



Fuzzy Set Based Multiobjective Allocation of Resources and Its Applications

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Abstract—This paper presents results of research into the use of the Bellman-Zadeh approach to decision making in a fuzzy environment for solving multiobjective optimization problems. Its application conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions on the basis of analyzing associated *maxmin* problems. The use of the Bellman-Zadeh approach has served as a basis for solving a problem of multiobjective allocation of resources (or their shortages) and developing a corresponding adaptive interactive decision-making system (AIDMS1). Its calculating kernel permits one to solve *maxmin* problems using an algorithm based on a nonlocal search (modification of the Gelfand's and Tsetlin's "long valley" method). The AIDMS1 includes procedures for considering linguistic variables to reflect conditions that are difficult to formalize as well as procedures for constructing and correcting vectors of importance factors for goals. The use of these procedures permits one to realize an adaptive approach to processing information of a decision maker to provide successive improving of the solution quality. C++ windows of the AIDMS1 are presented for input, output, and special possibilities related to considering linguistic variables and constructing and correcting vectors of importance factors. The results of the paper are universally applicable and are already being used to solve power engineering problems. © 2006 Elsevier Ltd. All rights reserved.

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1. INTRODUCTION

Diverse types of uncertainty are often encountered in a wide range of problems related to the design, planning, and control of complex systems. Taking into account the uncertainty factor in building mathematical models serves as a means for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis. Investigations of recent years show the benefits of applying fuzzy set theory [1,2] to deal with diverse types of uncertainty. Its use in problems of optimization character offers advantages of both fundamental nature (the possibility of validly obtaining more effective, less "cautious" solutions) and of a computational character [3,4].

The uncertainty of goals is an important kind of uncertainty related to a multiobjective character of many optimization problems. It is possible to classify two major categories of problems, which need the use of a multiobjective approach:

- problems in which solution consequences cannot be estimated on the basis of a single criterion: these problems are associated with the analysis of models including economic as well as natural indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimations are hampered;
- problems that, from the substantial point of view, may be solved on the basis of a single criterion; however, if the uncertainty of information does not permit one to obtain a unique solution, it is possible to reduce these problems to multiobjective decision making because the use of additional criteria (including criteria of qualitative character) can serve as a convincing means to contract the decision uncertainty regions [3].

According to this, two classes of models (so called $\langle X, M \rangle$ and $\langle X, R \rangle$ models [4]) may be constructed. The present paper is related to analyzing $\langle X, M \rangle$ models, when a vector of objective functions $F(X) = \{F_1(X), \dots, F_q(X)\}$ is considered, and the problem consists of simultaneous optimizing all objective functions, i.e.,

$$F_p(X) \rightarrow \underset{X \in L}{\text{extr}}, \quad p = 1, \dots, q, \quad (1)$$

where L is a feasible region in R^n .

The first step in solving problem (1) is associated with determining a set of Pareto solutions $\Omega \subseteq L$ [5]. This step is useful; however, it does not permit one to obtain unique solutions. It is necessary to choose a particular Pareto solution on the basis of information of a decision maker (DM). There are three approaches to using this information [6]: *a priori*, *a posteriori*, and adaptive. The most preferable approach is the adaptive one. When using this approach, the procedure of successive improving of the solution quality is realized as a transition from $X_\alpha^0 \in \Omega \subset L$ to $X_{\alpha+1}^0 \in \Omega \subset L$ with considering information I_α of the DM. The solution search may be presented in the following form:

$$X_1^0, F(X_1^0) \xrightarrow{I_1} \dots \xrightarrow{I_{\alpha-1}} X_\alpha^0, F(X_\alpha^0) \xrightarrow{I_\alpha} \dots \xrightarrow{I_{\omega-1}} X_\omega^0, F(X_\omega^0). \quad (2)$$

Process (2) serves for two types of adaptation: computer to preferences of the DM and DM to the problem. The first type of adaptation is based on information received from the DM. The second type of adaptation is realized as a result of carrying out several steps $X_\alpha^0, F(X_\alpha^0) \xrightarrow{I_\alpha} X_{\alpha+1}^0, F(X_{\alpha+1}^0)$, which permit the DM to understand the correlation between its own needs and possibilities of their satisfaction by model (1).

When analyzing multiobjective problems, it is necessary to solve some questions related to normalizing criteria, selecting principles of optimality, and considering priorities of criteria. Their

solution and, therefore, development of multiobjective methods is carried out in the following directions [7–9]: scalarization techniques, imposing constraints on criteria, utility function method, goal programming, and using the principle of guaranteed result. Without discussion of these directions, it is necessary to point out that an important question in multiobjective optimization is the solution quality. It is considered as high if levels of satisfying criteria are equal, or close to each other (harmonious solutions) if we do not differentiate the importance of objective functions [10]. From this point of view, the validity and advisability of the direction related to the principle of guaranteed result [4,10] should be recorded.

The lack of clarity in the concept of “optimal solution” is the basic methodological complexity in solving multiobjective problems. When applying the Bellman-Zadeh approach to decision making in a fuzzy environment [1,2,11], this concept is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions. The Bellman-Zadeh approach permits one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multiobjective models [4,10]. Finally, its use allows one to preserve a natural measure of uncertainty in decision making and to take into account indices, criteria, and constraints of qualitative character.

