QCD factorization for spin-dependent cross sections in DIS and Drell–Yan processes at low transverse momentum

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Abstract

Built on a recent work on the quantum chromodynamic (QCD) factorization for semi-inclusive deep-inelastic scattering (DIS), we present a set of factorization formulas for the spin-dependent DIS and Drell–Yan cross sections at low transverse momentum. The result can be used to extract transverse-momentum dependent parton distribution and fragmentation functions from relevant experimental data.

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1. Polarized hard scattering has becoming an important tool to learn about the internal structure of hadrons. In recent years, inclusive and semi-inclusive polarized deep-inelastic scattering (DIS) has helped to unravel the quark helicity distributions in the nucleon [1]. At present the polarized Relativistic Heavy Ion Collider at Brookhaven National Laboratory is producing data from polarized proton–proton collisions, which will provide a direct measurement of the polarized gluon distribution [2].

An important class of polarized experiments involves measurement of transverse momentum of the order $\Lambda_{\rm QCD}$. For example, in semi-inclusive deep-inelastic scattering (SIDIS) at moderate energy (for example, at HERMES kinematics), the tagged final-state hadron has a transverse-momentum peaked at a few hundred MeV. In Drell–Yan process also at moderate center-of-mass energy, Drell–Yan pairs typically have a transverse momentum of the same order of magnitude. Theoretical study of these processes began with the classical work of Collins and Soper in which a nearly back-to-back hadron pair is produced in e^+e^- collisions [3]. A factorization theorem for the process was established, which involves a new class of non-perturbative hadronic observables depending on the transverse-momentum of hadrons and/or partons: the transverse-momentum dependent (TMD) fragmentation

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functions and parton distributions. In a previous publication [4], we have extended the QCD factorization theorem to the case of the semi-inclusive DIS. The correction to the factorization is on the order of $P_\perp^2/Q^2$ and M_h^2/Q^2, where $P_\perp$ is the transverse momentum of the produced hadron and M_H is a hadron mass scale. In this Letter, we extend factorization further to a class of spin-dependent DIS and Drell–Yan processes. Since the details are similar, here we focus mostly on the final result, omitting technicalities which can be found in the above references. The result can be used to extract a new class of TMD parton functions from relevant experimental data, just like the standard QCD factorization theorems allow extraction of the Feynman parton distributions from hard scattering data.

Before proceeding future, let us remark that there have been many studies in the literature on the same processes at large, but not too large transverse-momentum ($Q \gg P_\perp \gg \Lambda_{\text{QCD}}$), where Q is hard-collision scale (for example, the virtual-photon mass). In this case, the cross section can in principle be calculated using the conventional perturbative QCD method with the integrated Feynman parton distributions. However, the hard part contains the large double logarithms of the type $\alpha_s \ln^2 P_\perp^2/Q^2$, which must be resummed to make reliable predictions [5,6]. The formalism developed by Collins and Soper in the case of e^+e^- annihilation [3] and followed in our previous paper, is ideal for making this type of resummation. In fact, an application to the unpolarized Drell–Yan process was first made by Collins, Soper and Sterman (CSS) [7]. Various applications of CSS formalism to Drell–Yan, heavy boson productions, and semi-inclusive DIS scattering have been developed in the literature [8–13].

2. Let us first define a new class of non-perturbative hadronic matrix elements, the spin and transverse-momentum dependent parton distributions and fragmentation functions, which one hopes to learn from high-energy scattering. As illustrated in Fig. 1, we consider a hadron with momentum P^μ moving in the z-direction, and is polarized with a spin vector S^μ (dimensionless and $S \cdot P = 0$). In the limit $P^3 \to \infty$, the P^μ is proportional to the light-cone vector $p^\mu = \Lambda(1, 0, 0, 1)$, where Λ is a mass dimension-1 parameter. The conjugation light-cone vector is $n = (1/2\Lambda)(1, 0, 0, -1)$, such that $p^2 = n^2 = 0$ and $p \cdot n = 1$. We use the light-cone coordinates $k^\pm = (k^0 \pm k^3)/\sqrt{2}$, and write any four-vector k^μ in the form of $(k^-, \vec{k}) = (k^-, k^+, \vec{k}_\perp)$, where $\vec{k}_\perp$ represents two perpendicular components (k^x, k^y). Let $(xP^+, \vec{k}_\perp)$ represent the momentum of a parton (quark or gluon) in the hadron as shown in Fig. 1. In a non-singular gauge (e.g., Feynman gauge), the TMD parton distributions can be defined through the following density matrix [14,15]

$$
\mathcal{M}^\pm(x, k_\perp, \mu, x\zeta, \rho) = p^+ \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{k}_\perp}
$$

