Three-dimensional critical slip surface locating and slope stability assessment for lava lobe of Unzen volcano

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Abstract: Even Unzen volcano has been declared to be in a state of relative dormancy, the latest formed lava lobe No.11 now represents a potential slope failure mass based on the latest research. This paper concentrates on the stability of the lava lobe No.11 and its possible critical sliding mass. It proposes geographic information systems (GIS) based three-dimensional (3D) slope stability analysis models. It uses a 3D locating approach to identify the 3D critical slip surface and to analyze the 3D stability of the lava lobe No.11. At the same time, the new 3D approach shows the effectiveness in selecting the range of the Monte Carlo random variables and locating the critical slip surface in different parts of the lava lobe No.11. The results are very valuable for judging the stability of the lava lobe and assigning the monitoring equipments.

Key words: three-dimensional (3D) slope stability; limit equilibrium equation; Unzen volcano; lava lobe; geographic information systems (GIS)

1 Introduction

Unzen volcano, located in Nagasaki prefecture of Japan, abruptly started to erupt after 198 years of dormancy in November 1990 [1]. Following around 8 major incidents up to 1995, it displayed varying aspects of volcanic activities, ranging from early phreatic eruption to the successive extrusion and growth of lava lobe, and the formation of pyroclastic flowed. However, it has been now, once more, declared to be in a state of relative dormancy [1]. In the past 20 years, the slope failures of lava lobes occurred and killed 44 persons. Based on the latest research, the latest formed lava lobe No.11 now represents a potential slope failure mass. This paper concentrates on the stability of the lava lobe No.11 and its possible critical sliding surface.

Presently, the majority of slope stability analyses are performed using a two-dimensional (2D) limit equilibrium method within the domain of geotechnical engineering, while the factor of safety is commonly assessed using a 2D representation of the slope, for example, an “equivalent” plane-strain problem is postulated and analyzed. The results of the 2D analysis are usually conservative. Although it is more “expensive”, the three-dimensional (3D) analysis tends to give higher values of the factor of safety. The failure surface is presumed to be infinitely wide in the 2D model, negating the 3D effects caused by the infinite width of the sliding mass. A summary of the studies concerning 3D slope stability leads to a conclusion that the 3D factor of safety exceeds the 2D equivalent one, provided that the 2D factor of safety is calculated for the most critical 2D section [2]. It will be shown herein that the percentage difference between the 2D and 3D analyses may be as large as 30% (the differences are 3%–30% and have an average of 13.9% [3]). Thus a 3D analysis is the preferred means of conducting slope stability analyses.

Since the mid-1970s, the development and the application of 3D stability models [2] have attracted a growing interest. However, although several 3D methods of analyses have been proposed in geomechanical literatures [4–12], a practical 3D slope stability analysis method and related computer...
programs are still urgently required.

In this study, as a new contribution and a following up to former researches [4, 5, 13], the geographic information systems (GIS) grid-based data are analyzed with four proposed column-based 3D slope stability analysis models [5–8]. The correspondingly new GIS grid-based 3D deterministic models are assessed in order to calculate the factor of safety. At the same time, a new developed GIS-based program, SlopeGIS3D, will be used to evaluate the 3D stability of the lava lobe No.11 of Unzen volcano [1]. This practical application of the 3D slope stability assessment will illustrate the effectiveness of SlopeGIS3D in selecting the range of the Monte Carlo random variables and locating the critical slip surface of the lava lobe.

2 GIS grid-based 3D models and critical slip surface locating

Using the functions of the GIS spatial analysis, all input data (such as elevation, inclination, slope, groundwater, strata, slip surface, and mechanical parameters) for calculating the factor of safety are available with respect to each grid pixel, while all slope-related data are grid-based. Figure 1 shows a real slope mass and its abstracted GIS layers. In the GIS, the reality of a landslide is abstracted to the GIS layers for each topographic and geological theme, and each layer represents each theme: ground surface, strata, weak discontinuities, groundwater, and slip surface, respectively. By inputting these data into a deterministic model of slope stability, the value of factor of safety can be calculated.