The present paper is dedicated to applying the Bellman-Zadeh approach to solving a problem of multiobjective allocation of resources (or their shortages).

2. BELLMAN-ZADEH APPROACH AND MULTIOBJECTIVE OPTIMIZATION PROBLEMS

When using the Bellman-Zadeh approach, each objective function $F_p(X)$ is replaced by a fuzzy objective function or a fuzzy set

$$A_p = \{X, \mu_{A_p}(X)\}, \quad X \in L, \quad p = 1, \dots, q, \tag{3}$$

where $\mu_{A_p}(X)$ is a membership function of A_p [1,2].

A fuzzy solution D with setting up the fuzzy sets (3) is turned out as a result of the intersection $D = \bigcap_{p=1}^q A_p$ with a membership function

$$\mu_D(X) = \min_{p=1, \dots, q} \mu_{A_p}(X), \quad X \in L. \tag{4}$$

Its use permits one to obtain the solution proving the maximum degree

$$\max \mu_D(X) = \max_{X \in L} \min_{p=1, \dots, q} \mu_{A_p}(X) \tag{5}$$

of belonging to the fuzzy solution D and reduced problem (1) to

$$X^0 = \arg \max_{X \in L} \min_{p=1, \dots, q} \mu_{A_p}(X). \tag{6}$$

To obtain (6), it is necessary to build membership functions $\mu_{A_p}(X)$, $p = 1, \dots, q$, reflecting a degree of achieving “own” optima by $F_p(X)$, $X \in L$, $p = 1, \dots, q$. This condition is satisfied by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{F_p(X) - \min_{X \in L} F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{7}$$

for maximized objective functions or by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{\max_{X \in L} F_p(X) - F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{8}$$

for minimized objective functions. In (7) and (8), $\lambda_p, p = 1, \dots, q$, are importance factors for the corresponding objective functions.

The construction of (7) or (8) demands to solve the following problems:

$$F_p(X) \rightarrow \min_{X \in L}, \tag{9}$$

$$F_p(X) \rightarrow \max_{X \in L}, \tag{10}$$

providing $X_p^0 = \arg \min_{X \in L} F_p(X)$ and $X_p^{00} = \arg \max_{X \in L} F_p(X)$, respectively.

Thus, the solution of problem (1) on the basis of the Bellman-Zadeh approach demands analysis of $2q + 1$ monoobjective problems (9), (10), and (5), respectively.

Since the solution X^0 is to belong to $\Omega \subseteq L$, it is necessary to build

$$\bar{\mu}_D(X) = \bigwedge_{p=1}^q \mu_{A_p}(X) \bigwedge \mu_{\pi}(X) = \min \left\{ \min_{p=1, \dots, q} \mu_{A_p}(X), \mu_{\pi}(X) \right\}, \tag{11}$$

where $\mu_{\pi}(X) = 1$ if $X \in \Omega$ and $\mu_{\pi}(X) = 0$ if $X \notin \Omega$.

The procedures for solving problem (5), discussed below, provide the line in obtaining $X^0 \in \Omega \subseteq L$ in accordance with (11). Thus, it can be said about equivalence of $\bar{\mu}_D(X)$ and $\mu_D(X)$. It permits one to give up the necessity of implementing a cumbersome procedure for building the set $\Omega \subseteq L$.

Finally, the existence of additional conditions (indices, criteria, and/or constraints) of qualitative character, defined by linguistic variables [1,2], reduces (6) to

$$X^0 = \arg \max_{X \in L} \min_{p=1, \dots, q+s} \mu_{A_p}(X), \tag{12}$$

where $\mu_{A_p}(X), X \in L, p = q+1, \dots, s$ are membership functions of fuzzy values [1,2] of linguistic variables which reflect these additional conditions.

There is a theoretical basis (for example, [12,13]) of the validity of applying the *min* operator in (4)–(6). However, there exist many families of aggregation operators [1,14] that may be used in place of the *min* operator. Considering this, it is possible to generalize (4) as follows:

$$\mu_D(X) = \text{agg}(\mu_{A_1}(X), \mu_{A_2}(X), \dots, \mu_{A_q}(X)), \quad X \in L. \tag{13}$$

Despite that some properties of the aggregation operators have been established, there is no clear and intuitive interpretation of these properties, nor unifying interpretation of the operators themselves [14]. It is possible to state the following question: among many types of aggregation operators, how is one selected, which is adequate for a particular problem? Although some selection criteria are suggested in [1], the majority of them deal with empirical fit. Thus, it is possible to assert that the selection of the operators, in large measure, is based on experience. Considering this, below we discuss computing experiments associated with using not only the *min* operator but the *product* operator as well. The last operator has found applications in decision-making problems. Its use reduces (4) to