$$
\times \frac{\langle PS|\bar{\psi}_q(\xi^-, 0, \vec{b}_\perp)\mathcal{L}_v^\dagger(\pm\infty; \xi^-, 0, \vec{b}_\perp)\mathcal{L}_v(\pm\infty; 0)\psi_q(0)|PS\rangle}{S^\pm(\vec{b}_\perp, \mu^2, \rho)}, \tag{1}
$$

where ψ_q is the quark field, and the Dirac and color indices of the quark fields are implicit. The $+$ $(-)$ superscript is appropriate for DIS (Drell–Yan) process [15,16]. v^μ is a time-like dimensionless ($v^2 > 0$) four-vector with zero transverse components ($v^-, v^+, \vec{0}$) and $v^- \gg v^+$. Thus the v^μ is a quasi-light-cone vector, approaching n^μ. The
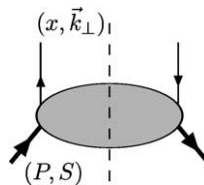

Fig. 1. Transverse momentum dependent quark distributions in the nucleon.

variable ζ^2 denotes the combination $(2P \cdot v)^2/v^2 = \zeta^2$. $\mathcal{L}_v$ is a gauge link along v^μ

$$\mathcal{L}_v(\pm\infty; \xi) = \exp\left(-ig \int\limits_0^{\pm\infty} d\lambda\, v \cdot A(\lambda v + \xi)\right). \tag{2}$$

Here the non-light-like gauge link is introduced to regulate the light-cone singularities. We avoid the use of singular gauges (e.g., the light-cone gauge) because in those gauges the gauge potential may not vanish at infinity and gauge links at infinity might be necessary to define gauge-invariant parton distributions [16].

In the above definition, we have derived a soft factor defined as [4]

$$S^\pm(\vec{b}_\perp, \mu^2, \rho) = \frac{1}{N_c}\langle 0|\mathcal{L}_{\tilde{v}il}^\dagger(\vec{b}_\perp, -\infty)\mathcal{L}_{vlj}^\dagger(\pm\infty; \vec{b}_\perp)\mathcal{L}_{vjk}(\pm\infty; 0)\mathcal{L}_{\tilde{v}ki}(0; -\infty)|0\rangle, \tag{3}$$

where $i, j, k, l = 1, 2, 3$ are color indices and new quasi-light-cone vector $\tilde{v}^\mu = (\tilde{v}^-, \tilde{v}^+, \vec{0})$ has been introduced with $\tilde{v}^- \ll \tilde{v}^+$. The ρ parameter is defined as $\rho = \sqrt{v^-\tilde{v}^+/v^+\tilde{v}^-} \gg 1$. The above soft factor will also be present in the factorization theorems below.

The leading order expansion of the density matrix $\mathcal{M}$ contains eight quark distributions [17,18]

$$\begin{aligned}
\mathcal{M} = \frac{1}{2}\Bigg[&q(x, k_\perp)\slashed{p} + \frac{1}{M}\delta q(x, k_\perp)\sigma^{\mu\nu}k_\mu p_\nu + \Delta q_L(x, k_\perp)\lambda\gamma_5\slashed{p} + \frac{1}{M}\Delta q_T(x, k_\perp)\gamma_5\slashed{p}(\vec{k}_\perp \cdot \vec{S}_\perp) \\
&+ \frac{1}{M}\delta q_L(x, k_\perp)\lambda i\sigma_{\mu\nu}\gamma_5 p^\mu k_\perp^\nu + \delta q_T(x, k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu S_\perp^\nu \\
&+ \frac{1}{M^2}\delta q_{T'}(x, k_\perp)i\sigma_{\mu\nu}\gamma_5 p^\mu\left(\vec{k}_\perp \cdot \vec{S}_\perp k_\perp^\nu - \frac{1}{2}\vec{k}_\perp^2 S_\perp^\nu\right) + \frac{1}{M}q_T(x, k_\perp)\epsilon^{\mu\nu\alpha\beta}\gamma_\mu p_\nu k_\alpha S_\beta\Bigg],
\end{aligned} \tag{4}$$

where M is the nucleon mass. We have omitted $\pm$ labels and the arguments ζ and μ for the distributions at the right side of the equation. The convention for γ_5 and ϵ-tensor follows that of [19]. In principle, there are Lorentz structures depending on v^μ and $\tilde{v}^\mu$; they can taken to be n^μ and p^μ, respectively, when no light-cone singularity is present. The polarization vector S^μ has been decomposed into a longitudinal component S_L^μ and a transverse one $S_\perp^\mu$

$$S^\mu = S_L^\mu + S_T^\mu = \lambda\left(\frac{p^\mu}{M} - \frac{M}{2}n^\mu\right) + S_T^\mu, \tag{5}$$

where λ is the helicity. The notations for the distributions follow Ref. [20], which are different than those in [17,18].