\[
SF_{3D} = \frac{\sum_{j} \sum_{i} \left[ c' A + [(W + P) \cos \theta - U] \tan \varphi' \right]}{\sum_{j} \sum_{i} \left[ (W + P) \sin \theta_{\text{dir}} + kW \right] - E}
\]

where \( SF_{3D} \) is the 3D factor of safety of slope; \( W \) is the weight of one column; \( A \) is the area of the slip surface; \( c' \) is the effective cohesion; \( \varphi' \) is the effective friction angle; \( \theta \) is the dip angle (the normal angle of slip surface); \( J \) and \( I \) are the numbers of row and column of the grid in the range of slope failure (in this study, a polygon feature will be used to confine the boundary of the sliding mass),

In this context, combining the GIS grid-based data with four proposed column-based 3D slope stability analysis models, their GIS grid-based 3D deterministic models are deduced to calculate the factor of safety:

1. The first one is based on Hovland’s model [6].
2. The second one is based on the algorithm of the 3D stability analysis method proposed by Hungr [7]. It is a 3D extending of 2D Bishop model [14], with its flexibility regarding to the type of slip surface that can be considered and has been widely addressed in geotechnical literatures.
3. The third one is based on the work of Hungr et al. [8]. This model is an extending of 2D Janbu’s simplified method without a correction factor that can be deduced from the horizontal force equilibrium equation along the slip direction.
4. The fourth model is based on the assumption that is same as that of Hovland’s model [6]. The basic algorithm is based on the former researches [4, 5], in which the external load and the seismic load are both considered.

Using the pixels in the range of sliding mass, the 3D factor of safety is deduced by force and/or moment equilibrium of each pixel-column (Fig.2). The equation of Hovland’s model [6] is deduced to a GIS grid form [4] as follows:

Fig.1 A slope failure mass and its abstracted GIS layers.

Fig.2 A slope failure mass and forces acting on a single grid-column.
respectively; \( U \) is the pore water pressure acting on the slip surface of each column; \( P \) is the vertical force acting on each column (the distributed force of upper load); \( k \) is the horizontal earthquake acceleration factor; and \( E \) is the result of all horizontal components of applied point loads (the reinforcement force is considered in this force).

In Fig.2, when considering the equilibrium equation of the vertical forces on one grid-column, the following equation can be obtained:

\[
N = \frac{P + W + QU - Re'}{\cos \theta + Q} \tag{2}
\]

where

\[
Q = (SF_{3D})^{-1} \tan \phi' \sin \theta_{avr} \tag{3}
\]

\[
R = (SF_{3D})^{-1} A \sin \theta_{avr}
\]

where \( \theta_{avr} \) is the apparent dip angle of main inclination direction of landslide.

Then, the equation for calculating the 3D factor of safety is deduced as the 3D Bishop extending model [7]:

\[
SF_{3D} = G \sum_{j} \sum_{i} \frac{(W + P - U \cos \theta) \tan \phi' + c' A \cos \theta}{\cos \theta + Q}
G = \left[ \sum_{j} \sum_{i} (W + P) \sin \theta_{avr} \right]^{-1} \tag{4}
\]

For the \( SF_{3D} \) is implicit in Eq.(4), it can be calculated using Eqs.(2) and (4) by an iterative procedure. Continuing the expression in 3D Bishop extending model, a 3D equivalent of the Janbu simplified method without a correction factor can be deduced from the horizontal force equilibrium equation along the slip direction [8]:

\[
SF_{3D} = \frac{\sum_{j} \sum_{i} (c' A + (N - U) \tan \phi') \cos \theta_{avr}}{\sum_{j} \sum_{i} [N \sin \theta \cos (Asp - AvrAsp) + kW] - E} \tag{5}
\]

where \( Asp \) is the dip direction, and \( AvrAsp \) is the main inclination direction of landslide.

