Exploiting the width difference in $B_s \to \phi \gamma$

Franz Muheim\textsuperscript{a}, Yuehong Xie\textsuperscript{a}, Roman Zwicky\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a} University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
\textsuperscript{b} IPPP, Department of Physics, University of Durham, Durham DH1 3LE, United Kingdom

\textbf{A R T I C L E  I N F O}

\textbf{Article history:}
Received 8 April 2008
Accepted 8 May 2008
Available online 17 May 2008
Editor: A. Ringwald

\textbf{A B S T R A C T}

The photon polarization in $B \to V \gamma$ is a sensitive probe of right-handed currents. In the time dependent decay rate of $B_s \to \phi \gamma$ the coefficients $S$ and $H$ in front of the sin$(\Delta m t)$ and the sin$(\Delta \Gamma_s / 2t)$ terms were studied. The coefficient $S$ is sensitive to right-handed currents. As compared to the $B_d$ system there is a sizable width difference in $B_s$ mesons which leads to the additional measurable observable $H$. We show with a Monte Carlo simulation that the expected resolution on $S$ and $H$ will be about 0.15 at the LHCb experiment for $\Delta \Gamma_s / \Gamma_s = 0.15$ and a data sample of 2 fb\textsuperscript{-1}. We also show that the observable $H$ can be measured from the untagged decay rate of $B_s$ mesons which has considerable experimental advantages as no flavour tag will be required. The resolution on $H$ is inversely proportional to the $B_s$ width difference $\Delta \Gamma_s$. These experimental prospects have to be compared with the Standard Model predictions $S_{\phi \gamma} = 0 \pm 0.002$ and $H_{\phi \gamma} = 0.047 \pm 0.025 \pm 0.015$ presented in this Letter. We also give the Standard Model prediction and the experimental sensitivity for the direct CP asymmetry in $B_s \to \phi \gamma$.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

Flavour changing neutral current (FCNC) decays are forbidden at tree level in the Standard Model (SM) and are therefore a sensitive probe of new physics (NP). Furthermore, the $V-A$ structure of the weak interactions can be tested in FCNC decays of the type $b \to (d, s) \gamma$, since the emitted photon is predominantly left-handed. The crucial point is that the weak force only couples to left-handed quarks. The structure of the weak interactions can be tested in FCNC decays of the type $b \to (d, s) \gamma$, since the emitted photon is predominantly left-handed. The crucial point is that the weak force only couples to left-handed quarks. The structure of the weak interactions can be tested in FCNC decays of the type $b \to (d, s) \gamma$, since the emitted photon is predominantly left-handed. The crucial point is that the weak force only couples to left-handed quarks. The structure of the weak interactions can be tested in FCNC decays of the type $b \to (d, s) \gamma$, since the emitted photon is predominantly left-handed. The crucial point is that the weak force only couples to left-handed quarks.

At the $B$ factories the exclusive radiative decays of the $B_s$ meson were studied. The coefficient $S$ in front of the sin$(\Delta m t)$ term in the time dependent CP asymmetry has been measured in $B_d \to K^{(*)} (\pi^0 \eta \pi^0)$ at the $B$ factories BaBar $S_{K^{(*)} \gamma} = -0.08 \pm 0.31 \pm 0.05$ [3] and Belle $S_{K^{(*)} \gamma} = -0.32^{+0.36}_{-0.33} \pm 0.05$ [4]. The average is $S_{K^{(*)} \gamma} = -0.19 \pm 0.23$ [5]. Recently Belle reported a measurement of $S_{\rho \gamma} = -0.83 \pm 0.65 \pm 0.18$ [6] in $B \to \rho^{(*)} \gamma$. Comparing the experimental values with theoretical predictions [2,7,8] it is clear that larger data samples are required before conclusions can be drawn.

The large production rate of $B_s$ mesons at the LHC opens up the possibility to study the $B_s$ system with high statistical precision. In this Letter we intend to argue that the $B_s \to \phi \gamma$ decay is a particularly promising channel to test the $V-A$ structure of the SM at the LHCb experiment. This method is independent of the actual value of the $B_s$ mixing angle, since there is a measurable coefficient in front of the sin$(\Delta \Gamma_s / 2t)$ term in the time dependent decay rate, which we shall denote by the letter $H$.