Out of the eight TMD distributions, three of them are associated with the $k_\perp$-even structure under the exchange $k_\perp \to -k_\perp$: $q(x, k_\perp)$, $\Delta q_L(x, k_\perp)$, and $\delta q_T(x, k_\perp)$, which correspond to the unpolarized, longitudinal polarized, and transversity distributions, respectively [21]. These distributions will survive after integrating over transverse momentum. The other five distributions are associated with the $k_\perp$-odd structures, and hence vanish when $k_\perp$ are integrated. Two of them, $q_T(x, k_\perp)$ and $\delta q(x, k_\perp)$, are odd under naive time-reversal transformation [22] and require hadronic final-state interactions to be non-zero [23].

If one is interested in production of hadrons, TMD fragmentation functions must be introduced. Following the above, we define the following density matrix for a quark fragmentation into a (pseudo)scalar hadron

$$\begin{aligned}
\mathcal{M}_h(z, P_{h\perp}, \mu, \hat{\zeta}/z, \rho) = \frac{n^-}{z}\int \frac{d\xi^-}{2\pi}\frac{d^2\vec{b}}{(2\pi)^2}e^{-i(k^+\xi^- - \vec{k}_\perp\cdot\vec{b}_\perp)}\sum_X\frac{1}{3}\sum_a\langle 0|\mathcal{L}_{\tilde{v}}(-\infty, 0)\psi_{\beta a}(0)|P_hX\rangle\gamma_{\alpha\beta}^+ \\
\times \langle P_hX|\bar{\psi}_{\alpha a}(\xi^-, \vec{b})\mathcal{L}_{\tilde{v}}^\dagger(\xi^-, \vec{b}, -\infty)|0\rangle/S(b_\perp, \mu, \rho),
\end{aligned} \tag{6}$$

where $\tilde{v}$ is mainly along the light-cone direction conjugating to P_h, $k^+ = P_h^+/z$ and $\vec{k}_\perp = -\vec{P}_{h\perp}/z$, and $a = 1, 2, 3$ is a color index. The variable $\hat{\zeta}$ is defined as $\hat{\zeta}^2 = 4(P_h \cdot \tilde{v})^2/\tilde{v}^2$. At leading order, we can have two fragmentation

functions in the expansion

$$\mathcal{M}_h = \frac{1}{2}\left[\hat{q}(x, p_\perp)\slashed{n} + \frac{1}{M}\delta\hat{q}(x, p_\perp)\sigma^{\mu\nu}p_{\mu\perp}n_\nu\right]. \tag{7}$$

The second one is (naively) time-reversal odd.

The $k_\perp$-even parton distributions satisfy the same Collins–Soper evolution equation respect to ζ, e.g., [3]

$$\zeta\frac{\partial}{\partial\zeta}f(x, b, \mu, x\zeta) = \big(K(b, \mu) + G(x\zeta, \mu)\big)f(x, b, \mu, x\zeta), \tag{8}$$

where $f = q$, Δq_L, and δq_T, and the sum $K + G$ is independent of ultraviolet scale μ. This is because the ζ-dependence arises from light-cone divergences that are independent of the spin structure. We have verified this explicitly at one-loop order. For $k_\perp$-odd distributions, similar equations have not yet been studied in the literature.

3. In [4], a factorization formula has been shown for the cross section of unpolarized semi-inclusive DIS, which involves the unpolarized TMD parton distributions and fragmentation functions. Let us first extend this result to polarized semi-inclusive DIS process. Our result is valid up to power corrections in $P_\perp^2/Q^2$, and can be used to extract the new non-perturbative hadronic matrix elements from relevant experimental data.

The differential cross section for semi-inclusive DIS reads

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2\vec{P}_{h\perp}} = \frac{2\pi\alpha_{\text{em}}^2}{Q^4}y\ell_{\mu\nu}W^{\mu\nu}(P, q, P_h), \tag{9}$$

where the leptonic tensor is

$$\ell^{\mu\nu} = 2\big(\ell^\mu\ell'^\nu + \ell^\mu\ell'^\nu - g^{\mu\nu}Q^2/2 - 2i\lambda_\ell\epsilon^{\mu\nu\alpha\beta}\ell'_\alpha\ell_\beta\big), \tag{10}$$

and λ_ℓ is the helicity of the initial lepton, ℓ and ℓ' the initial and final momenta of the lepton, $q^\mu = \ell^\mu - \ell'^\mu$ the momentum of the virtual photon, P_h the momentum of the observed hadron. As usual, x_B is the Bjorken variable, y is the fraction of the lepton energy loss, and $Q^2 = -q^2$. The hadron tensor has the following expression in QCD

$$W^{\mu\nu}(P, q, P_h) = \frac{1}{4z_h}\sum_X\int\frac{d^4\xi}{(2\pi)^4}e^{iq\cdot\xi}\langle PS|J_\mu(\xi)|XP_h\rangle\langle XP_h|J_\nu(0)|PS\rangle, \tag{11}$$

where J^μ is the electromagnetic current of the quarks, X represents all other final-state hadrons other than the observed particle h. The variable z_h can be defined as $P \cdot P_h/P \cdot q$ or P_h^-/q^-.

