Seismic uplift capacity of shallow horizontal strip anchor under oblique load using pseudo-dynamic approach

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Abstract

In this paper, an analytical method to compute the uplift capacity of an obliquely loaded horizontal strip anchor under both static and seismic conditions is described using the limit equilibrium method. The distribution of the soil reactions on a simple planar failure surface is obtained through the use of Kötter's equation, and the pseudo-dynamic approach is used to obtain the net seismic vertical uplift capacity factor for the unit weight component of the soil ($F_{\gamma d}$). The results for the static and seismic vertical uplift capacity factors are determined for various combinations of input parameters, such as the load inclination, the soil friction angle, the embedment ratio, the soil amplification and both horizontal and vertical pseudo-dynamic seismic accelerations. It is observed that the orientation of the load significantly affects the seismic uplift capacity of the horizontal strip anchor. $F_{\gamma d}$ is seen to decrease with an increase in both horizontal and vertical seismic accelerations and soil amplification, whereas it is seen to increase with an increase in the embedment ratio and the soil friction angle, as expected. The results in terms of the non-dimensional net seismic uplift capacity factor are presented in graphical and tabular forms. The present results are compared and found to be in good agreement with similar results available in literature.

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1. Introduction

Ground anchors are commonly used as foundation systems for important structures requiring uplift resistance, such as transmission towers, pipe lines buried under water, sheet pile walls, etc. It is well known that horizontal anchors are subjected to the vertical uplift load, and that when it reaches the ultimate load condition, failure surfaces develop around the anchor plate. These failure surfaces reach the ground surface for shallow anchors. The problem becomes more complex and important under seismic conditions due to the devastating nature of earthquake forces on such ground anchors. The importance of the computation of the vertical uplift capacity of horizontal ground anchors is clear; it is an important topic for geotechnical engineers to address, under both static and seismic conditions. Hence, the effect of earthquakes on the uplift capacity of a strip anchor is studied in the present theory, as this knowledge is vital to the study of the behavior of strip anchors under seismic conditions.

For static conditions, numerical solutions for the uplift capacity of horizontal anchors were obtained using the limit equilibrium method (Vesic, 1971; Meyerhof and Adams, 1968). The finite element method was used with a limit analysis to
compute the uplift capacity of anchors (Kumar and Kouzer, 2008; Merifield and Sloan, 2006; Tagaya et al., 1983). Experimental results and analytical solutions were obtained using the limit equilibrium method and a limit analysis (Rowe and Davis, 1982; Murray and Geddes, 1987; Tagaya et al., 1988; Dickin and Laman, 2007). Analytical models were presented by some researchers (Chattopadhayay and Pise, 1986; Deshmukh et al., 2011), whereas Kumar (1999) and Subba Rao and Kumar (1994) used an upper bound limit analysis to predict the pullout capacity of ground anchors, and Honda et al. (2011) employed a two-dimensional distinct element analysis to obtain the uplift capacity of belled and multi-belled piles in sand. Series of centrifuge tests were presented by Hugo et al. (2010) to determine the effect of gapping on the uplift capacity of a shallow skirted foundation. Miyata et al. (2011) obtained the accuracy of a single square anchor plate for obtaining the capacity in a multi-anchor wall system by applying two models.

Few researchers have obtained the vertical uplift capacity of horizontal plate anchors under seismic conditions (Kumar, 2001; Choudhury and Subba Rao, 2004; Rangari et al., 2011a). Based on field and laboratory tests, a semi-empirical relation was developed for the breakout resistance of a horizontal strip anchor under an oblique load by Meyerhof (1973). Das and Seeley (1975), on the other hand, obtained model test results for a square anchor in dry sand subjected to inclined loads and presented in graphical form.

A pseudo-dynamic approach, developed by Steedman and Zeng (1990) and upgraded by Choudhury and Nimbalarkar (2005), was used by Ghosh (2009) to obtain the seismic uplift capacity of a horizontal plate anchor by considering the effect of seismic amplification using an upper bound limit analysis and assuming a simple planar failure surface.

In a pseudo-static analysis, the dynamic loading induced by an earthquake is considered as time-independent, which ultimately assumes that the magnitude and the phase of the acceleration are uniform throughout the depth of the soil layer. Also, in a pseudo-static analysis, the effect of the amplification of vibrations cannot be considered; this effect generally occurs towards the ground surface and depends on various soil parameters, such as damping and the shear modulus of the soil (Steedman and Zeng, 1990). Rectifying these shortcomings of the pseudo-static approach, Steedman and Zeng (1990) proposed a pseudo-dynamic approach, which considers the effect of the finite shear wave velocity, horizontal seismic acceleration and the amplification of vibrations, to obtain the active earth pressure on retaining walls for a particular value of soil friction angle. Later, Choudhury and Nimbalarkar (2005) modified the approach further to consider the effect of vertical acceleration also due to vertically propagating primary waves along with horizontal seismic acceleration; they
considered the effects of time, the phase difference and the effect
of amplification in both shear and primary waves propagating
through the soil.