At the level of the QCD calculation the decay $B_s \to \phi \gamma$ is very similar to $B_d \to K^{(*)} \gamma$. Compared to the $B_d$ meson, the new elements of the $B_s$ meson are the small mixing phase $\phi_s$ and the large width difference $\Delta \Gamma_s$ of the $B_s$ meson, which will play a central role in this Letter. The SM predictions for the mixing angles and widths are

\begin{equation}
\phi_s \simeq -2 \lambda^2 \eta \simeq -2^\circ, \quad \phi_d \simeq 2 \beta \simeq 43^\circ, \quad \Delta \Gamma_s^H = 0.107 \pm 0.065, \quad \Delta \Gamma_s^d = (40.9^{+8.8}_{-8.9}) \times 10^{-4},
\end{equation}

where the values of the widths are taken from the recent update of Ref. [9]. The Wolfenstein parameters $\lambda \simeq 0.227(1)$ and $\eta \simeq 0.34(4)$ are taken from [10]. While the width and phase of the $B_d$ meson are precisely measured and consistent with the SM within uncertainties [10], the knowledge of the $B_s$ width and the mixing phase is still poor. The DØ experiment finds $\phi_s = -0.70^{+0.47}_{-0.39}$ and $\Delta \Gamma_s = 0.13 \pm 0.09$ ps\textsuperscript{-1} [11]. Combining this result of $\Delta \Gamma_s$ with...
other measurements, the Heavy Flavour Averaging Group quotes $\Delta \Gamma_t = 0.071^{+0.053}_{-0.052}$ and $\Delta \Gamma_t = 0.104^{+0.076}_{-0.064}$ [5].

In this Letter we will show that the experimental resolution is independent of the actual value for the coefficients $S$ and $H$ of the $\sin(\Delta m t)$ and $\sin(\Delta \Gamma/2 t)$ terms. Therefore it is crucial that either $S$ or $H$ is sizable in order to detect NP from enhanced right-handed currents as opposed to NP in the mixing. In the SM the short distance contribution dominates which has a single weak phase which is exactly cancelled by the mixing phase. Since $S$ and $H$ are proportional to the sine and cosine it is more likely that NP will be sizable in $S$ rather than in $H$. In the $B_{s}$ system only $S$ is measurable, since the width is too small, but fortunately $S$ is sizable because the phases from the mixing and the short distance process do not cancel. We refer the reader to Appendix A for formulae on $S$ and $H$ in terms of two weak amplitudes which go beyond the simplified discussion in this introduction.

The Letter is organised as follows. Definitions of the observables and theory predictions including the non-local charm loop contribution [7] are presented in Section 2. Further useful formulae are compiled in Appendix A. The extraction of the observables from the time dependent decay rates is discussed in Section 3 and a Monte Carlo simulation for the experimental accuracy is presented in Section 4. The Letter ends with conclusions in Section 5.

2. Time dependent CP-violation in $B_s \rightarrow \phi \gamma$

The normalised CP asymmetry, for $B_s \rightarrow \phi \gamma$ is defined as follows

$$A_{CP}(B_s \rightarrow \phi \gamma) = \frac{\Gamma[B_s \rightarrow \phi \gamma] - \Gamma[B_s \rightarrow \phi \gamma]}{\Gamma[B_s \rightarrow \phi \gamma] + \Gamma[B_s \rightarrow \phi \gamma]}$$

(2)

where the left- and right-handed photon contribution are added incoherently $\Gamma[B_s \rightarrow \phi \gamma] = \Gamma[B_s \rightarrow \phi \gamma] + \Gamma[B_s \rightarrow \phi \gamma]$. Neutral mesons, such as the $B_s$, exhibit a time dependence in the CP asymmetry through mixing, if the particle and the antiparticle allow for a common final state. In $B_s \rightarrow \phi \gamma$ this amounts to $B_s \rightarrow \phi \gamma \leftarrow B_s$.

The general time evolution of the decay rates parameterised in terms of the amplitudes can be found in [10]. The ratio of coefficients $p$ and $q$

$$\left( \frac{q}{p} \right) = \left[ \frac{q}{p} \right]_s e^{i \phi_s},$$

(4)

relating the physical and the flavour eigenstates, characterises the mixing of the $B_s$ mesons. In the SM the $B_s$ mixing phase $\phi_s$ is small when compared to the mixing phase in $B_d$ mesons, cf. Eq. (1). Please note that we have implicitly assumed the standard convention $CP(B_s) = -|B_s|$ in the equation above. The absolute value of $\langle q/p \rangle_s$ can be determined experimentally from the semileptonic CP asymmetry. The measurement of the latter [11] indicates that the quantity is very close to unity, $1 - |q/p|_s = (0.05 \pm 0.45) \times 10^{-3}$.