$$\mu_D(X) = \prod_{p=1, \dots, q} \mu_{A_p}(X) \tag{14}$$

and permits one to construct the problem

$$\max \mu_D(X) = \max_{X \in L} \prod_{p=1, \dots, q} \mu_{A_p}(X) \tag{15}$$

to find

$$X^0 = \arg \max_{X \in L} \prod_{p=1, \dots, q} \mu_{A_p}(X). \tag{16}$$

3. MULTIOBJECTIVE ALLOCATION OF RESOURCES

The statement of the problem of multiobjective allocation of resources or their shortages (these problems are equivalent from the substantial, mathematical, and information points of view) among consumers (departments, regions, projects, etc.) supposes the possibility to use diverse types of objective functions (linear, fractional, quadratic, etc.) [15] in (1) defined in a feasible region

$$L = \left\{ X \in R^n \mid 0 \leq x_i \leq A_i, \sum_{i=1}^n x_i = A \right\}, \quad (17)$$

where $X = (x_1, \dots, x_n)$ is sought for a vector of limitations (for definiteness) for consumers, A_i is the permissible value of limitation for the i^{th} consumer, A is a total value of limitations for all consumers considered in planning or control.

To describe a general scheme of solving the problem formalized within the framework of model (1),(17), it is advisable to introduce into consideration a linguistic variable [1,2] *Q-limitation for consumer* to provide the DM with the possibility to consider conditions that are difficult to formalize. Thus, the general scheme assumes the availability of a procedure for building a term-set $T(Q)$ of the linguistic variable and membership functions for its fuzzy values. In addition, if the solution X_α^0 with the values $\mu_{A_p}(X_\alpha^0)$, $p = 1, \dots, q$, is not satisfactory, the DM has to have the possibility to correct it, passing to $X_{\alpha+1}^0$ with changing the importance of one or more objective functions. Thus, the general scheme also assumes the availability of the procedure for constructing and correcting the vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of importance factors.

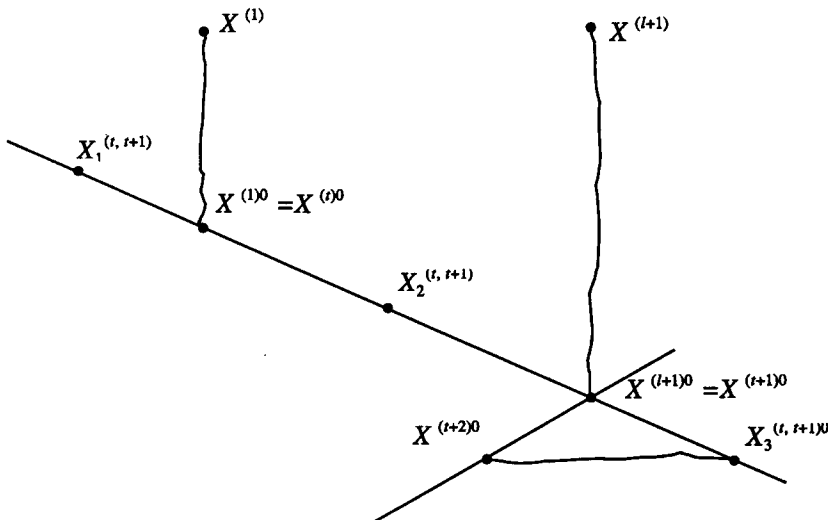
The general scheme of solving problem (1),(17), which has served for implementing an adaptive interactive decision-making system (AIDMS1) described below, is associated with the following sequence of blocks.

1. Solution of problems (9) and (10) to obtain X_p^0 , $p = 1, \dots, q$, and X_p^{00} , $p = 1, \dots, q$, respectively.
2. Construction of the membership functions defined by (7) or (8).
3. Construction of an initial vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of the importance factors.
4. Analysis of the availability of initial conditions defined by the linguistic variables. If these conditions are lacking, then go to Block 8; otherwise go to Block 5.
5. Verification of compatibility of the initial conditions and, if necessary, their correction.
6. Solution of problem (5) with the goal to obtain X_α^0 defined by (11).
7. Analysis of the current solution X_α^0 . If the DM is satisfied by the solution, then go to Block 10; otherwise go to Block 8, taking $\alpha := \alpha + 1$.
8. Correction of the vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of the importance factors.
9. Insertion of additional conditions defined by the linguistic variables; then go to Block 5.
10. Calculations are completed because the solution X^0 is obtained.

The main functions of a calculating kernel of the AIDMS1 are associated with obtaining X_p^0 , $p = 1, \dots, q$, and X_p^{00} , $p = 1, \dots, q$, defined by solving problems (9) and (10) and with obtaining X^0 in accordance with (6). The solution of problems (9) and (10) creates no difficulties. The maximization of (11) is based on a nonlocal search that is a modification of the Gelfand's and Tsetlin's "long valley" method [16].

Experimental calculations show that variables of (4) can be divided into two groups: inessential and essential. The change of inessential variables leads to essential variations of (4). The change of essential variables leads to inessential variations of (4). Thus, a structure of (4) may be considered as a multidimensional "long valley". If we use direct search methods [9], this circumstance requires the ascent from different initial points X_p^0 (Pareto points), if we minimize $F_p(X)$, or X_p^{00} (Pareto points), if we maximize $F_p(X)$, to find the most convincing solution X^0 . This explains the use of a nonlocal search, which can be presented as follows.

