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Electroweak corrections to three-jet production in e^+e^- annihilation

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ABSTRACT

We compute the electroweak $\mathcal{O}(\alpha^3\alpha_s)$ corrections to three-jet production and related event-shape observables at electron–positron colliders. We properly account for the experimental photon isolation criteria and for the corrections to the total hadronic cross section. Corrections to the three-jet rate and to normalised event-shape distributions turn out to be at the few-percent level.

Precision QCD studies at electron–positron colliders rely on the measurement of the three-jet production cross section and related event-shape observables. The deviation from simple two-jet configurations is proportional to the strong coupling constant α_s, so that by comparing the measured three-jet rate and related event shapes (see, e.g., Ref. [1]) with the theoretical predictions, one can determine α_s. Including electroweak coupling factors, the leading-order (LO) contribution to this process is of order $\alpha^2\alpha_s$.

Owing to recent calculational progress, the QCD predictions for event shapes [2,3] and three-jet production [4,5] are accurate to next-to-next-to-leading order (NNLO, $\alpha^2\alpha_s^3$) in QCD perturbation theory. Depending on the observable under consideration, the numerical magnitude of the NNLO corrections varies between three and twenty percent. Inclusion of these corrections results in an estimated residual uncertainty of the QCD prediction from missing higher orders at the level of below five percent for the event-shape distributions, and below one percent for the three-jet cross section. At this level of theoretical precision, higher-order electroweak effects could be of comparable magnitude. At present, only partial calculations of electroweak corrections to three-jet production and event shapes are available [6], which cannot be compared with experimental data directly. In this work, we present the first calculation of the NLO electroweak $(\alpha^3\alpha_s)$ corrections to three-jet observables in e^+e^- collisions including the quark–antiquark–photon $(q\bar{q}\gamma)$ final states. Note that the QCD corrections to these final states are of the same perturbative order as the genuine electroweak corrections to quark–antiquark–gluon $(q\bar{q}g)$ final states. Since photons produced in association with hadrons can never be fully isolated, both types of corrections have to be taken into account.

Event-shape measurements at LEP usually rely on a standard set of six variables y, defined, for example, in Ref. [7]: thrust T, C-parameter, heavy jet mass ρ, wide and total jet broadenings B_W and B_T, and two-to-three-jet transition parameter in the Durham algorithm Y_3. The experimentally measured event-shape distribution

$$\frac{1}{\sigma_{had}}\frac{d\sigma}{dy}$$

is normalised to the total hadronic cross section. In the perturbative expansion, it turns out to be most appropriate to consider the

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expansion of this ratio, which reads to NNLO in QCD and NLO in the electroweak theory

$$\frac{1}{\sigma_{\rm had}}\frac{{\rm d}\sigma}{{\rm d}y} = \left(\frac{\alpha_{\rm s}}{2\pi}\right)\frac{{\rm d}\bar{A}}{{\rm d}y} + \left(\frac{\alpha_{\rm s}}{2\pi}\right)^2\frac{{\rm d}\bar{B}}{{\rm d}y} + \left(\frac{\alpha_{\rm s}}{2\pi}\right)^3\frac{{\rm d}\bar{C}}{{\rm d}y} + \left(\frac{\alpha}{2\pi}\right)\frac{{\rm d}\delta_\gamma}{{\rm d}y} + \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right)\frac{{\rm d}\delta_{\rm EW}}{{\rm d}y}, \quad (1)$$

where the fact is used that the perturbative expansion of $\sigma_{\rm had}$ starts at order α^2. The calculation of the QCD coefficients $\bar{A}$, $\bar{B}$, and $\bar{C}$ is described in Refs. [2,3]. The LO purely electromagnetic contribution δ_γ arises from tree-level $q\bar{q}\gamma$ final states without a gluon. The NLO electroweak coefficient $\delta_{\rm EW}$ receives contributions from the $\mathcal{O}(\alpha)$ correction to the hadronic cross section,