Using the grid database of surface [4, 5], strata, groundwater, fault, slip surface, and a GIS grid-based equation, all the resistant and sliding forces are referred to the possible sliding direction, but not necessary to the \( Y \)-direction used in Hovland’s model [6]:

\[
SF_{3D} = \frac{\sum_{j} \sum_{i} (c' A + (W + P) \cos \theta - U) \tan \phi' \cos \theta_{avr}}{\sum_{j} \sum_{i} [(W + P) \sin \theta_{avr} \cos \theta_{avr} + kW] - E} \tag{6}
\]

The search is performed by means of minimizing the 3D factor of safety using the Monte Carlo random simulation method for detecting the 3D critical slip surface. The initial slip surface is assumed as the lower part of an ellipsoid, and then each randomly produced slip surface is changed according to different stratum strengths and conditions of weak discontinuities. Finally, the critical slip surface is obtained and consequently a relative minimization of the 3D factor of safety is achieved [13].

The object of the critical slip surface is fulfilled by trial searching and 3D factor of safety calculation, in which five parameters of size and posture of an ellipsoid are selected as random variables for Monte Carlo simulation: three axial parameters, \( a, b, c \); the central point \( C \) and the inclination angle, \( \theta \), of the ellipsoid (Fig.3). If randomly produced slip surface based on the lower part of an ellipsoid is lower than a weak discontinuity or the confinement of the hard stratum, priority will be given to the weak discontinuity of the hard stratum as one part of the assumed slip surface. Figure 3 shows an assumed slip surface composed of one part of the ellipsoid and one part of the weak discontinuity.

![Fig.3 An ellipsoid for slip surface.](image-url)
The central point $C$ of the ellipsoid is first set to be the centroid of the search limit or a researcher-selected point, and then in each trial searching, the random walking will change the central point.

The inclination direction of the ellipsoid is set to be the same as the direction of the slope, and the inclination angle $\theta$ of the ellipsoid is basically set according to the slope angle. If a slope has the complicated topographic characteristics, the inclination parameter of an ellipsoid is set to be the main inclination of the slope, as shown in Fig.4.

Five randomly proposed parameters are assumed to be in a uniform distribution. The random variables with a uniform distribution are calculated using the random variables in the range of $[0, 1]$, which can be obtained by the method of multiplicative congruity:

$$
y_i = ay_i \text{mod}(m)
$$

$$
r_i = y_i / m
$$

(8)

where $m$ is the module, and $r_i$ is the random variable of the uniform distribution within the range of $[0, 1]$. By setting an initial value of $y_0$, each random variable $r_i$ can be obtained. The random variable is then calculated by

$$
x_i = r_i(b - a) + a
$$

(9)

where $x_i$ is the random variable within the range of $[a, b]$.

3 3D slope stability assessment of lava lobe No.11 of Unzen volcano

3.1 Basic information and GIS data processing

Unzen volcano is located in Nagasaki prefecture of Japan (Fig.5(a)). In this study, we select the latest formed lava lobe No.11 as the object (Fig.5(b)) to evaluate its stability and to locate the critical slip surface. Figure 6 shows a photo of the lava lobe No.11 of Unzen volcano.

The topographic data are calculated using aerial photography, and the difference in such photos before and after each eruption is compared to allow each lava lobe to be detected, and at the same time, the topographic data for each occasion can be determined [14]. The lava lobe No.11 was formed between April 1993 and April 1994, and its shape can be revealed from a comparison between the two aerial photos taken in February 1993 and September 1994, respectively, revealing the 3D shape of the lava lobe No.11. The digital elevation model (DEM) data on each occasion, meanwhile, are deduced from the aerial photo by means of the following steps:

1. Selecting data: using local triangle net and level points.
2. Triangle measurement: air triangle surveying.
3. Grid measurement: in the range of $1000 \times 1000$ m, the values of each grid in $X^*$, $Y^*$, and $Z$-directions are obtained (the grid size is 20 m).
4. Digital photos georeference: georeferencing the aerial photos and corresponding to the actual site.
5. Coordinate conversion: changing to a common coordinate system.
(6) Forming TIN and converting into grid data: converting the TIN dataset to a grid raster dataset. The ground surfaces of each eruption are then abstracted as a GIS grid dataset. The adjacent interface is considered as the possible slip surface, and then the interface of two lava lobes formed in February 1993 and September 1994, respectively, as a weak layer, is considered as a possible slip surface, and the geomechanical parameters of the lava layer and the interface are listed in Table 1.