1. The sequence $\{X^{(l)}\}$, $l = 1, \dots, q$, is built from points X_p^0 , if we minimize $F_p(X)$, or X_p^{00} , if we maximize $F_p(X)$, obtained as a result of execution of Block 1 of the general scheme. This sequence has the following property: $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(l)}) \geq \min_{1 \leq p \leq q} \mu_{A_p}(X^{(l+1)})$, $l = 1, \dots, q - 1$.
2. The local search for X^0 is carried out from $X^{(l)}$ ($l = 1$). As a result of this search, we obtain a point $X^{(1)0}$ with corresponding $\mu_{A_p}(X^{(1)0})$, $p = 1, \dots, q$.
3. The local search for X^0 is carried out from $X^{(l+1)}$. As a result of this search, we obtain a point $X^{(l+1)0}$ with corresponding $\mu_{A_p}(X^{(l+1)0})$, $p = 1, \dots, q$.
4. Analysis is executed:
 - (a) if $X^{(1)0} \neq X^{(l+1)0}$, then go to Operation 5;
 - (b) if $X^{(1)0} = X^{(l+1)0}$ for $l \neq q - 1$, then go to Operation 3, taking $l := l + 1$;
 - (c) if $X^{(1)0} = X^{(l+1)0} = X^{(q)0}$, then go to Operation 8, taking $X^0 = X^{(1)0}$.
5. A line between points $X^{(t)0}$ and $X^{(t+1)0}$ is "built" to generate points $X_s^{(t,t+1)}$, $s = 1, 2, 3$ (see Figure 1). Among them (if they are acceptable from the point of view of constraints (17)), a point $X^{(t,t+1)0} = \arg \max_t \min_{1 \leq p \leq q} \mu_{A_p}(X_s^{(t,t+1)})$ is selected to define a direction for a future search.
6. The next local search for X^0 is carried out from $X^{(t,t+1)0}$. As a result of this search, we obtain a point $X^{(t+2)0}$ (see Figure 1).
7. Analysis is executed: if three "last" points $X^{(t)0}$, $X^{(t+1)0}$, and $X^{(t+2)0}$ differ on $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t)0})$, $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+1)0})$, and $\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)0})$ less than the accuracy desired, then go to Operation 8, taking $X^0 = \arg \max[\min_{1 \leq p \leq q} \mu_{A_p}(X^{(t)0}), \min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+1)0}), \min_{1 \leq p \leq q} \mu_{A_p}(X^{(t+2)0})]$; otherwise go to Operation 5, taking $X^{(t)0} := X^{(t,t+1)0}$ and $X^{(t+1)0} := X^{(t+2)0}$.
8. Calculations are completed because the solution $X^0 \in \Omega \subseteq L$ is obtained.

Figure 1. Nonlocal search for X^0 .

The execution of Operations 2, 3, and 6 of the algorithm is possible on the basis of any search method (in particular, a modification of the univariate method [9] was implemented within the framework of the AIDMS1). If $X^{(m)}$ is a current point, the transition to $X^{(m+1)}$ is expedient if

$$(\forall p = 1, \dots, q) : \mu_{A_p}(X^{(m+1)}) \geq \min_{1 \leq p \leq q} \mu_{A_p}(X^{(m)}). \quad (18)$$

In contrast, if

$$(\exists p = 1, \dots, q) : \mu_{A_p}(X^{(m+1)}) < \min_{1 \leq p \leq q} \mu_{A_p}(X^{(m)}), \quad (19)$$

the transition to $X^{(m+1)}$ is not expedient from the point of view of maximizing (4). This way of evaluating the expediency of the transition to the next point $X^{(m+1)}$ leads to solution (6) that is Pareto, if all inexpedient transitions are rejected.

The AIDMS1 includes the procedure for constructing and correcting the term-set $T(Q)$ and membership functions for fuzzy values of the linguistic variable Q -*limitation for consumer*. The initial term-set available for the DM is $T(Q) = \langle \textit{near}, \textit{approximately}, \textit{slightly less}, \textit{considerably less}, \textit{slightly more}, \textit{and considerably more} \rangle$. The corresponding membership functions are the following:

$$\mu(x_i) = e^{-k(T_i - x_i)^2}; \quad (20)$$

$$\mu(x_i) = \begin{cases} 1 - e^{-k(T_i - x_i)^2}, & x_i \leq T_i, \\ 0, & x_i > T_i; \end{cases} \quad (21)$$

$$\mu(x_i) = \begin{cases} 1 - e^{-k(T_i - x_i)^2}, & x_i \geq T_i, \\ 0, & x_i < T_i, \end{cases} \quad (22)$$

where k is a coefficient defined by a given solution accuracy; T_i is a "specific value" which is related to the condition that is to be taken into account.

Membership function (20) corresponds to the terms *near* and *approximately*, (21) to *slightly less* and *considerably less*, and (22) to *slightly more* and *considerably more*.

The availability of d additional conditions, defined by membership functions (20)–(22), leads to $p = 1, \dots, q + d$ in (3)–(5) and (11).

