



Inflation and reheating in induced gravity

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ABSTRACT

Inflation is studied in the context of induced gravity (IG) $\gamma\sigma^2R$, where R is the Ricci scalar, σ a scalar field and γ a dimensionless constant. We study in detail cosmological perturbations in IG and examine both a Landau–Ginzburg (LG) and a Coleman–Weinberg (CW) potential toy models for small field and large field (chaotic) inflation and find that small field inflationary models in IG are constrained to $\gamma \lesssim 3 \times 10^{-3}$ by WMAP 5-yrs data. Finally we describe the regime of coherent oscillations in induced gravity by an analytic approximation, showing how the homogeneous inflaton can decay in its short-scale fluctuations when it oscillates around a non-zero value σ_0 .

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Beginning with the association of the gravitational coupling with a scalar field [1] many attempts have been made to relate the gravitational constant to dynamics. Indeed starting from an attempt to relate it to one loop effects in some fundamental interaction [2] induced gravity (IG) theories $\gamma\sigma^2R$ have been developed [3–5]. In such theories σ acquires a non-zero vacuum expectation value through the spontaneous breaking of scale invariance arising through the presence of a condensate [3] or quantum effects (radiative corrections) [6]. Further such theories can be generalized [7] leading to a viable dark energy model [8].

In IG one may also use σ to achieve inflation. “Old inflation” in such a context is not satisfactory [9] and small field and large field (chaotic) inflation appear more promising. In particular for the last case with $\gamma \gg 1$ the constraints on σ are such that it may even be compatible with spontaneous symmetry breaking in the usual particle physics context [10–13].

In this Letter we analyze the slow-roll predictions for single-field inflation in IG and compare them with the recent WMAP 5-yrs data [14]. We also give an analytic approximate solution for the coherent oscillation regime during which reheating in IG takes place.

Let us start by considering the IG action

$$S = \int d^4x \sqrt{-g} \left[\frac{\gamma}{2} \sigma^2 R - \frac{g^{\mu\nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right] \quad (1)$$

where γ is a dimensionless and positive definite parameter and we assume a spatially flat Robertson–Walker background $g_{\mu\nu} = \text{diag}(-1, a^2(t), a^2(t), a^2(t))$. The variation of the above action leads to the following set of independent equations

$$H^2 = \frac{1}{3\gamma\sigma^2} \left[\frac{\dot{\sigma}^2}{2} + V(\sigma) \right] - 2H \frac{\dot{\sigma}}{\sigma}, \quad (2)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\dot{\sigma}^2}{\sigma} = -\frac{V_{\text{eff},\sigma}}{1+6\gamma} \quad (3)$$

where $H = \dot{a}/a$, $V_{\text{eff},\sigma} = dV/d\sigma - 4V/\sigma$ and the dot is the derivative with respect to cosmic time.

Inflation generically occurs during the slow-rolling of σ , for which Eqs. (2), (3) reduce to:

$$H^2 \simeq \frac{V}{3\gamma\sigma^2}, \quad 3H\dot{\sigma} \simeq -\frac{V_{\text{eff},\sigma}}{1+6\gamma}. \quad (4)$$

The ratio $-\dot{H}/H^2$ obtains contributions not only from the square of the velocity of the scalar field as in Einstein Gravity (EG) but also from other terms:

$$\dot{H} = -\frac{1}{2\gamma} \frac{\dot{\sigma}^2}{\sigma^2} + 4H \frac{\dot{\sigma}}{\sigma} + \frac{V_{\text{eff},\sigma}}{\sigma(1+6\gamma)}. \quad (5)$$

It is now useful to introduce the hierarchy of Hubble flow functions [15]: $d \ln |\epsilon_n| / dN \equiv \epsilon_{n+1}$ with $n \geq 0$, $\epsilon_0 \equiv H(t_i)/H$, t_i is some initial time and $N \equiv \ln \frac{a}{a(t_i)}$ is the number of e-folds. In scalar-tensor gravity it is also necessary to introduce another hierarchy associated with σ : $d \ln |\delta_n| / dN \equiv \delta_{n+1}$ with $n \geq 0$, $\delta_0 \equiv \sigma/\sigma(t_i)$. Thus in IG the cosmological perturbations depend on these two hierarchies which completely specify the background evolution. As

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usual, one can replace Eqs. (2), (3) with an equivalent set written in terms of the two hierarchies. However the two hierarchies are not independent and the following relation holds:

$$\epsilon_1 = \frac{\delta_1}{1 + \delta_1} \left(\frac{\delta_1}{2\gamma} + 2\delta_1 + \delta_2 - 1 \right). \quad (6)$$

Inflation in IG has a richer phenomenology than in EG: inflation could occur for large δ_1 and a super-inflationary stage [17] could take place for $|\delta_1| \ll 1$, ($\epsilon_1 = -\dot{H}/H^2 < 0$) in the Jordan frame with $\delta_1 > 0$ and $2\delta_1 + \delta_2 + \delta_1/(2\gamma) < 1$ or vice versa.