Kötter (1903) was used by researchers (Balla, 1961; Matsuo,
1967, 1968) to obtain the soil resistance on the failure surface.
Recently, the limit equilibrium method has been effectively applied
along with Kötter's equation to obtain the vertical uplift capacity of
horizontal strip anchors (Deshmukh et al., 2011; Rangari et al.,
2011a). However, the use of Kötter (1903) for obtaining the
seismic uplift capacity of horizontal strip anchors under an oblique
load is still scarce in literature and remains mostly unnoticed.

Hence, in this paper, an attempt has been made to obtain the
uplift capacity of a horizontal strip anchor under both static and
seismic conditions by employing Kötter's equation (1903) and by
taking the forces on the soil column above the anchor into
consideration. It is well known that the problem of the computa-
tion of the seismic uplift capacity of horizontal strip anchors is the
application of the seismic passive resistance on retaining walls for
the negative wall friction condition (Choudhury and Subba Rao,
2002, 2005a; Rangari et al., 2013), whereas the case of the
positive wall friction condition is used for bearing capacity
analyses (Subba Rao and Choudhury, 2005; Choudhury and
Subba Rao, 2005b). A pseudo-dynamic approach is used with the
limit equilibrium method for a simple planar failure surface to
obtain the seismic uplift capacity of an anchor with negative wall
friction angle \( \delta = \frac{2 \phi}{3} \) (Meyerhof and Adams, 1968; Choudhury
and Subba Rao, 2004; Rangari et al., 2011a, 2011b, 2013), where \( \phi \) is the soil friction angle.

2. Method of analysis

The net seismic uplift capacity factor for a shallow horizontal
strip anchor, subjected to an inclined load, is obtained using the
limit equilibrium method and assuming a simple planar failure

\[ F_{pd} = \frac{1}{k_h} \left[ \frac{1}{k'_{v}} + \frac{1}{k_{u}} \right] \]

where \( k_h \) is the horizontal seismic coefficient, \( k'_{v} \) is the
vertical seismic coefficient, and \( k_u \) is the uplift coefficient.

\[ k_u = \frac{1}{2} \left( 1 + \tan^2 \phi \right) \]

and

\[ k_h = \frac{1}{2} \left( 1 + \tan^2 \phi \right) \]

\[ k'_{v} = \frac{1}{2} \left( 1 + \tan^2 \phi \right) \]

with \( \phi \) being the soil friction angle.

![Fig. 3. Variation in \( F_{pd} \) for various values of \( k_h \) and \( k'_{v} \) for \( \phi = +10^\circ \) for (a) \( \varepsilon = 1 \), (b) \( \varepsilon = 3 \), (c) \( \varepsilon = 5 \) and (a) \( \varepsilon = 7 \) with \( f = 1.0 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).](image-url)
surface and a pseudo-dynamic approach for seismic forces. The soil is assumed as a rigid cohesionless material satisfying the Mohr–Coulomb failure criterion. It is assumed that the input values for the soil parameters, such as unit weight $\gamma$ and soil friction angle $\phi$, are not affected under seismic conditions. The smooth anchor plate is considered by disregarding its mass in the analysis. The marginal effect ($<4\%$) of the roughness of the plate anchor on the calculated uplift capacity of a shallow horizontal strip anchor for all soil friction angles and embedment ratios was observed by Merifield and Sloan (2006); it justifies the assumption of the smooth anchor plate used in the present study. It is also assumed that the soil lying below the surface of the anchor does not offer any resistance to the uplift capacity.

Fig. 1 shows the model considered for the horizontal plate anchor with an oblique load under seismic conditions. A horizontal plate anchor (CD) of width $B$ is embedded at a depth $H$ from the horizontal ground surface. $W$ is the weight of failure wedge ACDB, $R_1$ and $R_3$ are the reactions on planar failure surfaces BD and AC, respectively, and $Q_h$ and $Q_v$ are the horizontal and vertical inertia forces acting in horizontal and vertical directions, respectively.

Kötter’s equation (1903) for the passive state of earth pressure in a cohesionless soil is employed to obtain the distribution of soil reactions on a curved surface and can be expressed as

$$\frac{dp}{ds} + 2\rho \tan \phi \frac{d\alpha}{ds} = \gamma \sin(\alpha + \phi)$$

where

$dp$ = differential reaction pressure on failure surface
$ds$ = differential length of failure surface
$\phi$ = soil friction angle

Fig. 4. Variation in $F_{\mu}$ for various values of $\epsilon$ and $k_f$ for $\varphi = -10^0$ for (a) $\epsilon = 1$, (b) $\epsilon = 3$, (c) $\epsilon = 5$ and (a) $\epsilon = 7$ with $f = 1.0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$. 