Furthermore, the AIDMS1 includes several procedures for forming and correcting the vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of importance factors. These procedures are oriented to the individual DM as well as to the group DP. In particular, one of the procedures is associated with processing of the results of paired qualitative comparisons of the importance for different goals. The use of this type of information is rational because psychological experiments show that the DM is faced with difficulties in directly estimating the importance factors. In accordance with [17], the DM is to indicate which among two goals is more important and to estimate his or her perception of the distinction degree using a scale which includes the following ranks: *identical significance*, *weak superiority*, *strong superiority*, *evident superiority*, and *absolute superiority*. The comparisons allow one to construct the matrix $[b_{pt}]$, $p, t = 1, \dots, q$. The components of the eigenvector corresponding to the maximum eigenvalue of the matrix can serve as estimates for λ_p , $p = 1, \dots, q$.

3.1. Computing Implementation

The AIDMS1 has been developed in the C++ programming language and is executed in the graphical environment of the *Microsoft Windows*® Operating System. In this section, we list several typical windows that appear in the process of multiobjective resource shortage allocation.

The initial window (see Figure 2) permits one to start the decision-making process by clicking database.

The database window (see Figure 3) is destined for loading information available in the database by clicking load or for preparing and memorizing input information by clicking save. In the second case, number of functions (q), number of variables (n), initial function information (c_{pi} , $p = 1, \dots, q$, $i = 1, \dots, n$), variable limitations (A_i , $i = 1, \dots, n$), and limitation (A) are to be defined. Besides, the variable increment (D_X) and the desired accuracy (Err) are to be defined as well. The screen in Figure 3 reflects input information for an example of multiobjective resource shortage allocation discussed below.

Clicking importance factors (see Figure 3), it is possible to construct or correct the vector $\Lambda = (\lambda_1, \dots, \lambda_q)$ of importance factors by pairs indicating which among two goals is more important and estimating the corresponding distinction degree using the rank scale given above. As an

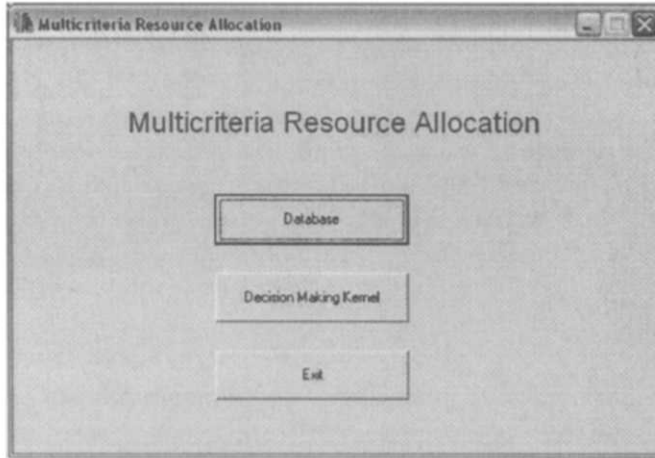


Figure 2. Initial window.

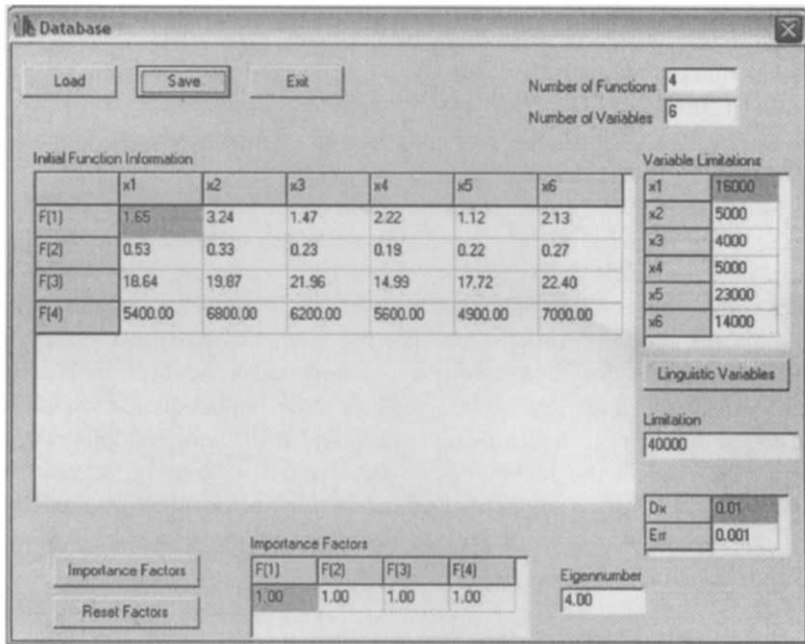


Figure 3. Database interface.

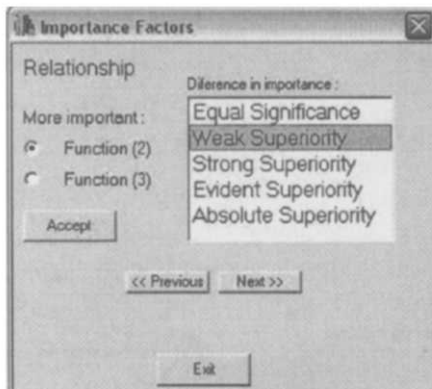


Figure 4. Importance factor interface.