Scalar curvature perturbations produced by quantum fluctuations of the inflaton during the accelerated stage are described by $\mathcal{R}(x) = -H\delta\sigma(x)/\dot{\sigma}$ in the uniform curvature gauge [16], where $\delta\sigma(x)$ is the scalar inflaton perturbation and is the correct field variable to quantize. The Fourier component $\delta\sigma_k$ of the inflaton fluctuation, in the IG context, has been shown to satisfy the differential equation [16]:

$$\delta\ddot{\sigma}_k + \left(3H + \frac{\dot{Z}}{Z} \right) \delta\dot{\sigma}_k + \left[\frac{k^2}{a^2} - \frac{1}{a^3 Z \sigma \delta_1} (a^3 Z (\sigma \delta_1) \dot{}) \right] \delta\sigma_k = 0 \quad (7)$$

where

$$Z = \frac{H^2 \sigma^2 (1 + 6\gamma)}{(\dot{\sigma} + H\sigma)^2} = \frac{1 + 6\gamma}{(1 + \delta_1)^2}. \quad (8)$$

Gravitational waves are also amplified from quantum fluctuations during inflation; in IG the Fourier modes of tensor perturbations satisfy:

$$\ddot{h}_{s,k} + (3H + 2H\delta_1)\dot{h}_{s,k} + \frac{k^2}{a^2} h_{s,k} = 0 \quad (9)$$

where $s = +, \times$ denotes the two polarization states. We define the power spectra of scalar curvature perturbations and tensor perturbation as

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} |\mathcal{R}_k|^2 \simeq \mathcal{P}_{\mathcal{R}}(k_*) \left(\frac{k}{k_*} \right)^{n_s - 1} \quad (10)$$

and

$$\mathcal{P}_h(k) \equiv \frac{2k^3}{\pi^2} (|h_{+,k}|^2 + |h_{\times,k}|^2) \simeq \mathcal{P}_h(k_*) \left(\frac{k}{k_*} \right)^{n_t}, \quad (11)$$

respectively, where k_* is a suitable pivot scale. It is important to stress that through a conformal transformation the action in Eq. (1) can be rewritten as EG involving the rescaled scalar field and metric with a different potential: whereas the spectra of primordial cosmological perturbations are conformal invariant, we prefer to work in the original frame in Eq. (1).

As in EG with a standard scalar field with exponential potential, exact inflationary solutions with $a(t) \sim t^p$ (with $t > 0$ and $p > 1$) exist for induced gravity with $V(\sigma) = \lambda_n \sigma^n/n$ [18] and

$$p = \frac{1}{\epsilon_1} = 2 \frac{1 + (n+2)\gamma}{(n-4)(n-2)\gamma}, \quad \sigma(t) = \frac{S}{t^{\frac{2}{(n-2)}}}, \quad (12)$$

with $4 < n < 4 + \sqrt{2(6+1/\gamma)}$ or $4 - \sqrt{2(6+1/\gamma)} < n < 2$ [19]. These are scaling solutions for which $\delta_2 = 0$ and

$$\delta_1 = -\frac{\gamma(n-4)}{1 + \gamma(n+2)}, \quad \epsilon_1 = \frac{\gamma(n-2)(n-4)}{2 + 2\gamma(n+2)}. \quad (13)$$

The poles in $n = 2, 4$ in the above equations correspond to de Sitter solutions having $a(t) \propto e^{Ht}$. The above scaling solution is also found for $a(t) \sim (-t)^p$ (with $t < 0$ and $p < 0$) but only for $2 < n < 4$: this solution characterizes a super-inflationary stage with