Integrating Eq. (1) and using the boundary conditions on failure surfaces BD and AC, reactions $R_1$ and $R_3$ can be obtained as follows (Rangari et al., 2011a):

\[
R_1 = \frac{1}{2} \gamma H^2 \sin (\alpha_1 + \phi) \quad \text{sin}^2 \alpha_1
\]

\[
R_3 = \frac{1}{2} \gamma H^2 \sin (\alpha_3 + \phi) \quad \text{sin}^2 \alpha_3
\]

Failure wedge ACDB, shown in Fig. 1, under the pseudodynamic approach, considers the propagation of finite shear wave velocity $V_s = \sqrt{G/\rho}$ and primary wave velocity $V_p = \sqrt{G(2-2\nu)/(\rho(1-2\nu))}$, where $G$ is the shear modulus, and $\rho$ and $\nu$ are the density and Poisson's ratio of the soil medium. The time of lateral shaking, $T = 2\pi/\omega$, is considered in the present analysis, where $\omega$ is the angular frequency of excitation.

A free body diagram, with the failure model considered for the present analysis, is shown in Fig. 2. For the seismic acceleration using the pseudo-dynamic method, both horizontal and vertical sinusoidal vibrations with amplitude of acceleration in the horizontal direction, $a_h = k_h g$, where $k_h$ is the horizontal coefficient of the seismic acceleration and $g$ is the acceleration due to gravity), and in the vertical direction, $a_v = k_v g$, where $k_v$ is the vertical coefficient of the seismic acceleration), are considered. The forces acting on failure block CDFE, failure wedge ACE and failure wedge BDF are shown in Fig. 2(a), (b) and (c), respectively.

![Fig. 5. Variation in $F_{\gamma d}$ for various values of $k_h$ and $k_v$ for $\phi = 20^\circ$ for (a) $\varepsilon = 1$, (b) $\varepsilon = 3$, (c) $\varepsilon = 5$ and (a) $\varepsilon = 7$ with $f = 1.0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$.](image-url)
Let us consider the free body diagram of wedge BDF (Fig. 2 (c)), subjected to both horizontal and vertical sinusoidal vibrations with amplitudes of acceleration \( a_{h1} (=k_h g) \) and \( a_{v1} (=k_v g) \), respectively. Steedman and Zeng (1990) proposed the equation of acceleration at depth \( z \) and time \( t \) acting in the horizontal direction by considering the effect of the finite shear wave velocity in terms of sinusoidal motion which was given by

\[
a_{h}(z, t) = k_h g \sin \omega \left( t - \frac{H - z}{V_s} \right)
\] (4a)

Later, Nimbalkar and Choudhury (2007) modified the basic equation for seismic acceleration to consider the effect of vertical acceleration, also due to vertically propagating primary waves along with horizontal seismic acceleration, and considered the effects of time and amplification \( f \) in both shear and primary waves propagating though the soil. In the present study, therefore, seismic horizontal and vertical accelerations at any depth \( z \) and time \( t \) below the ground surface with soil amplification factor \( f \) can be expressed as

\[
a_{h1}(z, t) = \left( 1 + \frac{H - z}{H} (f - 1) \right) \frac{a_{h1}}{V_s} \sin \omega \left( t - \frac{H - z}{V_s} \right)
\] (4b)

\[
a_{v1}(z, t) = \left( 1 + \frac{H - z}{H} (f - 1) \right) \frac{a_{v1}}{V_p} \sin \omega \left( t - \frac{H - z}{V_p} \right)
\] (5)

The mass of the small incremental part of thickness \( dz \) at depth \( z \) from the ground surface in failure wedge BDF is given by

\[
m_1(z) = \frac{\gamma (H - z)}{g \tan \alpha_1} dz
\] (6)

The total weight \( W_1 \) of failure wedge BDF is derived from Eq. (6) and yields

\[
W_1 = \frac{1}{2} \frac{\gamma H^2}{\tan \alpha_1}
\] (7)

![Fig. 6. Variation in \( F_{\mu\lambda} \) for various values of \( k_h \) and \( k_v \) for \( \varphi = -20^\circ \) for (a) \( \varepsilon = 1 \), (b) \( \varepsilon = 3 \), (c) \( \varepsilon = 5 \) and (a) \( \varepsilon = 7 \) with \( f = 1.0 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).]
The horizontal seismic inertia force acting on soil wedge BDF can be written as

\[
Q_{h1} = m_{1}(z)k_{h}(z, t) = \int_{0}^{H} \frac{\gamma g}{\tan \alpha_{1}} \left( 1 + \frac{H-z}{H} (f-1) \right) k_{h} g \sin \omega \left( t - \frac{H-z}{V_{s}} \right) dz
\]  

Integrating Eq. (8a), the total horizontal seismic inertia force can be expressed as

\[
Q_{h1} = \frac{\gamma k_{h}}{\tan \alpha_{1} 4\pi^{2}} [2\lambda \pi H \cos \omega \zeta + \lambda^{2} [\sin \omega \zeta - \sin \omega \zeta] + \frac{\gamma k_{h}(f-1)}{\tan \alpha_{1} 4\pi^{2}} \left( 2\pi^{2} \lambda^{2} H \cos \omega \zeta + 2\pi \lambda^{2} \sin \omega \zeta \right) + \frac{\lambda^{3}}{H} \cos \omega \zeta - \cos \omega \zeta ]
\]  

where \( \lambda = TV_{s} \) is the wavelength of the vertically propagating shear wave and \( \zeta = t - (H/V_{s}) \).