<table>
<thead>
<tr>
<th>Medium</th>
<th>c (kPa)</th>
<th>$\phi$ (°)</th>
<th>$\gamma$ (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lava layer</td>
<td>1 500</td>
<td>30</td>
<td>22.1</td>
</tr>
<tr>
<td>Interface</td>
<td>140</td>
<td>32.9</td>
<td>—</td>
</tr>
</tbody>
</table>

3.2 Suitable random variable selection in Monte Carlo simulation

The basic Monte Carlo simulation method has been reported in a former research [13], and here the method will be applied to this practice problem in order to detect the critical slip surface and the sliding masses.

For the critical slip surface identification, a test for the suitable Monte Carlo random calculation time is performed with a trial calculation frequency of up to 1 000 times. The resultant minimum 3D factors of safety of each trial calculation are illustrated in Fig. 7. In this case, considering the time consumed and effectiveness, the minimum factor of safety can be obtained by around 300 times of trial calculation times. In the following random variables studies, the calculation times for the Monte Carlo simulation are set at 300.

Fig. 7 Resultant minimum factor of safety and random Monte Carlo calculation times.

In the previous study [13] for explaining the relationship between $a/b$ and the critical 3D factor of safety, different ratios of $a$ to $b$ are selected to locate the critical slip surface and to calculate the 3D factor of safety. When $a/b$ is smaller than 0.8, the 3D factor of safety will increase sharply corresponding to the decreasing $a/b$. Conversely, if $a/b$ exceeds 0.8, the 3D factor of safety will decrease slowly with an increase in $a/b$ until the 3D factor of safety will approach the 2D factor of safety. With the increasing $a/b$, the problem of 3D factor of safety calculation approaches the plain-strain assumption that is used to calculate the 2D factor of safety.

For application study, the relationship between $a/b$ and the minimum factor of safety has been studied under two conditions (Fig. 8). If maintaining the parameters within a certain range, with the increasing values of $a/b$, the minimum factor of safety will increase. However, remaining parameter $b$ constant, with increasing values of $a/b$, the minimum factor of safety will decrease since the randomly selected sliding mass will approach a 2D case [13]. This study reveals that the ratio of $a$ to $b$, 0.5–0.6, will be a suitable value for effectively locating the critical slip surface.

Fig. 8 Relationship between minimum factor of safety and ratio of $a$ to $b$.

When calculating the 3D factor of safety, the force and moment equations for each column are based on the value of $A_{RASP}$, the average dip direction of the slip surface, in which the average dip direction is assumed to be the sliding direction. To confirm this assumption, the different ranges of $A_{RASP}$ were studied. For example, when $A_{RASP} = 90^\circ$ and the ranges of dip directions (aspects) were considered to be $89^\circ$–$91^\circ$, $80^\circ$–$100^\circ$ and $70^\circ$–$110^\circ$, the minimum factors of safety are 1.415, 1.437 and 1.490, respectively. At the same time, three cases provided a
broadly similar critical slip surface and the same critical direction (about 90°). This comparative study can confirm the assumption that AvrAsp can be regarded as the sliding direction.

A suitable range for the dip angle (slope angle) has also been studied. If AvrSlope = 33.5° and the ranges are set to be 25°–40°, 25°–33.5° and 33.5°–40°, the resultant minimum factors of safety are 1.412, 1.455 and 1.406 respectively. At the same time, using AvrSlope with ranges of 1 ± 10%, 1 ± 20%, 1 ± 30% and 1 ± 40%, the minimum factors of safety are 1.412, 1.407, 1.409, and 1.407, respectively. This result indicates that in the range of 1 ± 20%, it cannot result in a lower factor of safety. Then, we set the range of 1 ± 20% of AvrSlope for effectively locating the critical slip surface.