$$\sigma_{\rm had} = \sigma_0\left[1 + \left(\frac{\alpha}{2\pi}\right)\delta_{\sigma,1}\right], \quad (2)$$

and from the genuine $\mathcal{O}(\alpha^3\alpha_s)$ contribution to the event-shape distribution

$$\frac{1}{\sigma_0}\frac{{\rm d}\sigma}{{\rm d}y} = \left(\frac{\alpha_{\rm s}}{2\pi}\right)\frac{{\rm d}\bar{A}}{{\rm d}y} + \left(\frac{\alpha_{\rm s}}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right)\frac{{\rm d}\delta_{\bar{A}}}{{\rm d}y},$$

such that

$$\frac{{\rm d}\delta_{\rm EW}}{{\rm d}y} = \frac{{\rm d}\delta_{\bar{A}}}{{\rm d}y} - \delta_{\sigma,1}\frac{{\rm d}\bar{A}}{{\rm d}y} \quad (3)$$

yields the full NLO electroweak correction. Both terms are to be evaluated with the same event-selection cuts. As shown in the following, many of the numerically dominant contributions, especially from initial-state radiation, cancel in this difference.

In the experimental measurement of three-jet observables at electron–positron centre-of-mass energy $\sqrt{s}$, several cuts are applied to reduce the contributions from photonic radiation. In our calculation, we apply the criteria used in the ALEPH analysis [7]. Very similar criteria were also applied by the other experiments [8]. Particles contribute to the final state only if they are within the detector acceptance, defined by the production angle relative to the beam direction, $|\cos\theta| < 0.965$. Events are accepted if the reconstructed invariant mass squared s' of the final-state particles is larger than $s_{\rm cut} = 0.81$ s. To reduce the contribution from hard photon radiation, the final-state particles are clustered into jets using the Durham algorithm with resolution parameter $y_{\rm cut} = 0.002$. If one of the resulting jets contains a photon carrying a fraction $z_\gamma > z_{\gamma,{\rm cut}} = 0.9$ of the jet energy, it is considered to be an isolated photon, and the event is discarded. The event-shape variables are then computed in the centre-of-mass frame of the final-state momenta, which can be boosted relative to the e^+e^- centre-of-mass frame, if particles are outside the detector acceptance.

In the computation of the $\mathcal{O}(\alpha)$ corrections to the total hadronic cross section, we include the virtual electroweak corrections to $q\bar{q}$ final states, and the real radiation corrections from $q\bar{q}\gamma$ final states, provided the above event-selection criteria are fulfilled. The corrections to the event-shape distributions receive contributions from the virtual electroweak corrections to the $q\bar{q}g$ final state, the virtual QCD corrections to the $q\bar{q}\gamma$ final state, and from the real radiation $q\bar{q}g\gamma$ final state. To separate the divergent real radiation contributions, we used both the dipole subtraction method [9,10] and phase-space slicing [11], resulting in two independent implementations. Soft singularities are present in the virtual and real corrections. They are regularized dimensionally or with infinitesimal photon and gluon masses, and cancel in the sum. Collinear singularities from photon radiation off the incoming leptons (initial-state radiation, ISR) are only partially cancelled. The left-over collinear ISR singularity is regularized by the electron mass and absorbed into the initial-state radiator function, which we consider either at fixed order, or in a leading-logarithmic (LL) resummation [12]. Owing to the specific nature of the event selection, also photon radiation off the outgoing quarks (final-state radiation, FSR) is only partially cancelled. The left-over FSR singularity arises from the isolated photon definition, which vetoes on photon jets with $z_\gamma > z_{\gamma,{\rm cut}}$. This singularity is absorbed into the photon fragmentation function, which we apply in the fixed-order approach of Ref. [13]. For the non-perturbative contribution to this function, we use the $\mathcal{O}(\alpha)$ two-parameter fit of ALEPH [14]. The fragmentation contribution derived in Ref. [13] is based on phase space slicing and dimensional regularization. We recomputed this contribution using subtraction and mass regularization [10].