Similarly the vertical seismic inertia force acting on soil wedge BDF can be expressed as

\[
Q_{v1} = m_{1}(z)k_{v}(z, t) = \int_{0}^{H} \frac{\gamma g}{\tan \alpha_{1}} \left( 1 + \frac{H-z}{H} (f-1) \right) k_{v} g \sin \omega \left( t - \frac{H-z}{V_{p}} \right) dz
\]  

After integrating Eq. (9a), the total vertical seismic inertia force, one can obtain

\[
Q_{v1} = \frac{\gamma k_{v}}{\tan \alpha_{1} 4\pi^{2}} [2\pi \eta H \cos \omega \psi + \eta^{2} (\sin \omega \psi - \sin \omega \psi) + \frac{\gamma k_{v}(f-1)}{\tan \alpha_{1} 4\pi^{2}} \left( 2\pi^{2} \eta \pi H \cos \omega \psi + 2\pi \eta^{2} \sin \omega \psi \right) + \frac{\eta^{3}}{H} \cos \omega \psi - \cos \omega \psi ]
\]  

where \( \eta = TV_{p} \) is the wavelength of the vertically propagating primary wave through the soil wedge and \( \psi = t - (H/V_{p}) \).

Fig. 7. Variation in \( F_{pd} \) for various values of \( k_{h} \) and \( k_{v} \) for \( \phi = +10^\circ \) for (a) \( \epsilon = 1 \), (b) \( \epsilon = 3 \), (c) \( \epsilon = 5 \) and (a) \( \epsilon = 7 \) with \( f = 1.4 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).
Using the force equilibrium equation in horizontal and vertical directions, the seismic passive resistance \( P_{p\gamma d1} \) can be expressed as

\[
P_{p\gamma d1} = [R_1 \sin(\alpha_1 + \phi) - Q_{h1} \cos \phi - W_1 \sin \phi] / \cos(\delta + \phi) \tag{10}
\]

\[
P_{p\gamma d1} = [-R_1 \cos(\alpha_1 + \phi) - Q_{h1} \sin \phi + W_1 \cos \phi - Q_{v1} \cos \phi] / \sin(\delta + \phi) \tag{11}
\]

Consider the free body diagram of failure wedge ACE (Fig. 2(a)). The forces acting on wedge ACE are the \( W_3 \) weight of failure wedge ACE, and \( Q_{h3} \) and \( Q_{v3} \) are the seismic inertia force components in horizontal and vertical directions, respectively.

The horizontal seismic inertia force acting on failure wedge ACE is given by the following integral:

\[
Q_{h3} = \int_0^H \gamma g \left( \frac{H-z}{\tan \alpha_3} \right) \left\{ 1 + \frac{H-z}{f} \right\} k_{h3} \sin \omega \left( t - \frac{H-z}{V_p} \right) \, dz \tag{12a}
\]

After integrating Eq. (12a), the total horizontal seismic inertia force can be expressed as

\[
Q_{h3} = \frac{\gamma k_h}{\tan \alpha_3} \left[ 2\lambda \pi H \cos \omega \zeta + \lambda^2 \left( \sin \omega \zeta - \sin \omega t \right) \right] + \frac{\gamma k_h (f-1)}{\tan \alpha_3} \left[ 2\lambda^2 \lambda^2 \cos \omega \zeta + 2\pi \lambda^2 \sin \omega \zeta \right]
\]

\[
+ \frac{H}{H} \left( \cos \omega t - \cos \omega \zeta \right) \tag{12b}
\]

Similarly, the vertical seismic inertia force is given by the integral

\[
Q_{v3} = \int_0^H \gamma g \left( \frac{H-z}{\tan \alpha_3} \right) \left\{ 1 + \frac{H-z}{f} \right\} k_{v3} \sin \omega \left( t - \frac{H-z}{V_p} \right) \, dz \tag{13a}
\]

The total vertical seismic inertia force can be obtained by integrating Eq. (13a) as

\[
Q_{v3} = \frac{\gamma k_v}{\tan \alpha_3} \left[ 2\pi \eta H \cos \omega \psi + \eta^2 \left( \sin \omega \psi - \sin \omega t \right) \right]
\]

Fig. 8. Variation in \( F_{\gamma d} \) for various values of \( k_h \) and \( k_v \) for \( \phi = -10^\circ \) for (a) \( \epsilon = 1 \), (b) \( \epsilon = 3 \), (c) \( \epsilon = 5 \) and (a) \( \epsilon = 7 \) with \( f = 1.4 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).
\[ P_{p_{eff}} = R \sin (\alpha + \varphi) + W \sin \varphi \]

\[ P_{p_{eff}} = [R \cos (\alpha + \varphi) + W \cos \varphi - Q_h] \sin (\delta - \varphi) \]

The mass of the elementary strip is given by

\[ m_3(z) = \frac{\gamma Bdz}{g} \]

