dilaton mass, according to

$$\delta\ddot{f} + 3H\delta\dot{f} + m_d^2 \delta f = \frac{2\epsilon}{1 + 12\epsilon}\rho_m. \qquad (27)$$

Assuming that this NR matter dominates as the main component of the dark matter, one derives a $\delta f/f$, hence $-\delta G/G$, variation between epochs of NR matter appearance and NR matter dominance $\approx 16\epsilon/((1 + 12\epsilon)N_d)$, where N_d is the relativistic degrees of freedom contributing to the energy density at NR matter appearance. Thus, this fraction can be made small, and furthermore its change after NR matter dominance is small, although the variation can be made larger to accommodate some nonstandard varying G. We shall discuss elsewhere how much G varies when the dark matter is made of the newdiron.

In summary, the Dirac's large number hypothesis has been resurrected along with inflation. Further-more, a class of multi-dilaton models give a possibility of solving the problem of how the present dark energy density becomes of order $(\text{TeV}^2/m_{\text{pl}})^4$.

Interesting details and some extensions of the present model will be presented in separate publication.

References

[1] P.M.A. Dirac, Nature 139 (1937) 323.
[2] C. Brans, H. Dicke, Phys. Rev. 124 (1961) 925;
    See also S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, Wiley, New York, 1972.
[3] A.H. Guth, Phys. Rev. D 23 (1981) 347;
    A.D. Linde, Phys. Lett. B 129 (1983) 177.
[4] A. Zee, Phys. Rev. Lett. 42 (1979) 417.
[5] D. La, P.J. Steinhardt, Phys. Rev. Lett. 62 (1989) 376.
[6] L. Kofman, A. Linde, A.A. Starobinsky, Phys. Rev. Lett. 73 (1994) 3195;
    M. Yoshimura, Prog. Theor. Phys. 94 (1995) 873;
    H. Fujisaki, K. Kumekawa, M. Yamaguchi, M. Yoshimura, Phys. Rev. D 53 (1996) 6805, hep-ph/9508378.