The Feynman diagrams for the virtual corrections are generated with FeynArts [15,16]. Using two independent inhouse Mathematica routines, one of which builds upon FormCalc [17], each diagram is expressed in terms of standard matrix elements and coefficients of tensor integrals. The tensor integral coefficients are numerically reduced to standard scalar integrals using the methods described in Refs. [19,20]. The scalar master integrals are evaluated using the methods and results of Refs. [21–23], where UV divergences are regularized dimensionally. For IR divergences two alternative regularizations are employed, one that is fully based on dimensional regularization with massless light fermions, gluons, and photons, and another that is based on infinitesimal photon and gluon masses and small fermion masses. The loop integrals are translated from one scheme to the other as described in Ref. [24].

The Z-boson resonance is described in the complex-mass scheme [25,26], and its mass is fixed from the complex pole. The electromagnetic couplings appearing in LO are parametrized in the G_μ scheme, i.e., they are fixed via

$$\alpha = \alpha_{G_\mu} = \sqrt{2}G_\mu M_{\rm W}^2\left(1 - M_{\rm W}^2/M_{\rm Z}^2\right)/\pi\,.$$

As the leading electromagnetic corrections are related to the emission of real photons, we fix the electromagnetic coupling appearing in the relative corrections by $\alpha = \alpha(0)$, which is the appropriate choice for the leading photonic corrections. Accordingly the cross section for $e^+e^- \to q\bar{q}g$ is proportional to $\alpha_{G_\mu}^2\alpha_{\rm s}$ while the electroweak corrections to this process are proportional to $\alpha(0)\alpha_{G_\mu}^2\alpha_{\rm s}$.

We performed two independent calculations of all ingredients resulting in two independent Fortran codes, one of them being an extension of Pole [18].

We use the following values of the electroweak and QCD parameters:

$$\begin{aligned} &G_\mu = 1.16637\times10^{-5}\ {\rm GeV}^{-2}, && M_{\rm H} = 120\ {\rm GeV}, \\ &\alpha(0) = 1/137.03599911, && \alpha_{\rm s}(M_{\rm Z}) = 0.1176, \\ &m_{\rm e} = 0.51099892\ {\rm MeV}, && m_{\rm t} = 171.0\ {\rm GeV}. \end{aligned} \quad (4)$$

Because we employ a fixed width in the resonant W- and Z-boson propagators in contrast to the approach used at LEP to fit the W and Z resonances, where running widths are taken, we have to convert the "on-shell" values of $M_V^{\rm LEP}$ and $\Gamma_V^{\rm LEP}$ ($V = {\rm W,Z}$), resulting from LEP, to the "pole values" denoted by M_V and Γ_V, leading to [27]:

$$\begin{aligned} &M_{\rm W} = 80.375\ldots\ {\rm GeV}, && \Gamma_{\rm W} = 2.140\ldots\ {\rm GeV}, \\ &M_{\rm Z} = 91.1535\ldots\ {\rm GeV}, && \Gamma_{\rm Z} = 2.4943\ldots\ {\rm GeV}. \end{aligned} \quad (5)$$

In the final state we take all light quarks into account, including b quarks.

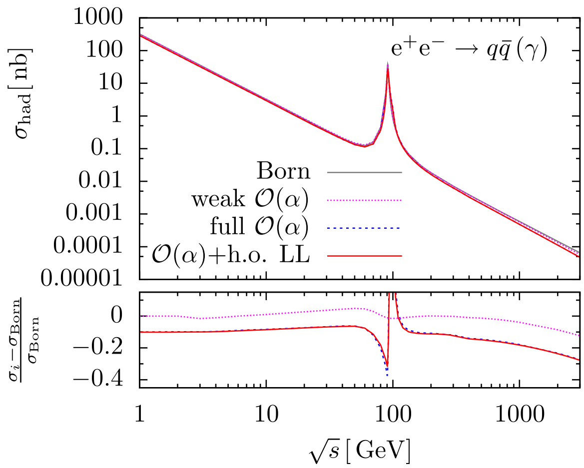

Fig. 1. Total hadronic cross section $\sigma_{\rm had}(\sqrt{s})$.