The total weight of failure block CDFE, \( W_2 \), is obtained from Eq. (16), namely,

\[ W_2 = \gamma BH \]

The total horizontal seismic inertia force acting on soil block CDFE can be obtained as

\[ Q_{h2} = \int_0^H \frac{\gamma Bdz}{g} \left\{ 1 + \frac{H-z}{H} (f-1) \right\} \sin \omega \left( \frac{t - \frac{H-z}{V_s}}{V_s} \right) \]

\[ = \frac{\gamma Bk_h}{4\pi^2} \left[ 2\pi \lambda (\cos \omega \zeta - \cos \omega t) + 2\pi \lambda (f-1) \times \cos \omega \zeta \right. \\
\left. + \lambda^2 (f-1) (\sin \omega \zeta - \sin \omega t) \right] \]

Fig. 9. Variation in \( F_{\gamma d} \) for various values of \( k_h \) and \( k_v \) for \( \varphi = +20^\circ \) for (a) \( \epsilon = 1 \), (b) \( \epsilon = 3 \), (c) \( \epsilon = 5 \) and (a) \( \epsilon = 7 \) with \( f = 1.4 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).
The total vertical seismic inertia force acting on soil block CDEF can be given by

\[ Q_v = \int_0^H \frac{\gamma B dz}{g} \left\{ 1 + \frac{H - z}{H} (f - 1) \right\} k_v \sin \omega \left( t - \frac{H - z}{V_p} \right) \]

\[ = \frac{\gamma B k_v}{4 \pi^2} \left[ 2 \pi \eta (\cos \omega \psi - \cos \omega t) + \frac{\eta^2 (f - 1) (\sin \omega \psi - \sin \omega t)}{H} \right] \]

(19)

The seismic passive resistance acting on the imaginary faces of retaining walls (Choudhury and Subba Rao, 2004), DF and CE, are \( P_{pd1} \) and \( P_{pd3} \), respectively. As mentioned by Meyerhof and Adams (1968), the anchor uplift capacity is an application of the retaining wall problem under the negative wall friction angle case of passive earth pressure under the static condition due to the movement of the anchor block. Similar consideration was given by Choudhury and Subba Rao (2004) for the pseudo-static seismic analysis and applied in the present pseudo-dynamic analysis for imaginary wall movement.

The gross seismic pullout load \( (P_{ud}) \) of an anchor is given by

\[ P_{ud} = P_{pd1} \sin (\delta + \phi) + P_{pd3} \sin (\delta - \phi) + W_2 \cos \phi - Q_v \cos \phi - Q_h \sin \phi \]

(20)

The net ultimate seismic uplift capacity of anchor \( q_{udnet} \) is given by

\[ q_{udnet} = \frac{P_{ud} - [W_2 \cos \phi - Q_v \cos \phi - Q_h \sin \phi]}{B} \]

(21)

On simplification, \( q_{udnet} \) can be written in the following form:

\[ q_{udnet} = 0.5 \gamma BF_{pl} \]

(22)

Fig. 10. Variation in \( F_{pl} \) for various values of \( k_h \) and \( k_v \) for \( \phi = -20^\circ \) for (a) \( \varepsilon = 1 \), (b) \( \varepsilon = 3 \), (c) \( \varepsilon = 5 \) and (a) \( \varepsilon = 7 \) with \( f = 1.4 \), \( H/\lambda = 0.3 \) and \( H/h = 0.16 \).
The net seismic uplift capacity factor in the above expression \( F_{rd} \) can be given as

\[
F_{rd} = \epsilon^2 K_{pyd} \left[ \tan (\delta + \phi) + \tan (\delta - \phi) \right] - 2 \epsilon \sin \phi \tan (\delta - \phi) \\
+ \frac{k_v \sin \phi}{2\pi^2} \left[ 2\pi\eta(\cos \omega t - \cos \omega t_0) + 2\pi\eta(\phi - 1) \cos \omega t \right] \\
+ \frac{H}{\phi} \left[ 2\pi \lambda (\cos \omega t - \cos \omega t_0) + 2\pi \lambda (\phi - 1) \right] \\
\times \cos \omega \zeta + \frac{\lambda^2 (\phi - 1)(\sin \omega \zeta - \sin \omega t_0)}{H} \right] \tan (\delta - \phi)
\]

(23)

where \( K_{pyd} \) is the net seismic passive earth pressure coefficient for the unit weight component and the embedment ratio of the anchor, \( \epsilon = H / B \).

The net seismic uplift capacity factor \( F_{rd} \) can be obtained from the above expression for different combinations of load inclination, soil friction angle, seismic acceleration coefficients in both vertical and horizontal directions, soil amplification factor and embedment ratio of a shallow horizontal plate anchor subjected to an oblique load.

In the present analysis, the value of \( V_p/V_s = 1.87 \) is considered by using the relationship between the primary and the shear wave velocities with Poisson’s ratio of the soil (Das, 1993). It can be observed from Eq. (23) that \( F_{rd} \) is a function of the angle of the failure plane, \( \alpha/T \), \( H/\lambda \) and \( H/\eta \). The minimum value for \( F_{rd} \) is obtained by optimizing \( F_{rd} \) with respect to the angle of the failure planes and \( \alpha/T \).