With $|q/p|_s = 1$ the CP asymmetry (2) assumes the following generic time dependent form

$$A_{CP}(B_s \rightarrow \phi \gamma)(t) = \frac{S}{C} \sin(\Delta m t) - \frac{C}{S} \sin(\Delta m t) \cos(\Delta \Gamma/2 t) - H \sin(\Delta \Gamma/2 t).$$

(5)

The mass difference and the width difference are defined as $\Delta m_t = m_H - m_L > 0$, $\Delta \Gamma_t = \Gamma_L - \Gamma_H$, where the subscripts $H$ and $L$ stand for heavy and light respectively. The definition of the width difference corresponds to a positive value in the SM, i.e. $\Delta \Gamma_t^{SM} > 0$.

In terms of the left- and right-handed amplitudes,

$$A_{L(R)} = A(B_s \rightarrow \phi \gamma L(R))$$

(6)

the observables $C$, $S$ and $H$ assume the following form

$$C = \left( |A_L|^2 + |A_R|^2 \right) / \left( |A_L|^2 + |A_R|^2 \right),$$

$$S = \left| A_L \right|^2 / \left( |A_L|^2 + |A_R|^2 \right),$$

$$H = \left| A_R \right|^2 / \left( |A_L|^2 + |A_R|^2 \right).$$

(7)

The amplitudes are parameterised in terms of the CKM phases according to Eq. (A.3) in Appendix A, although with a different normalisation,

$$A_{L(R)} = \frac{G_F}{\sqrt{2}} \left[ 1 - \frac{em_b}{2\pi^2} T_2^{\phi \gamma - \phi \gamma} 0 \right] \times \left( \lambda_0 a_{L(R)}^u + \lambda_2 a_{L(R)}^u + \lambda_4 a_{L(R)}^u \right) S_{L(R)},$$

(8)

where $m_b$ is the b quark mass, $G_F$ is the Fermi constant, $\lambda_0 = V_{tb}^* V_{ub}$ are CKM factors with $U = \{u, c, t\}$ and $T_2 = 0.31$ (4) is a pfugerm form factor [12] whose value was updated in [8]. The left-right-projectors are

$$S_{L(R)} = \left( \begin{array}{c} e^{i \phi} \pi \end{array} \right) \left( \begin{array}{c} c \eta \pi \phi \gamma \end{array} \right).$$

(9)

where $\pi \gamma$ and $\eta$ are the photon and $\phi$ polarisation vectors and $p$ and $q$ are the photon and $\phi$ four-momentum, respectively. The decomposition in Eq. (8) is ambiguous since the three generation unitarity $\lambda_0 + \lambda_2 + \lambda_4 = 0$ allows us to reshuffle terms from one amplitude into the other. Often it is convenient to eliminate one amplitude by invoking the unitarity relation, e.g. formulae in Appendix A. For notational clarity we shall quote,

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left( \sum_{U=u,c} \lambda_0 \left( Q_1 U^0 + Q_2 U^0 \right) + \lambda_2 \sum_{i=3,8} Q_1 Q_i \right),$$

(10)

the total $b \rightarrow s \gamma$ effective Hamiltonian. In the SM the leading operator,

$$Q_2 = \frac{e}{8\pi^2} \left[ m_b \phi \gamma \pi \mu \nu \pi \phi \gamma \right] \left( 1 + \gamma_5 \right) b + m_b \phi \gamma \pi \mu \nu \pi \phi \gamma \left( 1 - \gamma_5 \right) b \right),$$

(11)

is due to short distance quark processes. This leads to a particular chiral pattern [2] due to the $V-A$ structure of the weak interactions. Namely, the $B_s (B_s)$ meson decays predominantly into a left- (right-)and handed photon whereas the decay of the $B_s (B_s)$ meson into the left- (right-)handed photon is suppressed by an $m_s/m_b$ chirality factor,

$$a_L^u = C_7 \left( \frac{1}{m_b/m_s} \right) + 0 \left( \frac{1}{m_b/m_s} \alpha_c \right).$$

(12)

Due to the interference of mixing and decay in $B_s \rightarrow \phi \gamma$, a single weak decay amplitude proportional to $\lambda_2$ is exactly cancelled by the mixing phase,

$$H_{Q_2} = \frac{2m_b}{m_b} \cos(\phi_s - \phi_2) = \frac{2m_b}{m_b} \cos(\phi_s - \phi_2) = 0.$$