In a coordinate system in which the virtual-photon and hadron momenta are collinear and along the z-direction, the above hadronic tensor $W^{\mu\nu}$ has the following leading-twist structures

$$\begin{aligned}
2W^{\mu\nu} = &-g_\perp^{\mu\nu}F_{UU}^{(1)} + i\lambda_\ell\epsilon_\perp^{\mu\nu}\big(\lambda F_{LL} + S_\perp^i\hat{P}_{h\perp}^i F_{LT}\big) + \big(g_\perp^{\mu\nu} - \hat{P}_{h\perp}^\mu\hat{P}_{h\perp}^\nu - \hat{P}_{h\perp}^\nu\hat{P}_{h\perp}^\mu\big)F_{UU}^{(2)} \\
&+ \lambda\hat{P}_{h\perp\alpha}\epsilon_\perp^{\alpha(\mu}\hat{P}_{h\perp}^{\nu)}F_{UL} - g_\perp^{\mu\nu}\epsilon^{\alpha\beta}S_{\perp\alpha}\hat{P}_{h\perp\beta}F_{UT}^{(1)} \\
&+ \big(S_{\perp\alpha}\epsilon_\perp^{\alpha(\mu}\hat{P}_{h\perp}^{\nu)} - g_\perp^{\mu\nu}\epsilon^{\alpha\beta}S_{\perp\alpha}\hat{P}_{h\perp\beta}\big)\big(F_{UT}^{(2)} - F_{UT}^{(3)}/2\big) + \hat{P}_{h\perp\alpha}\epsilon_\perp^{\alpha(\mu}\hat{P}_{h\perp}^{\nu)}(\hat{\vec{P}}_{h\perp}\cdot\vec{S}_\perp)F_{UT}^{(3)}, \tag{12}
\end{aligned}$$

where $g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\nu$ and $\epsilon_\perp^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu}p_\alpha n_\beta$, and $\hat{P}_{h\perp}$ is the unit vector along $\vec{P}_{h\perp}$. $F_{I_1 I_2}$ are *structure functions* depending on x_B, z_h, $P_{h\perp}$, and Q^2, where I_1 represents the polarization of the incident lepton and I_2 that of the target hadron (U: unpolarized, L: longitudinally-polarized, T: transversely-polarized). For example, F_{UU} denotes the unpolarized structure function, F_{UT} means the target is transversely polarized while the beam lepton is unpolarized.

Substituting the hadronic tensor into the differential cross section, we get

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2\vec{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\bigg[\big(1 - y + y^2/2\big)x_B\big(F_{UU}^{(1)} + \sin(\phi_h - \phi_S)|S_\perp|F_{UT}^{(1)}\big)$$

$$+ \lambda_\ell y(1 - y/2)x_B\big(\lambda F_{LL} + \cos(\phi_h - \phi_S)|S_\perp|F_{LT}\big)$$

$$+ (1 - y)x_B\bigg(-\cos(2\phi_h)F_{UU}^{(2)} + \lambda\sin(2\phi_h)F_{UL} + |\vec{S}_\perp|\sin(\phi_h + \phi_S)F_{UT}^{(2)}$$

$$+ \frac{1}{2}|\vec{S}_\perp|\sin(3\phi_h - \phi_S)F_{UT}^{(3)}\bigg)\bigg], \tag{13}$$

where $s = (P + \ell)^2$, and ϕ_S and ϕ_h are azimuthal angles for the transverse polarization vector of the initial hadron and the transverse momentum of the final state hadron, respectively. By studying the angular dependence of the cross section, one can isolate the different structure functions.

Following [4], we can factorize the structure functions into TMD parton distributions and fragmentation functions, and soft and hard parts. For example, the double polarized structure function F_{LL} has the following factorization form

$$F_{LL}\big(x_B, z_h, Q^2, P_{h\perp}\big)$$

$$= \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, d^2\vec{\ell}_\perp\, \Delta q_L\big(x_B, k_\perp, \mu^2, x_B\zeta, \rho\big)\hat{q}\big(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\big)S^+\big(\vec{\ell}_\perp, \mu^2, \rho\big)$$

$$\times H_{LL}\big(Q^2/\mu^2, \rho\big)\delta^2(z\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}), \tag{14}$$

where we have chosen a coordinate system in which $\zeta^2 = (Q^2/x_B^2)\rho$ and $\hat{\zeta}^2 = (Q^2 z_h^2)\rho$. Similarly, we can have for the other structure functions

$$F_{UU}^{(2)} = \int \frac{2\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp} - \vec{k}_\perp \cdot \vec{p}_\perp}{MM_h}\delta q(x_B, k_\perp)\delta\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{UU}^{(2)}(Q^2),$$