### 2.1. Determination of critical angle of failure plane

The critical angle of failure planes depends on the soil friction angle and remains constant under static conditions, whereas under seismic conditions, it shifts with changes in the input parameters, like \( k_h \), \( k_v \) and soil amplification in addition to \( \phi \). Hence, a trial and error procedure is used in the present analysis to find the critical angle of the failure planes for different combinations of seismic acceleration coefficients in both horizontal and vertical directions with the amplification factor and the inclination of load for various values of the soil friction angle. The trial values obtained for \( \alpha_1 \) and \( \alpha_3 \) must satisfy the condition whereby the two computed values for \( P_{pyd} \), from Eqs. (10) and (11), and \( P_{pyd} \), from Eqs. (14) and (15), are the same. The critical values for \( \alpha_1 \) and \( \alpha_3 \) are obtained until a convergence is reached to the third decimal for various values of \( \phi \) with different combinations of \( k_h \), \( k_v \), \( \phi \) and \( f \).

### 3. Results and discussion

The equation proposed in the case of cohesionless soil to avoid the phenomenon of shear fluidization for various combinations of \( k_h \) and \( k_v \) (Richards et al., 1990) and modified by Ghosh (2009) for the amplification effect, \( f \), for the values of \( \phi \), is considered and given by

\[
\phi \geq \tan^{-1} \left[ \frac{f k_h}{1 - f k_v} \right]
\]

(24)

The results for \( F_{rd} \) for a horizontal strip anchor subjected to an oblique load are presented in a combination of graphs and tables. The different input parameters, such as \( \phi \), \( q \), \( k_h \), \( k_v \), \( \epsilon \), \( f \) and their variations, are considered in the present study.

Figs. 3 and 4 show the variations in \( F_{rd} \) with \( k_h \) for various combinations of \( k_v \) and different values for \( \phi \) with \( f = 1.0 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \) for \( q = +10 \) and \( q = -10 \), respectively. Similarly, Figs. 5 and 6 show the variations in \( F_{rd} \) with \( k_h \) for these combinations.
various combinations of $k_v$ and different values for $\phi$, $\epsilon$ with $f=1.0$, $H/\lambda=0.3$ and $H/h=0.16$ for $\phi=+20^\circ$ and $\phi=-20^\circ$, respectively. Again, Figs. 7 and 8 are drawn for $\phi=\pm 10^\circ$, respectively, whereas Figs. 9 and 10 are plotted for $\phi=\pm 20^\circ$, respectively, to show the variations in $F_{yd}$ with $k_v$ for various combinations of $k_v$, and different values for $\phi$, $\epsilon$ with $f=1.4$, $H/\lambda=0.3$ and $H/h=0.16$. Fig. 11 shows the effect of the amplification factor on $F_{yd}$ for different values of $\phi$, $\epsilon$ and thereafter, $F_{yd}$ increases with an increase in $\phi$ (Fig. 12). A significant increase in $F_{yd}$ values with an increase in the negative inclination of the load is observed (Fig. 13). The values for $F_{yd}$ decrease with an increase in the amplification factor and the inclination of the load in either direction for all the combinations of seismic acceleration coefficients in both horizontal and vertical directions. For example, referring to Table 1, for the case of $\phi=30^\circ$, $k_h=0.1$, $k_v=0.0k_h$, $\epsilon=5$ and $f=1.4$, it is observed that the net seismic uplift capacity factor decreases by 5.86% for $\phi=+10^\circ$ and increases by 10.76% for $\phi=-10^\circ$ if compared with the values for $F_{yd}$ for $\phi=0^\circ$ and the difference increases with an increase in $k_h$ and $\phi$ in both directions of load inclination. Again, from Figs. 5 and 9, for the case of $\phi=40^\circ$, $k_h=0.2$ and $k_v=0.5k_h$, it is seen that the value for $F_{yd}$ with $f=1.0$ is 1.09 times more than the value with $f=1.4$ for $\phi=+20^\circ$.

It is observed that the inclinations of failure planes AC and BD keep changing with changes in the soil friction angle, both seismic acceleration coefficients in horizontal and vertical directions and the amplification factor. From Table 2, it is seen that $\alpha_3$ increases and $\alpha_1$ decreases with an increase in $k_h$ in the critical direction, as shown in Fig. 1. Similarly, both $\alpha_3$ and $\alpha_1$ increase significantly with an increase in the seismic acceleration coefficient in the vertical direction. Referring to Table 3, for $\phi=30^\circ$ and $\phi=40^\circ$, $k_h=0.1$ and $f=1.0$, $\alpha_1$ and $\alpha_3$.