Note that at this stage the CP asymmetry pattern is analogous to $B_s \rightarrow J/\psi \phi$ up to the chiral suppression of the interference term. The formula for $S$ was presented in the original paper [2]. Later it
was pointed out by Grinstein et al. [13] that QCD alters the V–A pattern and that the current operator $Q_L^5$,  
$$Q_L^5 = \frac{3}{2} \gamma \gamma_\mu (1 - \gamma_5) U \gamma_\mu (1 - \gamma_5) b, \quad U = [u, c],$$  
(13)  
might lead to sizable corrections in part due to its large Wilson coefficient $|C_2| \approx 3|C_7|$. The dominant contribution corresponds to the physical process of emission of a collinear gluon from the long distance charm loop into the vector meson final state. In Ref. [14] the charm loop was expanded to leading order in $1/m_c^2$, for which a large uncertainty was attributed, and the remaining matrix element was estimated with light-cone sum rules (LCSR). The contribution turned out to be relatively small, suppressed by large loop factors. In Ref. [7] the charm loop is calculated to all orders in $1/m_c^2$, for which a large uncertainty was attributed, and the remaining matrix element was estimated with light-cone sum rules (LCSR). The contribution turned out to be relatively small, suppressed by large loop factors. In Ref. [7] the charm loop is calculated to all orders in $m_c$ within the framework of the light-cone expansion. The closure to the charm threshold results in a large strong phase. The expansion in the charm mass does not reveal the phase and is not convergent when higher orders are taken into account. Nevertheless the first order and the all order result differ by less than a factor of two which is well within the uncertainty attributed in [14]. The numerical result is [7]  
$$\left( \frac{\sigma_c^{R}}{\sigma_K^{R}} \right) = C_2 \frac{Q_c}{L_{c,K}(0)} \left( \frac{L_{c,R}(0)}{L_{c,L}(0)} \right), \quad Q_c = 2 \frac{2}{3},$$  
(14)  
where $Q_c$ is the charge of the charm quark and  
$$L_{c,L}(0) = (4.8 \times 10^{-3} \pm 7\%) e^{i(255\pm15)\circ},$$  
$$L_{c,R}(0) = (1.8 \times 10^{-3} \pm 7\%) e^{i(106\pm15)\circ}.\quad (15)$$  

For the results up quark loops, due to $Q_u^5$, (13), can be found in Ref. [8]. They are generally not sizable in $b \to s$ transitions because of the CKM hierarchy $|\lambda_t| \ll |\lambda_c| \approx |\lambda_s|$. The contributions in (15) will have a minor impact on the observables $S$ and $H$ because they are almost imaginary and the left-handed one is larger than the right-handed one. It is therefore natural to ask whether these patterns will remain for contributions other than short distance, such as the emission of the gluon from the $B$ meson or hard spectator interactions beyond the leading $1/m_b$ term. In Ref. [7] the emission of a soft gluon from the $B$ meson to the charm quark loop is studied. Using an analogous notation as above it is found that,  
$$L_{c,L}^{R} = 0.03(20) \times 10^{-3}$$  
and  
$$L_{c,R}^{R} = 0.43(3) \times 10^{-3}.$$  
These contributions are real and the left-right hierarchy appears to be inverted. We will take these contributions as an estimate of the uncertainty due to non-short distance contributions.

We will now turn to the results of the parameters $S$ and $H$. We use the formulae given in Appendix A in Eq. (A.5) and obtain  
$$H = 0.047(1 \pm 17\% m_s \pm 10\% m_d \pm 14\% |\lambda_t| \pm 5\% |\lambda_c|),$$  
$$S = 0 \pm 0.002,$$  
(16)  
where we have indicated parametric relative uncertainties for the strange quark mass $m_s(2$ GeV) = 100(20) MeV, further long distance contributions mentioned above and for the collinear gluon $|L_{c,L}^{R}, \delta_{c,R}^g|$ as given in Eq. (15). The uncertainty of the latter is small because the imaginary part does not contribute to the time dependent CP asymmetry when it interferes with the dominant and real $a_1^H$ in Eq. (12). In other words the strong phase difference is nearly ninety degrees and gives a small contribution when the cosine is taken, cf. formula (A.5) in Appendix A. The leading contribution to the observable $S$ is given by $2\text{Re}[a_1^H a_1^S] |\mu_b/\lambda_s| \sin(\gamma)$, cf. using the notation in (8) in the formulae given in (A.5) in Appendix A. From this expression it is seen that $S$ is CKM and helicity suppressed resulting in a vanishingly small value. For the uncertainty we assume that the helicity suppression of charm and up contributions is not larger than the one of the leading operator $Q_7^S$ (12). The uncertainty for $S$ and $H$ caused by the form factor $T_1$ and the Wilson coefficients $C_2 = 1.03$ and $C_7 = -0.31$ are negligible due to cancellation in the ratio.