$$F_{UL} = \int \frac{2\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp} - \vec{k}_\perp \cdot \vec{p}_\perp}{MM_h}\delta q_L(x_B, k_\perp)\delta\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{UL}(Q^2),$$

$$F_{LT} = \int \frac{\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M}\Delta q_T(x_B, k_\perp)\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{LT}(Q^2),$$

$$F_{UT}^{(1)} = \int \frac{\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M}q_T(x_B, k_\perp)\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{UT}^{(1)}(Q^2),$$

$$F_{UT}^{(2)} = \int \frac{\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M}\delta q_T(x_B, k_\perp)\delta\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{UT}^{(2)}(Q^2),$$

$$F_{UT}^{(3)} = \int \frac{4(\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp})^2\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp} - |\vec{k}_\perp|^2\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp} - 2\vec{k}_\perp \cdot \vec{p}_\perp\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M^2 M_h}$$

$$\times \delta q_T'(x_B, k_\perp)\delta\hat{q}(z_h, p_\perp)S^+(\vec{\ell}_\perp)H_{UT}^{(3)}(Q^2), \tag{15}$$

where the simple integral symbol represents a complicated integral used above, i.e., $\int = \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, d^2\vec{\ell}_\perp \times \delta^2(z\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp})$. And for simplicity, we also omit the explicit sum over all flavors weighted with their charge square, and the explicit μ, ρ and ζ dependence for the parton distributions and fragmentation functions, as well as the soft and hard factors.

From the above factorization formulas in the transverse momentum space, we can also obtain the factorization form in the impact parameter space. For example,

$$\tilde{F}_{UT}^{(1)}(b) = \frac{-1}{M z_h} \partial_b^i \big[\big(\partial_b^i q_T(x_B, z_h b) \big) \hat{q}(z_h, b) S^+(b) H_{UT}^{(1)}(Q^2) \big], \tag{16}$$

where $\tilde{F}_{UT}^{(1)}(b) = \int d^2 P_{h\perp} e^{i \vec{P}_{h\perp} \cdot \vec{b}} |\vec{P}_{h\perp}| F_{UT}^{(1)}(P_{h\perp})$, and other quantities depending on $\vec{b}$ are obtained from the Fourier transformation of the corresponding momentum-dependent ones. The convolution in transverse momentum space becomes a simple product in the impact parameter space, while the $\vec{k}_\perp$ moment integral generates a derivative on $\vec{b}$.

An explicit one-loop calculation for F_{UU} and F_{LL} can be done. The hard parts at one-loop order have the following form

$$H_{UU}^{(1)} = H_{LL} = \frac{\alpha_s}{2\pi} C_F \left[\big(1 + \ln \rho^2 \big) \ln \frac{Q^2}{\mu^2} - \ln \rho^2 + \frac{1}{4} \ln^2 \rho^2 + \pi^2 - 4 \right]. \tag{17}$$

To get the next-to-leading order corrections for the hard parts related to $k_\perp$-odd distributions, one has to consider two-loop calculations for scattering off an elementary quark target.

In the above discussions, we consider the factorization of the leading contributions in the expansion of $P_{h\perp}^2 / Q^2$. However, the power corrections in the semi-inclusive DIS [17] might have sizable effects at moderate Q^2 range. They are not included in our factorization formalism. On the other hand, it will be interested to extend our factorization formalism to include the subleading power contributions in SIDIS, just like what have been done for the subleading power correction to the inclusive DIS and DY cross sections [24]. This however is beyond the scope of the present Letter.

One can apply the above formalism to study the polarized cross sections and asymmetries in the SIDIS, and compare theoretical predictions with experimental measurements if the non-perturbative parton functions are known from solving non-perturbative QCD. In practice, we can treat them as unknown inputs, and fit to the experimental data. The parton functions determined phenomenologically can be used to make predictions for similar processes. Note that in the TMD quark distributions and fragmentation functions there is a double logarithmic dependence on the hadron energy (in terms of $\ln^2 b^2 \zeta^2$) which is controlled by the Collins–Soper evolution equations as shown in Eq. (8). To get the reliable prediction for the cross sections for the SIDIS, one has to solve these evolution equations to resum the double logarithms. As we showed in the above the Collins–Soper evolutions for the $k_\perp$-even TMD quark distributions are the same. It will be interested to study the Collins–Soper evolution for all the leading-twist quark distributions including the $k_\perp$-odd ones. We leave this study in a separate publication.

Let us finally remark on the two special cases related to the above results. The first case concerns cross sections integrated over the transverse momentum of the hadrons [17]. Since the above factorization formulas are valid only when $P_{h\perp} \ll Q$, one cannot use them to integrate out all $P_{h\perp}$. To do that, one must have factorization formulas valid at all $P_{h\perp}$, including all power corrections. Alternatively, one can prove the factorization theorems in the $P_{h\perp}$-integrated form, which can be done, but is beyond the scope of this Letter.