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are 48.90 and 52.51 for $k_h=0.0 k_h$ increases to 52.25 and 55.90, respectively, for $k_h=0.5 k_h$. The inclination of the failure planes further increases in the presence of the soil amplification factor resulting in the formation of steeper failure planes. It is worth noting that the embedment ratio does not affect the geometry of the failure planes even under seismic conditions, which shows a resemblance to the observation under static conditions, whereas the reverse observation was proposed by Ghosh (2009). From Table 2, for $\phi=30^\circ$ and $\varphi=10^\circ$, $k_h=0.1$, $k_i=0.0$, $f=1.0$ and from Table 3 for $\phi=30^\circ$, $\varphi=30^\circ$, $k_h=0.1$, $k_i=0.0$, $f=1.0$ values of $\alpha_1$ and $\alpha_3$ are the same, i.e., 51.18 and 54.61, respectively. It is also observed from Tables 2 and 3 that the anchor inclination does not change the geometry of the failure planes, which shows a resemblance to the prediction by Das and Seeley (1975).

Fig. 11 shows the effect of the amplification factor on the values for the net uplift capacity factor for $\phi=30^\circ$ with $\varphi=+10^\circ$, $e=3.0$, $H/H_i=0.3$ and $H/H_i=0.16$ for different values of $k_h$ with $k_i=0.0 k_h$. It shows that $F_{pu}$ decreases continuously with an increase in the soil amplification factor and the seismic acceleration coefficient in the horizontal direction. Referring to Fig. 11, $F_{pu}$ decreases by 1.22% if the amplification factor ($f$) changes from 1 to 1.2, 1.35% for a change in $f$ from 1.2 to 1.4, 1.47% for a change in $f$ from 1.4 to 1.6, 1.62% when $f$ changes from 1.6 to 1.8, 1.65% for a change in $f$ from 1.8 to 2.0. From Figs. 4–10, it is also observed that $F_{pu}$ decreases with an increase in $k_i$ and $f$ for all possible inclinations of load. Hence, proper attention must be given by a geotechnical engineer to determine the effect of soil amplification on $F_{pu}$ as it may lead to the catastrophic failure of anchors under seismic conditions. Previously, very few researchers obtained the uplift capacity for an oblique loaded anchor under static conditions. The present study shows the importance of obtaining the uplift capacity under seismic conditions considering the effect of amplification.

4. Comparison of results

Table 4 shows a comparison of the breakout factor ($q_{lad}=P_{lad}/A_H$) in medium–dense sand under static conditions ($k_h=k_i=0.0$) for the test data of Das and Seeley (1975) with $B=0.064$ m and $\gamma=14.71$ kN/m$^3$, $\phi=31^\circ$. The shape factors proposed by Tagaya et al. (1988) are used for the comparison. Up to an embedment ratio of 2, the proposed theory follows the experimental results closely; thereafter, for $e=4.5$ it over predicts the results (by 15.21%). The shape factors used for the comparison are lower than the values recommended by Meyerhof and Adams (1968) and higher than those recommended by Dickin and Laman (2007).

The results of the present study and the test results obtained by Meyerhof (1973) for $\phi=43^\circ$ for a gross pullout load of a horizontal strip anchor subjected to an inclined load are given in Table 5. The results obtained in the present study are in favorable comparison with the experimental results and follow a similar trend, showing a constant decrease in the gross pullout load, except the marginal shift when $\varphi$ changes from $30^\circ$ to $40^\circ$. In the present study, the gross pullout load increases by 2.21% and decreases by 0.49% for Meyerhof (1973), when $\varphi$ changes from $30^\circ$ to $40^\circ$.

The experimental results of Dickin and Laman (2007), for loose sand with $B=1$ m, $\gamma=14.5$ kN/m$^3$, $\phi=35^\circ$ under static conditions, are compared with the predictions of theoretical

<table>
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<th>Meyerhof (1973)</th>
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</table>

Fig. 14. Comparison of breakout factor ($q_{lad}$) with experimental results of Dickin and Laman (2007) with $\gamma=14.5$ kN/m$^3$, $B=1$ m and $\phi=35^\circ$ under static conditions ($k_h=k_i=0.0$).

Table 4

| $\varphi$ | $\varepsilon=1$ | $\varepsilon=2$ | $\varepsilon=4.5$ | Present study | $\varepsilon=1$ | $\varepsilon=2$ | $\varepsilon=4.5$
<table>
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methods, as shown in Fig. 14. The solutions obtained by Deshmukh et al. (2011) and Kumar (2001), the lower bound results by Merifield and Sloan (2006) and the proposed method show a favorable agreement with the experimental results up to \( \varepsilon = 4.0 \); thereafter, they overpredict. The theory of Meyerhof and Adams (1968) overestimates the uplift capacity with the observed one. For a horizontal strip anchor subjected to vertical load under static conditions, Merifield and Sloan (2006) quantify the effect of anchor roughness, which is less than 4%, and hence, justifies the assumption of the smooth plate in the present study.

Table 6 shows a comparison of \( q_{ud} \) with the available results in literature under seismic conditions for \( \phi = 30^\circ \), \( \psi = 0^\circ \), \( f = 1.0 \), \( \varepsilon = 1 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \). It is observed that the present study shows a reasonable comparison with the results obtained by other researchers (Choudhury and Subba Rao, 2004; Kumar, 2001; Ghosh, 2009) for \( \varepsilon = 1 \) with various combinations for \( k_h \) and \( k_v \). The difference in the results obtained in the present study and those by Choudhury and Subba Rao (2004) is found to vary about 1.28% to –5%, –4.16% to 1% with Kumar (2001).