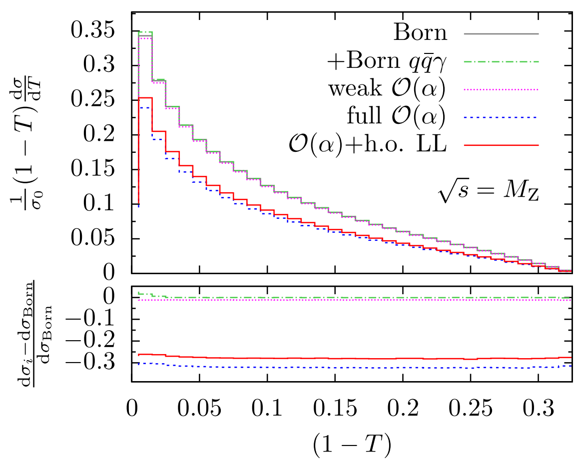

Fig. 2. Differential thrust distribution at $\sqrt{s} = M_Z$.

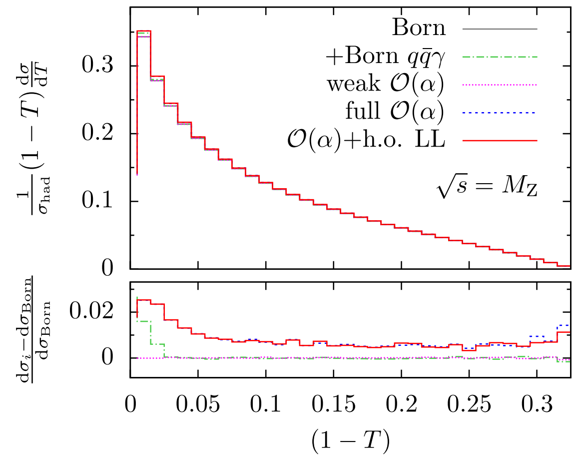

Fig. 3. Differential thrust distribution at $\sqrt{s} = M_Z$ normalised to $\sigma_{\rm had}$.

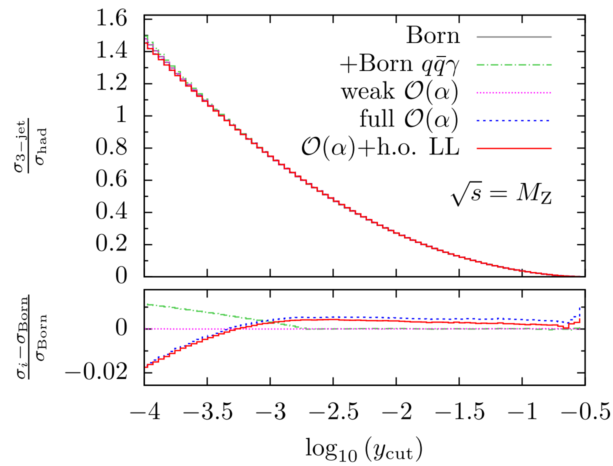

Fig. 4. Three-jet rate at $\sqrt{s} = M_Z$ normalised to $\sigma_{\rm had}$.

In Fig. 1, we display the total hadronic cross section $\sigma_{\rm had}$ for the above event-selection criteria including NLO electroweak corrections and the relative corrections separately. For the latter, "full" and "weak" refers to the electroweak NLO corrections with and without purely photonic corrections, respectively, and "h.o. LL" indicates the inclusion of the higher-order ISR effects. For most energies, the full $\mathcal{O}(\alpha)$ corrections are sizable and negative, ranging between -30% at the Z peak and about -10% at energies above and below. The numerically largest contribution is always due to ISR. However, above the Z resonance up to about 110 GeV the corrections are positive and of the order of the Born cross section due to the well-known radiative return phenomenon [28] that occurs in this region because of our choice of $s_{\rm cut}$. As it is not relevant experimentally, we did not fully resolve it in Fig. 1. Below 60 GeV and above 120 GeV the magnitude of the corrections is increased due to LL resummation of ISR, whereas it is decreased in the region in between. The virtual one-loop weak corrections (from fermionic and massive bosonic loops) yield only a moderate correction between -5 and $+5\%$. The increase for energies above 1 TeV is due to electroweak Sudakov logarithms.