Further uncertainties are coming from weak annihilation whose size does not contribute more than 5% [8,17] and contributions from the gluon penguin operator $Q_8$, where the gluon is emitted into the long distance photon wave function, are expected to be of the same size. Hard spectator corrections to the chirality structure are of order $O(m_t/m_b)$ and, taking into account the leading contribution from Ref. [17], are about 10% if they should contribute maximally to the right-handed amplitude. Another contribution comes from the gluon emission to the spectator quark which has been calculated in the perturbative QCD approach [16] and indicates a shift of $\Delta S_{\gamma} = -0.015$ which we translate into a one sided uncertainty for $\delta H_{\gamma} = 0.015$ for $H_{\gamma}$. Adopting a conservative estimate and adding the uncertainties in (16) linearly, another 10% for the further contributions mentioned above and the one sided spectator correction we arrive at our final estimate  
$$H_{\gamma} = 0.047 \pm 0.025 \pm 0.015_{(\omega_s)},$$  
$$S_{\gamma} = 0 \pm 0.002.$$

(17)

Without the inclusion of the charm loops the results is $H = 0.041$. The result for $H$ is new whereas $S$ is almost the same as $-0.001(1)$ predicted in [8] up to the contribution of the charm loop which changes due to the large strong phase found in (15) as compared to the real values in [8].

The CP asymmetry $C$ (5) is sensitive to novel weak phases rather than to right-handed currents. It is proportional to the sine of the weak and strong phase and is given by  
$$C_{\gamma} \simeq - \frac{2\text{Im}[\alpha_{e}^* \lambda_s]}{|\lambda_t|^2} \frac{\text{Im}[\alpha_d^* a_1^H]}{C_7^2} \approx 0.005(5),$$  
(18)  
where we have used the notation given in Eq. (8) with $a_1^H$ eliminated by use of the three generation unitarity relation. Note that the right-handed amplitudes are irrelevant since their contributions are of the order $O(m_t/m_b)$. The numerically relevant imaginary parts are due to charm loop contributions from the operator $Q_5^S$ (13). More specifically there are vertex corrections, hard spectator interactions and gluon emission into the final state. The first two contributions are taken from [17] and the gluon emission is given by $L_{c,R}^{g}$ in Eq. (15) and contributes about one third to the asymmetry. The CP asymmetry is small since it is CKM and $O(\omega_s)$ suppressed. For the uncertainty in Eq. (18) is due to the one given in Eq. (15) for the emission of the gluon into the final state and an assumed a similar precision for the short distance and hard spectator contributions.

After the theoretical prediction we will now turn in the next sections to the experimental prospects for measuring the observables $S$, $H$ and $C$.

3. Extraction of observables

The observables $S$, $H$ and $C$, appearing in the time dependent CP asymmetry (5), can be extracted from the time dependent decay rates. Without considering any experimental effects, the time dependent decay rate, $B(t)$, of a $B_s$ meson, produced at $t = 0$, is given by

---

3. The same expansion and local QCD sum rules were also used in Ref. [15] in the conjunction with the total branching fraction.
\[ B(t) = B_0 e^{-\frac{\Delta \Gamma_e t}{2}} \left[ \cosh \left( \frac{\Delta \Gamma_e t}{2} \right) - H \sinh \left( \frac{\Delta \Gamma_e t}{2} \right) \right] \\
+ C \cos(\Delta m_t t) - S \sin(\Delta m_t t) \]  
(19)

and the decay rate, \( \tilde{B}(t) \), of a \( B_s \) at \( t = 0 \) is given by

\[ \tilde{B}(t) = B_0 e^{-\frac{\Delta \Gamma_e t}{2}} \left[ \cosh \left( \frac{\Delta \Gamma_e t}{2} \right) - H \sinh \left( \frac{\Delta \Gamma_e t}{2} \right) \right] \\
- C \cos(\Delta m_t t) + S \sin(\Delta m_t t) \]  
(20)