The ultimate seismic uplift capacity factor, \( F_{c,e} = P_{ud}/B^2 \), using the upper bound solution, was obtained by Ghosh (2009) for a horizontal strip anchor using the pseudo-dynamic approach. Table 7 shows a comparison of \( F_{c,e} \), for which various values for \( k_h \) and \( k_v \) are considered for \( \phi = 30^\circ \), \( \psi = 0^\circ \), \( f = 1.0 \) and \( 1.4 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \). It is observed that the present study predicts lower values than Ghosh (2009) for the given combinations of \( k_h \) and \( k_v \) with various amplification factors. The percentage difference of the present theory, observed for \( f = 1.0 \) ranges between –1.78% and –5.27% for \( k_v = 0.0 k_h \); –1.78% to –6.15% for \( k_v = 0.5 k_h \) for different values of \( k_h \) and difference increases with an increase in \( f (= 1.4) \), with Ghosh (2009). This difference in results can be attributed to the inherent differences between the limit analysis method and the limit equilibrium method, thus showing the adequacy of the present theory.

5. Conclusions

The net seismic uplift capacity factor for an obliquely loaded horizontal strip anchor for the unit weight component of soil (\( F_{\gamma d} \)) is computed using the limit equilibrium method with a pseudo-dynamic approach. Kötter (1903) is employed to obtain the distribution of soil reaction on a simple planar failure surface. It is observed that \( F_{\gamma d} \) decreases for a certain positive inclination of load (\( \phi \approx +20^\circ \)); thereafter, it increases and a significant increase in the results was observed for the inclination of load in a negative direction under seismic conditions. \( F_{\gamma d} \) decreases significantly with an increase in the seismic acceleration coefficients in both vertical and horizontal directions and the amplification factor and increases with an increase in the embedment ratio and the soil friction angle, as expected. The values obtained from the present analysis are compared with the available specific results reported in literature under both static and seismic conditions and are found to be in good agreement with them. Hence, Kötter (1903) worked as a powerful tool in the present analysis, namely, to obtain the uplift capacity factor which shows the better agreement with the results available in literature, obtained by the pseudo-static, pseudo-dynamic approach, using various approaches like the complicated upper bound limit method, the log-spiral failure surface and even the experimental results.

### Table 6
Comparison of \( q_{ud} \) with the available results in literature under seismic conditions for \( \phi = 30^\circ \), \( \psi = 0^\circ \), \( f = 1.0 \), \( \varepsilon = 1 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).

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### Table 7
Comparison of \( F_{c,e} \) for \( f = 1.0 \) and \( 1.4 \), \( \phi = 30^\circ \), \( \psi = 0^\circ \), \( \varepsilon = 5 \), \( H/\lambda = 0.3 \) and \( H/\eta = 0.16 \).

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Note: ‘–’ shows the case of shear fluidization (Richards et al., 1990; Ghosh, 2009).
Nomenclature

\(a_h\) amplitude of acceleration in the horizontal direction
\(a_v\) amplitude of acceleration in the vertical direction
\(B\) width of ground anchor
\(d_p\) differential reaction pressure on failure surface
\(f\) amplification factor
\(F_{r,n}\) net seismic uplift capacity factor
\(F_{r,E}\) ultimate seismic uplift capacity factor
\(H\) depth of anchor from ground
\(G\) shear modulus
\(k_h\) seismic coefficient of acceleration in the horizontal direction
\(k_v\) seismic coefficient of acceleration in the vertical direction
\(L\) length of anchor
\(P_{ud}\) gross seismic pullout capacity
\(P_{p,h,1}, P_{p,h,3}\) seismic passive resistance acting on an imaginary faces of retaining wall
\(q_{ud}\) breakout factor
\(Q_h\) total horizontal inertia force
\(Q_v\) total vertical inertia force
\(R_{1,2}\) resultant soil reaction on failure planes
\(S\) length of failure plane
\(t\) time
\(T\) period of lateral shaking
\(V_p\) primary wave velocity
\(V_s\) shear wave velocity
\(W\) total weight of failure wedge
\(W_{1,2}\) and \(W_{3}\) weights of respective failure wedges
\(z\) depth of point below ground level
\(\phi\) soil friction angle
\(\delta\) wall friction angle
\(\alpha\) inclination of tangent on failure surface at the point of interest with horizontal
\(\alpha_1, \alpha_3\) angles of failure planes with horizontal
\(e\) embedment ratio
\(\gamma\) unit weight of soil
\(\omega\) angular frequency
\(\rho\) density of soil medium
\(\nu\) Poisson's ratio of soil medium
\(\lambda\) wavelength of the vertically propagating shear wave
\(\eta\) wavelength of the vertically propagating primary wave
\(\varphi\) angle of pullout load with vertical.

References