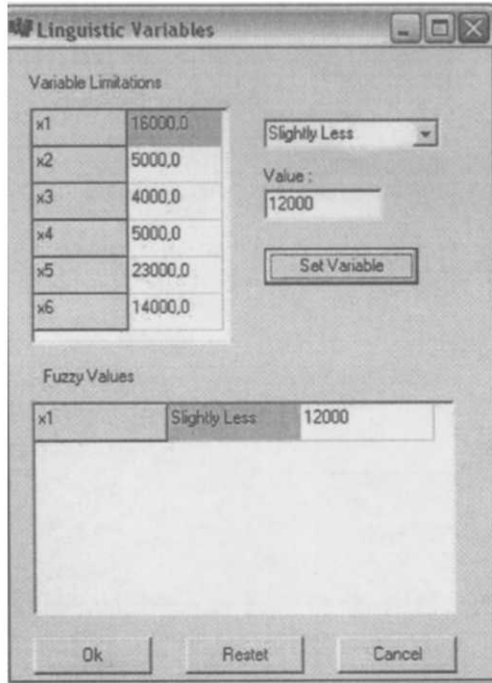


Figure 5. Linguistic variable interface.

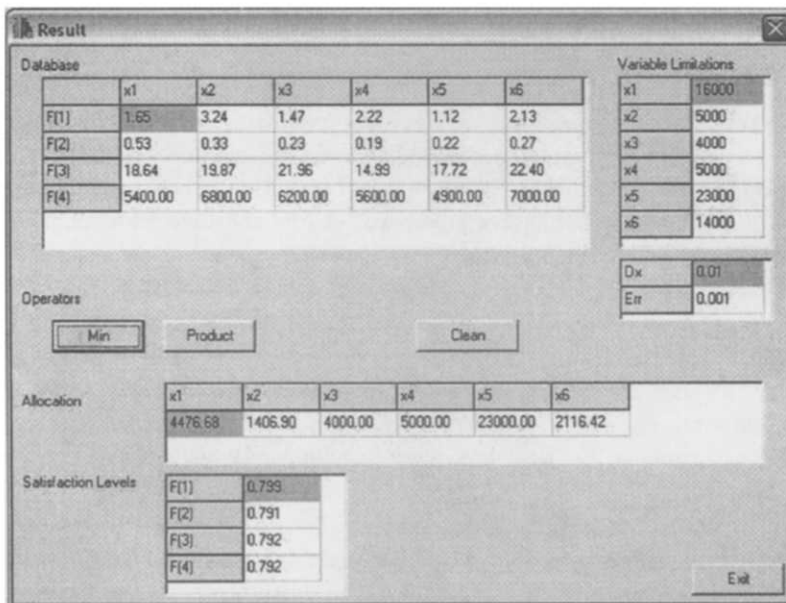


Figure 6. Execution interface with applying the *min* operator.

example, Figure 4 reflects the fact that the second objective function is more important than the third objective function with the rank *weak superiority*.

Clicking linguistic variables (see Figure 3), it is possible to consider the linguistic variable *Q-limitation for consumer*. As an example, Figure 5 demonstrates the application of the fuzzy value *slightly less* with respect to the magnitude 12,000.00 for the *limitation for consumer 1*.

The results of solving the problem of multiobjective resource shortage allocation presented in the database window (see Figure 3) on the basis of applying the *min* operator are given in

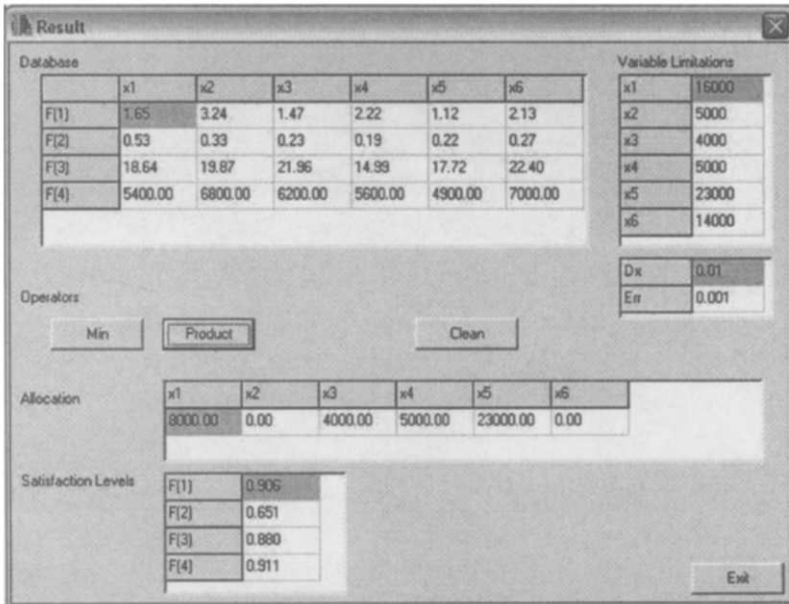


Figure 7. Execution interface with applying product operator.

Figure 6. The results of solving the same problem on the basis of applying the *product* operator are given in Figure 7.

3.2. Computing Experiments

The results presented above have served as a basis for solving several power engineering problems (multiobjective power system operation [18], tuning of fuzzy models associated with voltage and reactive power control [19], etc.). The results of computing experiments discussed below are related to the problem of multiobjective power and energy shortage allocation. This statement of the problem is justified by the following considerations.