where \( B_0 \) is the total decay rate. It is the quantity \( H \Delta \Gamma_e \), which can be experimentally measured since \( H \) \( \sinh(\Delta \Gamma_e/2) \approx H \Delta \Gamma_e/2 \) for small \( \Delta \Gamma_e \). Thus, the determination of \( H \) requires that the \( B_s \) width difference \( \Delta \Gamma_e \) be measured elsewhere. This can be achieved by the LHCb experiment which, using the \( B_s \rightarrow J/\psi \phi \) data sample, will be able to reach a statistical precision of \( \pm 0.0092 \) on \( \Delta \Gamma_e/\Gamma_e \) up to a sign-ambiguity [18]. Therefore, in this study we assume that \( \Delta \Gamma_e \) is precisely known. We only need to perform the study of the time dependent decay rates at one given value of \( \Delta \Gamma_e/\Gamma_e \) as the sensitivity on \( H \) is inversely proportional to the width difference \( \Delta \Gamma_e \).

While the determination of the coefficients \( S \) and \( C \) relies on the knowledge of the initial flavour of the \( B_s \) mesons, the extraction of the observable \( H \) does not require flavour tagging. The observable \( H \) can be measured from the untagged time dependent decay rate spectrum (19), (20) which, from an experimental point of view, makes this a very promising method. In the next section we will investigate prospects to measure these observables in future experiments.

4. Experimental prospects

In the Standard Model, the CP-averaged branching ratio of \( B_s \rightarrow \phi \gamma \) is predicted to be [8]:

\[ B(B_s \rightarrow \phi \gamma) = (39.4 \pm 10.7 \pm 5.3) \times 10^{-6} \]  
(21)

The CDF Collaboration searched for this decay in \( p \bar{p} \) collisions and set an upper limit of \( B(B_s \rightarrow \phi \gamma) < 1.9 \times 10^{-3} \) [19] at the 95% confidence level. Using a data sample of 23.6 fb\(^{-1}\) recorded at the \( \Upsilon(5S) \) resonance, which corresponds to about 2.6 millions of \( B_s \) mesons, the Belle Collaboration recently reported a measurement of \( B(B_s \rightarrow \phi \gamma) = (5.7^{+1.8+1.2}_{-1.5-1.7}) \times 10^{-5} \) with a significance of 5.5\( \sigma \) [20]. LHCb is a dedicated B physics experiment at the Large Hadron Collider, and is expected to start data taking in 2008 [21].

A data sample of \( \sim 2 \) fb\(^{-1}\), which the LHCb experiment expects to accumulate in a nominal year, corresponds to about \( 7 \times 10^{10} \) produced \( B_s \) mesons whose decay products will be inside the LHCb detector acceptance. This copious production rate for \( B_s \) mesons will open a window for the search of physics beyond the SM.

LHCb has performed a detailed Monte Carlo simulation to estimate the performance of the event reconstruction for the decay \( B_s \rightarrow \phi \gamma \) [22]. In a 2 fb\(^{-1}\) data sample about 11500 signal events are expected to pass the level 0 trigger [4] and the event selection criteria with an upper limit on background over signal ratio of \( B/S < 0.55 \) at 90% confidence level. The \( B_s \) mass resolution is about 70 MeV/c\(^2\). The flavour of a \( B_s \) meson at production can be inferred from the decay products of the opposite side b hadron or from the charge of the kaon accompanying the production of the signal \( B_s \) meson. Using simulated events, this tagging procedure yields an efficiency of 60% and a wrong-tag fraction of 30% at the LHCb experiment. The proper decay time resolution is estimated to be about 80 fs. In this study we take into account the high level trigger efficiency and conservatively assume a signal yield of 7700 signal events from an integrated luminosity of 2 fb\(^{-1}\) and a background over signal ratio of 0.62.

Based on these yields for 2 fb\(^{-1}\) of data and the experimental resolutions, a toy Monte Carlo approach is used to evaluate the statistical errors on \( C, S \) and \( H \). The distributions for the proper decay time, the reconstructed \( B_s \) mass, the cosine of the polar angle of the \( K^0 \) in the rest frame of the \( \phi \) meson (\( \cos \theta \)) and the flavour tag are described by a probability density function (PDF). In each toy experiment this PDF is used to generate and fit the data. Then this toy experiment is repeated many times to produce distributions for \( C, S \) and \( H \), from which the statistical precision can be determined.