The second case concerns the region of transverse momentum $\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$. In this region, the above factorization formula is still valid. However, because $P_{h\perp}$ is now hard, all structure functions can be further factorized in terms of ordinary twist-two and twist-three parton distributions. Using Collins and Soper equations, one can sum over large logarithms of type $\alpha_s \ln^2 Q^2 / P_{h\perp}^2$. This case, has been studied for example in [11,12]. Some of the relevant leading-order perturbative QCD calculations have been done in [25].

4.   In the Drell–Yan processes, we have two hadrons in the initial states: A and B. We consider the hadron A moving along z-direction while B along $(-z)$-direction. For the TMD parton distributions in hadron B, we introduce another non-light-like vector $\tilde{v} = (\tilde{v}^-, \tilde{v}^+, \vec{0})$ $(\tilde{v}^- \ll \tilde{v}^+)$ and the variable $\bar{\zeta}^2 = (2 P_B \cdot \tilde{v})^2 / \tilde{v}^2$, just like for the fragmentation functions in DIS. The TMD quark and anti-quark distributions for hadron B are similar to those of hadron A.

We study the production of a lepton pair with low transverse momentum in Drell–Yan process

$$A(P_1, S_1) + B(P_2, S_2) \rightarrow \gamma^*(q) + X \rightarrow \ell^+ + \ell^- + X, \tag{18}$$

where P_1 and P_2, S_1 and S_2 are momenta and polarization vectors of A and B, respectively. The differential cross sections for $\ell^+ \ell^-$ production reads

$$\frac{d\sigma}{d^4Q\, d\Omega} = \frac{\alpha_{\text{em}}^2}{2s\, Q^4} L_{\mu\nu} W^{\mu\nu}, \tag{19}$$

where $L_{\mu\nu}$ is the leptonic tensor. For the Drell–Yan production, we have $L^{\mu\nu} = 4(\ell_1^\mu \ell_2^\nu + \ell_1^\mu \ell_2^\nu - g^{\mu\nu} Q^2/2)$, where ℓ_i is the lepton's momentum. Ω is the solid angle of the lepton in the virtual photon rest frame. Q^μ is the momentum of the lepton pair; s is the total center-of-mass energy square. The hadronic tensor is defined as

$$W_{\mu\nu}(x_1, x_2, Q_\perp, S_1, S_2) = \int \frac{d^4\xi}{(2\pi)^4} e^{-iq\cdot\xi} \langle P_1 S_1 P_2 S_2 | J_\mu(\xi) J_\nu(0) | P_1 S_1 P_2 S_2 \rangle, \tag{20}$$

where J_μ is the electromagnetic current. Disregarding the contributions from $k_\perp$-odd parton distributions, the leading hadronic tensor can be decomposed into three terms

$$2W^{\mu\nu} = -g_\perp^{\mu\nu}(W_0 - \lambda_1\lambda_2 W_{LL}) - g_{TT}^{\mu\nu} W_{TT}, \tag{21}$$

where $g_{TT}^{\mu\nu} = \vec{S}_{1\perp} \cdot \vec{S}_{1\perp} g_\perp^{\mu\nu} + S_1^\mu S_2^\nu + S_1^\nu S_2^\mu$. The tensor structure W_0 denotes the unpolarized tensor, W_{LL} the double longitudinal polarized tensor, and W_{TT} the double transversely polarized tensor. We can choose a frame that the momenta P_1^μ, P_2^μ and Q^μ can be decomposed into the following forms

$$P_1^\mu = p^\mu + \frac{M^2}{2} n^\mu, \qquad P_2^\mu = \frac{s}{2} n^\mu + \frac{M^2}{s} p^\mu, \qquad Q^\mu = x_1 p^\mu + x_2 \frac{s}{2} n^\mu + Q_\perp^\mu. \tag{22}$$

The polarization vectors of hadrons can be decomposed as

$$S_1^\mu = \lambda_1 \left(\frac{p^\mu}{M} - \frac{M}{2} n^\mu \right) + S_{1\perp}^\mu, \qquad S_2^\mu = \lambda_2 \left(\frac{s}{2M} n^\mu - \frac{M}{s} p^\mu \right) + S_{2\perp}^\mu, \tag{23}$$

where λ_1 and λ_2 are the helicities of hadron A and B respectively, and $S_{i\perp}$ the relevant transverse polarization vectors.

Using the hadronic tensor, the differential cross section can be expressed as

$$\frac{d\sigma}{d^4Q\, d\Omega} = \frac{\alpha^2}{2s\, Q^2} \Big\{ (1 + \cos^2\theta) W_0(x_1, x_2, Q^2, Q_\perp) - (1 + \cos^2\theta) \lambda_1 \lambda_2 W_{LL}(x_1, x_2, Q^2, Q_\perp)$$
$$+ \sin^2\theta \cos(\phi_1 + \phi_2) |\vec{S}_{1\perp}| |\vec{S}_{2\perp}| W_{TT}(x_1, x_2, Q^2, Q_\perp) \Big\}, \tag{24}$$

where θ is the polar angle of the lepton momentum, and ϕ_1 and ϕ_2 are azimuthal angles of the nucleon transverse spin relative to the lepton plane. W_{LL} contributes to double longitudinal spin asymmetry and W_{TT} to double transverse spin asymmetries.