$\dot{H} > 0$ and ends up in a future singularity (the Ricci scalar grows with the arrow of time instead of decreasing). Let us note that inflation does not end for the above scaling solutions (just as in EG), but cosmological perturbations can be solved exactly on such backgrounds. The spectral index for scalar curvature perturbations is:

$$n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} = \frac{2\gamma(n-4)^2}{\gamma(n-4)^2 - 2(6\gamma+1)} \quad (14)$$

where $dn_s/d \ln k = dn_t/d \ln k = 0$ and $n_t = n_s - 1$. The exact tensor-to-scalar ratio is given by

$$r = \frac{\mathcal{P}_h(k)}{\mathcal{P}_{\mathcal{R}}(k)} = -\frac{8n_t}{1 - \frac{n_t}{2}} \quad (15)$$

which agrees with the consistency condition of power-law inflation in EG. On examining the behavior of (14) one has

$$n_s - 1 < 0 \quad \text{when } \rho \equiv \frac{\gamma(n-4)^2}{2(1+6\gamma)} < 1 \quad (16)$$

and $n_s - 1$ is close to scale invariance (zero) only for $\rho \sim 0$ and $n_s - 1 > 2$ for $\rho > 1$.

Let us now obtain general formulae for scalar and tensor spectra in the slow-roll regime. As for EG, the potential and its derivatives are given explicitly by the Hubble parameter and the coefficients of the hierarchies (ϵ_i, δ_i). On assuming $\delta_i \ll 1$, we can invert such relations obtaining (we only exhibit the first two):

$$\delta_1 \simeq -\gamma\sigma \frac{V_{\text{eff},\sigma}}{(1+6\gamma)V}, \quad (17)$$

$$\delta_2 \simeq -\gamma\sigma^2 \frac{V_{\text{eff},\sigma\sigma}}{(1+6\gamma)V} + \delta_1 \left(\frac{\delta_1}{\gamma} - 3 \right). \quad (18)$$

From Eq. (17) we immediately see that the first slow-roll parameter in IG is roughly the square root of what we expect from EG. Indeed through a conformal transformation to the Einstein frame one can show that $\epsilon_1^{\text{EF}} = \delta_1^2(1+6\gamma)/(2\gamma(1+\delta_1)^2)$. Further, from Eq. (17), it is easy to see that $\epsilon_1 \simeq -\delta_1$ to lowest order for large γ . The scalar power-spectrum is

$$\mathcal{P}_{\mathcal{R}}(k_*) \simeq \frac{AH_*^2}{4\pi^2(1+6\gamma)\delta_{1*}^2\sigma_*^2} \simeq \frac{AV_*^3(1+6\gamma)}{12\pi^2\gamma^3\sigma_*^6V_{\text{eff}\sigma*}^2} \quad (19)$$

with

$$A = [1 - 2\epsilon_{1*} + C(\delta_{1*} + \delta_{2*} + \epsilon_{1*})], \quad (20)$$

where $C = 2(2 - \ln 2 - b)$, b is the Euler-Mascheroni constant and all the above quantities with the subscript $*$ are evaluated when k_* crosses the Hubble radius. The scalar spectral index in Eq. (10) is

$$n_s - 1 = -2(\delta_{1*} + \delta_{2*} + \epsilon_{1*}) = \frac{2\gamma\sigma_*^2}{1+6\gamma} \left(\frac{V_{\text{eff},\sigma\sigma*}}{V_*} - \frac{3V_{\text{eff},\sigma*}^2}{2V_*^2} - \frac{3V_{\text{eff},\sigma*}}{\sigma_*V_*} \right). \quad (21)$$

Let us note that the above result for n_s agrees with the calculation in the Einstein frame, but does not agree with Ref. [20] where a term of order δ_i is omitted. We also note $V_{\text{eff},\sigma}$ is less than V to the addition of radiative corrections of logarithmic form to the tree potential.

Similarly, the tensor power spectrum is given by:

$$\mathcal{P}_h(k_*) \simeq \frac{2(A - C\delta_{2*})H_*^2}{\pi^2\gamma\sigma_*^2} \simeq \frac{2(A - C\delta_{2*})V_*}{3\pi^2\gamma^2\sigma_*^4} \quad (22)$$

and

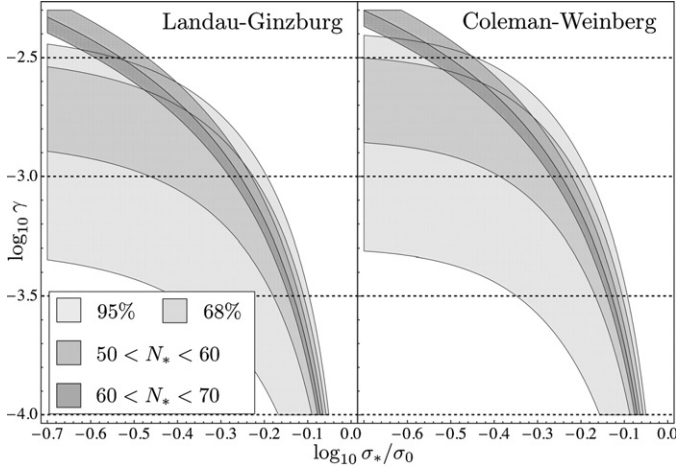


Fig. 1. The WMAP 5 + BAO + SNIa constraints on n_s [14] in absence of gravitational waves with the condition $50 < N_* < 70$.