The signal PDF is modelled using the theoretical distribution for each observable convoluted by the following detector effects: the \( B_s \) mass resolution, the proper decay time resolution, the reconstruction efficiency as a function of proper decay time, the tagging efficiency and the wrong-tag fraction. A simple model is employed to describe the background PDF. We assume that the background is uniformly distributed in the \( B_s \) mass and in \( \cos \theta \), and has an exponential proper decay time spectrum with an effective lifetime which is one third of the signal lifetime. The detector effects and the background distributions are assumed to be precisely known. The theoretical signal distributions contain the following physical parameters: \( C, S, H \); the \( B_s \) average decay width \( \Gamma_s \), the \( B_s \) width and mass difference \( \Delta \Gamma_e \) and \( \Delta m_t \), and the \( B_s \) mass \( m_{B_s} \). In the fit \( C, S \) and \( H \) are free parameters. All the other parameters are fixed to their input values which are given in Table 1. A possible \( B_s \rightarrow B_s \) production asymmetry and CP violation in the \( B_s \) mixing \((|q/p| \neq 1)\) are neglected in this study.

A number of 500 toy experiments are generated for each set of values for \( C, S \) and \( H \). For a baseline scenario we set the parameters \( C = 0, S = 0 \), and \( H = 0 \), which is close to the SM prediction. Using a maximum likelihood fit we then determine these parameters in each toy experiment.

Table 1

<table>
<thead>
<tr>
<th>( \Gamma_{\phi} )</th>
<th>( \Delta \Gamma_e )</th>
<th>( \Delta m_t )</th>
<th>( m_{B_s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67 ps(^{-1})</td>
<td>0.1 ps(^{-1})</td>
<td>17.0 ps(^{-1})</td>
<td>5369 MeV/c(^2)</td>
</tr>
</tbody>
</table>

The high level trigger is not taken into account in this reference.
The sizable lifetime difference of the $B_s$ meson allows us to measure the photon polarization in the time dependent decay rate of $B_s \rightarrow \phi \gamma$. In addition to measuring the coefficient $S$ of the $\sin(\Delta m t)$ term, which is already probed in $B_d \rightarrow K^{(*)0}(K_S \pi^0)\gamma$, there exists a measurable coefficient $H$ for the $\sinh(\Delta \Gamma/2t)$ term in the decay $B_s \rightarrow \phi \gamma$. Both $S$ and $H$ are sensitive to right-handed currents in $B \rightarrow V\gamma$ transitions.

The SM prediction, $S_{\phi \gamma} \sim 0 \pm 0.002$ and $H_{\phi \gamma} = 0.047 \pm 0.025 + 0.015$ (17) [7], is dominated by short distance penguins and under control due to the smallness of the charm loop contributions (15) originating from the current-current operator (13). We also give a prediction for the direct CP asymmetry, $C_{\phi \gamma} = 0.005(5)$ (18), which is sensitive to new weak phases rather than right-handed currents. In Section 4 we presented a toy Monte Carlo simulation for the time dependent decay rate of $B \rightarrow \phi \gamma$ for a data sample of 2 fb$^{-1}$ which will be recorded by the LHCb experiment. From this study we estimate an experimental sensitivity on $S$ of about 0.14. The sensitivity on $H$ is inversely proportional to the $B_s$ width difference $\Delta \Gamma_s$. For an anticipated relative width difference $\Delta \Gamma_s/\Gamma_s = 0.15$ a precision of 0.16 can be reached for the observable $H$. Note also that $H$ can be extracted from the untagged decay rate. Thus knowledge of the production flavour of the $B_s$ meson is not required (19), (20) which will facilitate this measurement. If either $S$ or $H$ is large in NP, the LHCb experiment will be able to observe it. It is likely that NP, in terms of right-handed currents, will enhance the observable $H$ rather than $S$. Therefore it is fortunate that the sizable width difference of the $B_s$ meson gives access to $H$ which makes the decay $B_s \rightarrow \phi \gamma$ an exciting channel to search for NP.

Acknowledgements

R.Z. is grateful to Patricia Ball for collaboration on related work, to Thorsten Feldmann for interesting discussions and Daniel Wyler for hospitality at the University of Zurich during the time when this work was finalised. He is supported in part by the Marie Curie...
research training networks contract Nos. MRTN-CT-2006-035482, FLAVIANET, and MRTN-CT-2006-035505, HEPTOOLS.