Following Ref. [4], we have the following factorization formulas for the above hadronic tensors

$$W_0(x_1, x_2, Q^2, Q_\perp)$$
$$= \sum_{q=u,d,s,\dots} \frac{e_q^2}{3} \int d^2\vec{k}_{1\perp}\, d^2\vec{k}_{2\perp}\, d^2\vec{\ell}_\perp\, q(x_1, k_{1\perp}, \mu^2, x_1\zeta, \rho) \bar{q}(x_2, k_{2\perp}, \mu^2, x_2\bar\zeta, \rho) S^-(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H_0(Q^2, \mu^2, \rho) \delta^2(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\ell}_\perp - \vec{Q}_\perp), \tag{25}$$

$$W_{LL}(x_1, x_2, Q^2, Q_\perp)$$

$$= \sum_{q=u,d,s,...} \frac{e_q^2}{3} \int d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{\ell}_\perp \, \Delta q_L(x_1, k_{1\perp}, \mu^2, x_1\zeta, \rho) \Delta\bar{q}_L(x_2, k_{2\perp}, \mu^2, x_2\bar{\zeta}, \rho) S^-(\vec{\ell}_\perp, \mu^2, \rho)$$

$$\times H_{LL}(Q^2, \mu^2, \rho)\delta^2(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\ell}_\perp - \vec{Q}_\perp), \tag{26}$$

$$W_{TT}(x_1, x_2, Q^2, Q_\perp)$$

$$= \sum_{q=u,d,s,...} \frac{e_q^2}{3} \int d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{\ell}_\perp \, \delta q_T(x_1, k_{1\perp}, \mu^2, x_1\zeta, \rho) \delta\bar{q}_T(x_2, k_{2\perp}, \mu^2, x_2\bar{\zeta}, \rho) S^-(\vec{\ell}_\perp, \mu^2, \rho)$$

$$\times H_{TT}(Q^2, \mu^2, \rho)\delta^2(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\ell}_\perp - \vec{Q}_\perp), \tag{27}$$

where we have chosen a special coordinate system: $\zeta^2 = (Q^2/x_1^2)\rho$ and $\bar{\zeta}^2 = (Q^2/x_2^2)\rho$ and $\rho = \sqrt{v^-\tilde{v}^+/v^+\tilde{v}^-}$. The above results are accurate at leading powers in $(Q_\perp^2/Q^2)$ for soft $Q_\perp \sim \Lambda_{\text{QCD}}$. Introducing the impact-parameter space representation, the above factorization formulas become simple products, e.g., for the unpolarized tensor

$$W_0(x_1, x_2, b, Q^2) = \sum_{q=u,d,s,...} \frac{e_q^2}{3} q(x_1, b, \mu^2, x_1\zeta, \rho)\bar{q}(x_2, b, \mu^2, x_2\bar{\zeta}, \rho)S^-(b, \mu^2, \rho)H_0(Q^2, \mu^2, \rho). \tag{28}$$

The other two hadronic tensors can be obtained similarly.

In the following, we will show that the above factorization formulas are valid at one-loop order, and the relevant hard parts emerge from the calculation. We first consider the soft factor in the factorization formulas. Since it has no spin dependence, it contributes equally to all three structure functions. From its definition Eq. (3), the one-loop contribution Drell–Yan soft factor is

$$S^-(\vec{\ell}_\perp, \mu^2, \rho) = \frac{\alpha_s C_F}{2\pi^2}\left[\ln\frac{4(v\cdot\tilde{v})^2}{v^2\tilde{v}^2} - 2\right]\left[\frac{1}{\ell_\perp^2 + \lambda^2} - \pi\delta^2(\ell_\perp)\ln\frac{\mu^2}{\lambda^2}\right]. \tag{29}$$

At this order, it is the same as that for the DIS process [4].