In the Monte Carlo simulation, each central point of the ellipsoid was randomly selected around the start point. As shown in Fig.9, the randomly selected central points are set around a start point. At the same time, we found that different start points resulted in different minimum factors of safety, which were dependent on the geological structure and the topographic conditions. This means that, prior to the calculation, the geological engineering and topographic conditions have to be carefully studied to determine a suitable start point.

![Fig.9](image)

Fig.9 Start point and random central points in the Monte Carlo simulations.

The resultant minimum factors of safety and calculation time have also been compared using different 3D models. For a certain case study, the differences are illustrated in Table 2. It can be seen that there is no apparent difference in the calculation time because of the same management method of related data for different models, while the iterative procedures for the 3D extended Bishop and Janbu models take a little more time. On the other hand, because it neglects the interactive forces of the column, Hovland’s model results in the lowest factor of safety.

### 3.3 Overall and partial stability of lava lobe No.11 of Unzen volcano

To evaluate the 3D stability of the lava lobe No.11 comprehensively, 4 searching ranges are selected to identify the 3D critical sliding masses. In Fig.10, range A is used to assess the possible slope failure of the lava lobe No.11 as a whole. At the same time, the ranges B, C and D (Figs.12–14) are set to analyze the critical sliding masses of the front toe of the lava lobe No.11. These 4 range settings assure that different sizes of critical sliding masses can be located in the lava lobe No.11.

![Fig.10](image)

Fig.10 Searching ranges for locating critical sliding masses.

### Table 2 Calculation time and minimum factors of safety for different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Calculation time (10⁻³ s)</th>
<th>Minimum factor of safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hovland’s model</td>
<td>1 074</td>
<td>1.243</td>
</tr>
<tr>
<td>3D extended Bishop model</td>
<td>1 116</td>
<td>1.372</td>
</tr>
<tr>
<td>3D extended Janbu model</td>
<td>1 168</td>
<td>1.322</td>
</tr>
</tbody>
</table>

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Taking the lava lobe No.11 as a whole, various cases with a range of different random variables have been calculated. The section of resultant critical sliding mass is illustrated in Fig.11(a) with a 3D view shown in Fig.11(b), and the minimum factors of safety are revealed as 1.322, 1.388 and 1.346, using Hovland’s model, 3D extended Bishop model and 3D extended Janbu model, respectively. Considering the possible influence of earthquake with the horizontal coefficient of $k = 0.05$, the 3D factors of safety will be 1.162, 1.235 and 1.159, respectively.

The former study [1] of the sliding patterns concluded that the front toe of the lava lobe No.11 was relatively unstable. Therefore, in this study, the relative potential of a minor slope failure in the front toe section was studied. The resultant minimum factors of safety are 1.508, 1.531 and 1.509 in the north (range $D$), middle (range $B$) and south (range $C$) sections of the toe, respectively. Considering the possible vertical cracks cutting through the lava lobe No.11, the minimum factors of safety would be 1.012, 1.221 and 1.210, respectively. At the same time, the same seismic load with the horizontal coefficient of $k = 0.05$ will result in the 3D minimum factors of safety of 1.322, 1.351 and 1.300, respectively.

Finally, the following conclusions can be obtained from the stability analyses of the lava lobe No.11 of Unzen volcano:

1. The study shows that the lava lobe No.11 is now in a stable condition based on the proposed geomechanical parameters.
2. A trial study is necessary to set a suitable range of random variables to effectively identify the critical slip surface.
3. If the possible vertical cracks are considered, the front toe of the lava lobe No.11 will be considered in a dangerous condition.
4. Moreover, if any slope failure in the front toe takes place, the whole stability of the lava lobe No.11 will be affected too.

4 Conclusions

Combining the GIS grid-based data with four proposed column-based 3D slope stability analysis models, the slope stability of Unzen volcano lava lobe has been evaluated. The results have illustrated the convenience of the data management in effectively selecting the range of the Monte Carlo random variables, and in locating the critical slip surface. These results will be a valuable reference to taking measures against the slope failure hazard and setting monitoring equipments.

Benefiting from the convenient functions of data management and the GIS spatial analysis, the new database approach will present a new challenge for the geotechnical researchers using traditional numerical methods for 3D slope stability assessment.

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References