Appendix A

In this appendix we shall derive the CP asymmetries in terms of two amplitudes, of different weak and strong phases. The algebra can easily be generalised to an arbitrary number of amplitudes. We extend the shorthand notation of Eq. (6) to

\[ \tilde{A}_{L(R)} \equiv \sum_i A^i_{L(R)} e^{3i\delta^i_{L(R)}}, \]

where \( i \) sums over the amplitudes. The weak phase \( \phi \) and the strong phase \( \delta \) have been separated leaving the remaining parameter \( A^i_{L(R)} \) real. In this notation the right-handed amplitude and the corresponding CP conjugate amplitudes become

\[ \tilde{A}_L = \sum_i A^i_L e^{3i\delta^i_L} \quad \tilde{A}_R = \sum_i A^i_R e^{3i\delta^i_R}, \]

where the CP-eigenvalue of the final state \( V \) and \( i = \{ u, c, t \} \) is the summation over the up-type quarks. For \( V = \{ \rho, \omega, \phi, K^*(K_S^0) \} \) the eigenvalue is \( \xi = 1 \) and for \( V = K^+(K_L^0) \) it is \( \xi = -1 \). In the SM there are three amplitudes at first, corresponding to the three up-type quarks \( u, c \) and \( t \)

\[ A = A^u + A^c + A^t = \lambda_u a^u + \lambda_c a^c + \lambda_t a^t, \]

where we have already separated the CKM parameters \( \lambda_{ij} = V^*_{ij} V_{ij} \). The parameters \( a^{u,c,t} \) are the same ones as in Eq. (8) up to the helicity specification and an irrelevant normalisation factor and differ from \( A^{u,c,t} \) by the inclusion of the strong phase. As discussed in the main text the three generation unitarity, \( \lambda_u + \lambda_c + \lambda_t = 0 \), may be used to reduce one amplitude, e.g.

\[ A = \lambda_t (a^t - a^u) + \lambda_u (a^u - a^c), \]

for the sake of more compact formulae. In the case where the two amplitudes are degenerate, e.g. \( a^u = a^c \), the amplitude reduces to a single term. This arises in the decay \( B \to \gamma V \) if the operators \( Q^{u,c,t}_{L,R} \) (13) are not treated separately. In terms of two amplitudes denoted by \( \{ u, t \} \) the CP asymmetries (7) assume the following form

\[ C = 4 \left( A^u A^t \sin(\phi^u - \phi^t) \sin(\delta^u - \delta^t) \right) \]

\[ H[S] = \frac{\xi}{4} \left( A^u A_k^t \cos(\delta^u - \delta_k^t) \cos[\sin(\phi^u - \phi^t - \phi^k)] + (u \leftrightarrow t) \right) \]

with the normalisation factor

\[ N = 2(A^u)^2 + (A^t)^2 + 2A^u A^t \cos(\delta^u - \delta^t) \cos(\phi^u - \phi^t) \]

Notice that the quantities \( H \) and \( S \) differ by a cosine and a sine of the weak phases only. In the case where there is only one amplitude the direct CP asymmetry \( C \) vanishes and the formulae for \( S \) and \( H \) reduce to

\[ H[S] = \frac{\pm 2A L A_k \cos(\delta^u - \delta_k^t) \cos[\sin(\phi^u - \phi^t - \phi^k)]}{(A^u)^2 + (A^t)^2}. \]

The formula for \( S \) reduces to the one given in [2] in the case where the strong phases \( \delta \) are set to zero. The celebrated formula for \( S_{B_d \to \phi K_S} = \sin(2\beta) \) is obtained by setting \( A_R = A_L \) and \( \xi = -1 \).

A few remarks concerning the SM are in order. The left- and right-handed phases are equal, \( \phi^u = \phi^SM \). The weak mixing phase, assuming \( |M_{d2}| \gg |M_{d3}| \) as valid in the SM [10], is approximately given by the top quark box diagram \( \phi_u \approx 2 \Phi[\lambda_t] = -2\lambda^2 \sim -0.035 \sim -2^\circ \).

The weak phases of the amplitudes are exactly given by \( \phi^u = \Phi[\lambda_t] \). Using the Wolfenstein parameterisation the SM the phases are:

\[ \phi_u \approx -2\lambda^2 \eta, \quad \phi^b_{u-s} \approx -\lambda^2 \eta, \quad \phi^b_{u-s} \approx -\gamma, \]

\[ \phi^c_{u-s} = O(\lambda^4). \]

In the \( B_s \) system the unitarity triangle follows the hierarchical pattern \( |\lambda_u| \approx |\lambda_c| \gg |\lambda_t| \) \( \to \). The term proportional to \( \lambda_u \) in (A.4) can be neglected in the case where the trigonometric function of the angles are of the same order, which is the case for \( H \) but not for \( S \).

References