$$n_t \simeq -2(\delta_{1*} + \epsilon_{1*}) \simeq -\frac{\delta_{1*}^2(1+6\gamma)}{\gamma}. \quad (23)$$

The above results lead to the standard tensor-to-scalar ratio $r_* \simeq -8n_{t*}$. We shall consider the following potentials leading to the spontaneous breaking of scale invariance and Newton's constant:

$$V_{CW} = \frac{\mu}{8}\sigma^4 \left(\ln \frac{\sigma^4}{\sigma_0^4} - 1 \right) + \frac{\mu}{8}\sigma_0^4 \quad (24)$$

where the breaking arises through a quantum effect, Coleman-Weinberg (CW) type, and

$$V_{LG} = \frac{\mu}{4}(\sigma^2 - \sigma_0^2)^2, \quad (25)$$

where the breaking arises through a condensate, Landau-Ginzburg (LG) type. Both large and small field configurations for the potentials (24), (25) can lead to predictions in agreement with observations, for suitable values of γ and the parameters of the potential, μ, σ_0 . In Fig. 1 we plotted the values of (σ^*, γ) in the small field regime and assuming that $k_* = aH$ for 50–70 e-folds, N_* , before inflation ends. Lighter regions represent the observational constraints on the spectral index n_s coming from the WMAP 5 + BAO + SNIa [14] and a 68% and 95% confidence level ($n_s = 0.963 \pm 0.014$ and $n_s = 0.963 \pm 0.028$), darker regions represent the two intervals $50 < N_* < 60$ and $60 < N_* < 70$ (darker area). Such a plot constraints γ to be $\lesssim 3 \times 10^{-3}$ in order to fit observations. We note that small field regime in IG leads to predictions very different from EG. For both CW and LG potentials we obtain $n_s - 1 \simeq -16\gamma/(1+2\gamma)$. For small field potentials as $V(\sigma) \propto 1 - (\sigma/\sigma_0)^n + \dots$ we obtain $n_s - 1 \simeq -16\gamma/(1+6\gamma)$. For large field configurations, we obtain $n_s - 1 \simeq -2/N_*$ ($n_s - 1 \simeq -1.5/N_*$) for LG (CW) which leaves γ unconstrained. The amplitude for scalar perturbations for the LG potential is

$$P_{\mathcal{R}}(k_*) \simeq \begin{cases} \frac{\mu}{3\pi^2\gamma} N_*^2, & \gamma \ll 1 \text{ (large and small field),} \\ \frac{\mu}{72\pi^2\gamma^2} N_*^2, & \gamma \gg 1 \text{ (large field).} \end{cases} \quad (26)$$

WMAP 5 + BAO + SNIa [14] require $P_{\mathcal{R}}(k_*) = (2.445 \pm 0.096) \times 10^{-9}$ [14]: from the above expression we observe that when $\gamma \gg 1$ μ need not be as small as the same self-coupling in EG [10–13]. For instance $\mu \sim 0.5$ is allowed for $\gamma \simeq 3 \times 10^4$; on requiring $\gamma\sigma_0^2 = M_{\text{pl}}^2$, it is clear that in general IG can incorporate the GUT symmetry breaking scale for inflation [10–13].

In Fig. 2 we compared the WMAP 5 + BAO + SNIa constraints at the 68% (dotted contour) and the 95% (continuous contour)