Different conceptions of load management (for example, discussed in [20,21]) may be united by the following: control action elaboration is performed on the two-stage bases. On the level of energy control centers, optimization of allocating power and energy shortages (natural or associated with the economic feasibility of load management) is carried out for different levels of territorial, temporal, and situational hierarchy of planning and operation. This allows one to draw up tasks for consumers. On their level, control actions are realized in accordance with these tasks.

Thus, the questions of power and energy shortage allocation are of a fundamental importance in a family of load management problems. These questions should be considered not only from the economical and technological points of view, but from the social and ecological points of view as well. Besides, it is necessary to account for considerations of creating incentive influences for consumers. Considering this, it should be pointed out that methods of power and energy shortage allocation, based on fundamental principles of allocating resources [22], have drawbacks [15,21]. Their overcoming is possible on the basis of formulating and solving the problems within the framework of multiobjective models. This permits one to consider and to minimize diverse consequences of power and energy shortage allocation and to create incentive influences for consumers.

Substantial analysis of the problems of power and energy shortage allocation, systems of economics management, as well as real, readily available reported and planned information has permitted the construction of a general set of goals. The list includes 17 types of goals. Without

listing all of them, it is possible to indicate the following goals:

1. primary limitation of consumers with lower cost of produced production and/or given services on consumed 1 kWh of energy (achievement of a minimal drop in total produced production and/or given services);
15. primary limitation of consumers with a lower value of the demand coefficient (primary limitation of consumers with greater possibilities of production out the peak time);
16. primary limitation of consumers with a lower duration of using maximum load in 24 hours (primary limitation of consumers with greater possibilities in transferring maximum load in the 24 hours interval);
17. primary limitation of consumers with a lower duration of using maximum load in month (quarter, year) (primary limitation of consumers with greater possibilities in transferring maximum load in the month (quarter, year) interval).

The general set of goals is sufficiently complete because it is directed to decreasing diverse negative consequences for consumers and creating incentive influences for them. This set is universal because it can serve as the basis for building models at different levels of load management hierarchy by aggregation of information and posterior decomposition of the problems in accordance with different indices. The concrete list of goals can be defined at every case by the DM, who can be individual or group (for example, it may be leading organizations of the country or state, a council of directors of enterprises, etc., whose decision regarding the concrete list of goals can be considered as the legislative one for the corresponding level).

Consider the solution of the problems of multiobjective power shortage allocation formalized within the framework of model (1) and (17) for six consumers for $A^1 = 40000$ kW and $A^2 = 60000$ kW taking into account four objective functions on the basis of the Bellman-Zadeh approach with using the *min* and *product* operators as well as the well-known Boldur's method (the scalarization approach) [23].

In our case, $x_i, i = 1, \dots, 6$, are limitations of power supply for consumers. The coefficients $c_{pi}, p = 1, 15, 16, 17, i = 1, \dots, 6$ (for linear objective functions of (1) reflecting the goals indicated above), are determined by specific characteristics of consumers. Table 1 provides initial information for the problems. As it was indicated above, this information corresponds to input information given in Figure 3.

Table 1. Initial information.

i	1	2	3	4	5	6
$c_{1,i}$, monetary units/kWh	1.65	3.24	1.47	2.22	1.12	2.13
$c_{15,i}$	0.53	0.33	0.23	0.19	0.22	0.27
$c_{16,i}$, hours	18.64	19.87	21.96	14.99	17.72	22.40
$c_{17,i}$, hours	5400	6800	6200	5600	4900	7000
A_i , kW	16000	5000	4000	5000	23000	14000

The results of the solution on the basis of the Bellman-Zadeh approach with using the *min* operator (X^0) and the *product* operator (X^{00}) as well as the Boldur method (X^{000}) are presented in Tables 2 and 3. The solutions based on the use of the *min* and *product* operators for $A^1 = 40000$ kW are presented in Figures 6 and 7 as well.

To reflect the quality of solutions obtained on the basis of different approaches, Table 4 includes the mean magnitudes of absolute values $\Delta(X)$ of deviations of membership function-levels (satisfaction levels) $\mu_{A_p}(X)$ from their mean values $\hat{\mu}_{A_p}(X)$ calculated as follows:

$$\Delta(X) = \frac{1}{4} \sum_{i=1}^4 |\mu_{A_p}(X) - \hat{\mu}_{A_p}(X)|,$$

Table 2. Power shortage allocation.

i	1	2	3	4	5	6
	4476.68	1406.90	4000.00	5000.00	23000.00	2116.00
	8000.00	0	4000.00	5000.00	23000.00	0
	8000.00	0	4000.00	5000.00	23000.00	0
	12868.07	2763.63	4000.00	5000.00	23000.00	12368.30
	15510.77	0	4000.00	5000.00	23000.00	12489.23
	16000.00	0	4000.00	5000.00	23000.00	12000.00

Table 3. Levels of the membership functions.

p	1	15	16	17
$\mu_{A_p}(X^{1,0})$	0.799	0.791	0.792	0.792
$\mu_{A_p}(X^{1,00})$	0.906	0.651	0.880	0.911
$\mu_{A_p}(X^{1,000})$	0.906	0.651	0.880	0.911
$\mu_{A_p}(X^{2,0})$	0.629	0.624	0.624	0.629
$\mu_{A_p}(X^{2,00})$	0.968	0.400	0.688	0.879
$\mu_{A_p}(X^{2,000})$	0.986	0.446	0.727	0.932

Table 4. Mean deviations.