One-loop Feynman diagrams for the Drell–Yan process initiated from a quark and anti-quark pair are shown in Fig. 2, which contain both virtual and real corrections. The virtual corrections are the same for all three structure functions

$$W_{0,LL,TT} = \frac{1}{3}\delta(x_1 - 1)\delta(x_2 - 1)\delta^2(\vec{Q}_\perp)\left(1 + 2(Z_F - 1) + 2(\hat{Z}_V' - 1)\right), \tag{30}$$
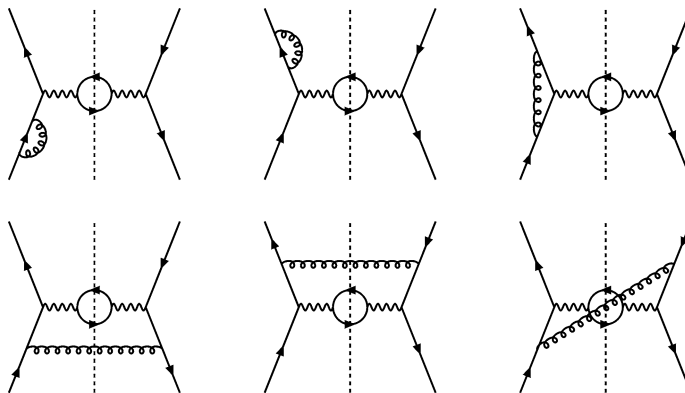

Fig. 2. One-loop corrections to DY processes.

where Z_F and Z_V come from self-energy and vertex corrections, respectively. Z_F is the same as that in DIS [4]. The vertex correction is different

$$\hat{Z}'_V = 1 - \frac{\alpha_s}{4\pi} C_F \left(\ln \frac{Q^2}{\mu^2} + \ln^2 \frac{Q^2}{m^2} + 2 \ln \frac{m^2}{\lambda^2} \ln \frac{Q^2}{m^2} - 4 \ln \frac{Q^2}{m^2} - \frac{4\pi^2}{3} \right), \tag{31}$$

which can be obtained through analytical continuation from space-like region ($q^2 < 0$) to time-like region ($q^2 > 0$). From the above, the one-loop TMD parton distributions, and the soft factor in Eq. (29), the one-loop contribution to the hard parts can be extracted

$$H_{0,LL,TT}\left(Q^2, \mu^2, \rho\right) = \frac{\alpha_s}{2\pi} C_F \left[\left(1 + \ln \rho^2\right) \ln \frac{Q^2}{\mu^2} - \ln \rho^2 + \frac{1}{4} \ln^2 \rho^2 + 2\pi^2 - 4 \right], \tag{32}$$

where we have chosen a coordinate system in which $x_1 \zeta = x_2 \tilde{\zeta}$ and therefore the dependence on the quasi-light-like vectors v and $\tilde{v}$ is simply through a combination $\rho = \sqrt{v^- \tilde{v}^+ / v^+ \tilde{v}^-}$. Note that the hard parts at this order are the same for all three structure functions and hence are spin-independent.

The real contributions are different for the three hadronic tensors

$$W_0 = \frac{\alpha_s C_F}{6\pi^2} \delta(1 - x_2) \left\{ \frac{1 - x_1}{q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2} - \frac{2x_1(1 - x_1)m^2}{(q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2)^2} \right.$$
$$+ \frac{2x_1}{(1 - x_1)_+} \frac{1}{q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2}$$
$$\left. + \delta(1 - x_1) \frac{1}{q_\perp^2 + \lambda^2} \ln \frac{Q^2}{q_\perp^2 + \lambda^2} \right\} + (x_1 \leftrightarrow x_2), \tag{33}$$

$$W_{LL} = \frac{\alpha_s C_F}{6\pi^2} \delta(1 - x_2) \left\{ \frac{1 - x_1}{q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2} - \frac{2m^2(1 - x_1)(1 - x_1 + x_1^2)}{(q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2)^2} \right.$$
$$+ \frac{2x_1}{(1 - x_1)_+} \frac{1}{q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2}$$
$$\left. + \delta(1 - x_1) \frac{1}{q_\perp^2 + \lambda^2} \ln \frac{Q^2}{q_\perp^2 + \lambda^2} \right\} + (x_1 \leftrightarrow x_2), \tag{34}$$

$$W_{TT} = \frac{\alpha_s C_F}{6\pi^2} \delta(1 - x_2) \left\{ \frac{2x_1}{(1 - x_1)_+} \frac{1}{q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2} - \frac{2x_1(1 - x_1)m^2}{(q_\perp^2 + x_1 \lambda^2 + m^2(1 - x_1)^2)^2} \right.$$
$$\left. + \delta(1 - x_1) \frac{1}{q_\perp^2 + \lambda^2} \ln \frac{Q^2}{q_\perp^2 + \lambda^2} \right\} + (x_1 \leftrightarrow x_2). \tag{35}$$

All these results can be reproduced by one-loop parton distributions for the quark and anti-quark and soft factors discussed before. Therefore, the factorization formulas for the Drell–Yan processes at low transverse momentum are correct at one-loop order. For an all-order proof, one can follow the same procedure outlined in [4].

Finally, we comment that when the transverse-momentum of the lepton pair is large compared to Λ_{QCD}, but much smaller than Q, the above formalism leads to a resummation of double logarithms as in the original paper by Collins, Soper, and Sterman [7]. For spin-dependent part, some studies along this direction can be found in Refs. [26,27].

Acknowledgements

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