Using the same event-selection cuts, Fig. 2 displays the differential thrust distribution at $\sqrt{s} = M_Z$, including NLO electroweak contributions. The distributions are weighted by $(1-T)$, evaluated at each bin centre. The Born contribution is given by the $\bar{A}$-term of (1), while the full $\mathcal{O}(\alpha)$ corrections contain the tree-level $q\bar{q}\gamma$ contribution δ_γ and the NLO electroweak contribution $\delta_{\bar{A}}$. Again, we observe large negative corrections due to ISR, and moderate weak corrections. The corrections are largely constant for $T < 0.95$, where the isolated photon veto rejects all contributions from $q\bar{q}\gamma$ final states. For $T > 0.95$, corresponding to the two-jet limit, we find a substantial contribution from $q\bar{q}\gamma$ final states already at LO (α^3). Moreover, it turns out that the electromagnetic corrections depend non-trivially on the event-selection cuts.

In expanding the corrections according to (1), and retaining only terms up to LO in α_s, we obtain the genuine electroweak corrections to normalised event-shape distributions, which we display for thrust at $\sqrt{s} = M_Z$ in Fig. 3. Again, the Born contribution is given by the $\bar{A}$-term, while the $\mathcal{O}(\alpha)$ corrections now consist of δ_γ and $\delta_{\rm EW}$. It can be seen very clearly that the large ISR corrections cancel between the event-shape distribution and the normalisation to the hadronic cross section, resulting in electroweak corrections of a few percent. Moreover, effects from ISR resummation are largely reduced as well. The purely weak corrections are below 0.5 per mille.

As shown in Fig. 4, we observe a similar behaviour for the three-jet rate in the region $y_{\rm cut} \gtrsim 0.002$. For $y_{\rm cut} \lesssim 0.002$, $q\bar{q}\gamma$ final states contribute and lead to a distortion of the shape. For $y_{\rm cut} \lesssim 0.0005$ the three-jet rate becomes larger than $\sigma_{\rm had}$. In this region, fixed order perturbation theory becomes unreliable due to large logarithmic corrections at all orders. By improving the LO QCD prediction used here with NLO and NNLO QCD corrections, which are large and negative [3,4] for small y_{cut}, it is possible to extend the range of validity of the fixed order predictions.

In Ref. [6], another calculation of electroweak corrections to three-jet observables was performed, which differs in two important aspects from the work presented here. It considered only the corrections to $q\bar{q}g$ final states, while $q\bar{q}\gamma$ final states at LO and NLO were not taken into account. To remove singularities associated with infrared gluons in $q\bar{q}\gamma g$ final states, event-shape observables were calculated from the reconstructed jet momenta and not from the parton momenta, as used in experiment and in our work. Moreover, the NLO electroweak corrections to the hadronic cross section were not taken into account, such that only unnormalised distributions were considered. Owing to these substantial differences, a direct comparison with the results of Ref. [6] is not possible. Taking care of the different renormalizaion of α, we do observe, however, in the unnormalized distribution, Fig. 2, that the relative size of the $\mathcal{O}(\alpha)$ weak and exact corrections, and of the LL-improved corrections to the thrust distribution agree at the percent level with the results of Ref. [6], except in the region $(1-T) < 0.05$, where $q\bar{q}\gamma$ final states contribute.

Data on event-shape distributions and jet cross sections have been corrected for photonic radiation effects modelled by standard LL parton-shower Monte Carlo programs. They can thus not be compared directly with the NLO electroweak corrections computed here. Incorporation of these corrections requires a more profound reanalysis of LEP data, in order to quantify the impact of the NLO electroweak corrections on precision QCD studies, such as the precise extraction of the strong coupling constant at NNLO in QCD [29].

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