Δ	A^1	A^2
$\Delta(X^0)$	0.003	0.003
$\Delta(X^{00})$	0.093	0.195
$\Delta(X^{000})$	0.093	0.186

where

$$\hat{\mu}_{A_p}(X) = \frac{1}{4} \sum_{i=1}^4 \hat{\mu}_{A_p}(X_i).$$

The data of Table 4 bring out that $X^0 \succ X^{00}$ and $X^0 \succ X^{000}$. The high quality of the solution X^0 is also confirmed by inequalities $\min_p \mu_{A_p}(X^0) > \min_p \mu_{A_p}(X^{00})$ and $\min_p \mu_{A_p}(X^0) > \min_p \mu_{A_p}(X^{000})$ observed for both cases.

To show the possibility of correcting solutions as a result of changing the importance of the objective functions, we can assume, for example, that the second objective function ($p = 15$) has the level "weak superiority" relative to other objective functions (other objective functions have the level "identical significance" relative to each other). These paired comparisons permit one to obtain $\lambda_1 = 0.67$, $\lambda_2 = 2.00$, $\lambda_3 = 0.67$, and $\lambda_4 = 0.67$. The corresponding solution for $A^1 = 40000$ kW is: $x_1^{1,0} = 2196.11$ kW, $x_2^{1,0} = 1262.74$ kW, $x_3^{1,0} = 4000.00$ kW, $x_4^{1,0} = 5000.00$ kW, $x_5^{1,0} = 23000.00$ kW, and $x_6^{1,0} = 4541.15$ kW with $\mu_{A_2}(X^{1,0}) = 0.944$ and $\mu_{A_1}(X^{1,0}) = 0.675$, $\mu_{A_3}(X^{1,0}) = 0.602$, $\mu_{A_4}(X^{1,0}) = 0.599$. It is possible to increase to a greater degree the importance of the second objective function ($p = 15$), utilizing, for example, the level "evident significance" relative to other objective functions. In this case, we have $\lambda_1 = 0.40$, $\lambda_2 = 2.80$, $\lambda_3 = 0.40$, and $\lambda_4 = 0.40$, and the solution is: $x_1^{1,0} = 1036.58$ kW, $x_2^{1,0} = 1079.98$ kW, $x_3^{1,0} = 4000.00$ kW, $x_4^{1,0} = 5000.00$ kW, $x_5^{1,0} = 23000.00$ kW, and $x_6^{1,0} = 5883.44$ kW with $\mu_{A_2}(X^{3,0}) = 0.980$ and $\mu_{A_1}(X^{3,0}) = 0.499$, $\mu_{A_3}(X^{3,0}) = 0.364$, $\mu_{A_4}(X^{3,0}) = 0.364$.

Let us consider the influence of the linguistic variable Q -limitation for consumer. For example, the introduction of the condition "considerably less than 5000 kW" for the fourth consumer leads to the change of the solution given in Table 2 to: $x_4^{1,0} = 1936.10$ kW and $x_1^{1,0} = 5313.92$ kW, $x_2^{1,0} = 3312.00$ kW, $x_3^{1,0} = 4000.00$ kW, $x_5^{1,0} = 23000.00$ kW, $x_6^{1,0} = 1999.98$ kW. At the same

time, the introduction of the condition “slightly less than 5000 kW” for the fourth consumer leads to the change of the solution given in Table 2 to: $x_4^{1,0} = 3257.52$ kW and $x_1^{1,0} = 4888.11$ kW, $x_2^{1,0} = 3750.00$ kW, $x_3^{1,0} = 4000.00$ kW, $x_5^{1,0} = 23000.00$ kW, $x_6^{1,0} = 1104.57$ kW.

4. CONCLUSIONS

When using the Bellman-Zadeh approach to decision making in a fuzzy environment, the concept of “optimal solution” is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions on the basis of analyzing associated *maxmin* models. The use of the Bellman-Zadeh approach has served as a basis for solving the problem of multiobjective allocation of resources (or their shortages) and developing a corresponding adaptive interactive decision-making system (AIDMS1). Some details of its implementation as well as its principal C++ windows have been presented. The calculating kernel of the AIDMS1 is based on a nonlocal search (modification of the Gelfand’s and Tsetlin’s “long valley” method) to solve *maxmin* problems. The peculiarities of this modification permit one to obtain solutions that are, indeed, Pareto. The AIDMS1 includes procedures for constructing linguistic variables (to consider conditions that are difficult to formalize) as well as for forming and correcting importance factors for goals. The use of these procedures permits one to realize an adaptive approach to provide successive improving of the solution quality on the basis of information of a decision maker. The results of some computing experiments have been presented to show the efficiency of using the Bellman-Zadeh approach with applying the *min* operator to solve the problem of multiobjective allocation of resources (or their shortages).

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