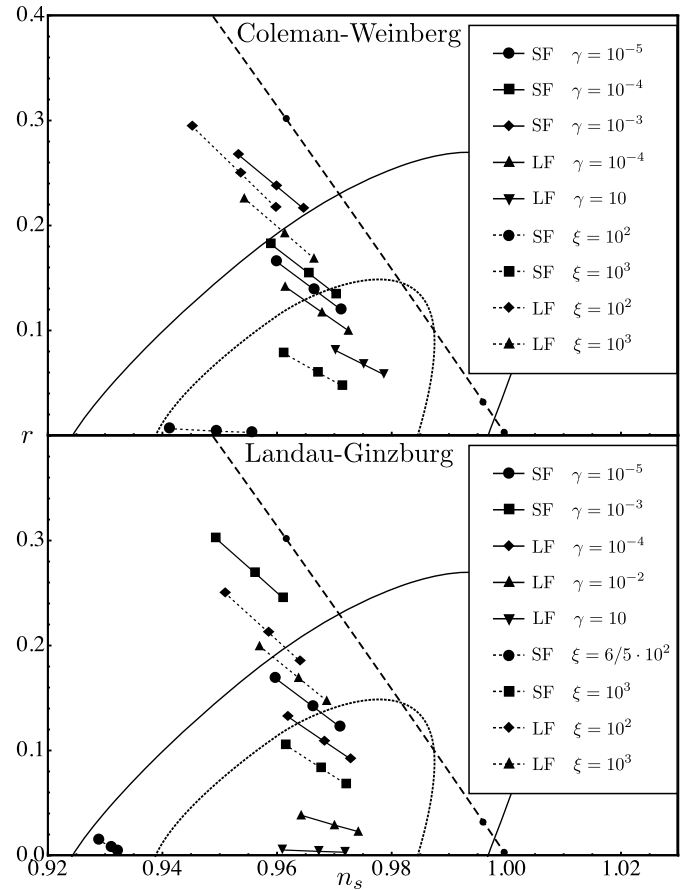


Fig. 2. The WMAP 5 + BAO + SNIa constraints [14] in the (n_s, r) plane. Each segment shows the uncertainty on N_* . The dashed line plots the consistency relation and the points on such a line are for $n=6$ and $\gamma = 10^{-2}, 10^{-3}, 10^{-4}$ from the left to the right.

confidence levels [14] with the prediction for the potentials (24), (25) both in IG (continuous lines) and in EG (dashed lines) framework: each point represents a different choice for γ (or $\xi \equiv \sigma_0^2/M_{\text{pl}}^2$ in EG) and N_* (identical markers, from left to right, identify $N_* = 50, 60, 70$). We observe that in IG for both the above potentials, the small field inflation with $\gamma \geq 10^{-3}$ is disfavored by observations confirming what was found in the n_s analysis. Again large field inflation in IG fits observations independently of γ in contrast with the same regime in EG which requires $\xi \geq 10^3$. It is however worth noting that both the potentials (24), (25), although very similar to a simple quartic potential in this regime, fit observations well, independently of μ and σ_0 , in contrast with such a potential in EG which leads to results lying far away from the 95% region in the same e-folds interval. Let us note that the $\gamma \ll 1$ limit leads to predictions very close to EG with large ξ .

The general slow-roll formula (at leading order) for the tensor-to-scalar ratio is:

$$r \simeq 8 \frac{V_{\text{eff},\sigma_*}^2}{V_*^2} \frac{\gamma \sigma_*^2}{1+6\gamma} = \frac{8M_{\text{pl}}^2}{1+6\gamma} \frac{V_{\text{eff},\sigma_*}^2}{V_*^2} \frac{\sigma_*^2}{\sigma_0^2} \quad (27)$$

where in the second line we have introduced the reduced Planck mass fixed after inflation by $M_{\text{pl}}^2 = \gamma\sigma_0^2$ (which of course does not apply to runaway potentials as the ones leading to exact solutions). Several comments are in order to compare the result in IG with

the standard $r_{EG} = 8M_{\text{pl}}^2 V_{\sigma_*}^2 / V_*^2$ in EG with the same potential V . As a first point, let us note that V_{eff,σ_*} occurs in the numerator instead of the standard first derivative of the potential. Secondly, the factor $(1+6\gamma)^{-1}$ can only decrease r with respect to EG, in particular for $\gamma \gg 1$. Thirdly, the factor σ_*^2 / σ_0^2 is a (de)amplification for (small) large field configurations. The first point has different effects for different potentials. For $V \propto \sigma^4$ in LF configurations, r is drastically reduced. Vice versa, for SF configurations $4V_*/\sigma_*$ can be much larger than V_{σ_*} , thus increasing r . All these effects are the explanation for the behavior observed for r in Fig. 2.

Let us now study the oscillating regime occurring for $\sigma \simeq \sigma_0$ and consider the symmetry-breaking potential in Eq. (25) for the sake of simplicity. We find an approximate analytic solution given by:

$$\sigma(t) = \sigma_0 + \frac{2}{t} \sqrt{\frac{\gamma}{3\mu}} \sin(\omega t) + \mathcal{O}\left(\frac{1}{t^2}\right), \quad (28)$$

$$H(t) \simeq \frac{2}{3t} \left[1 - \sqrt{\frac{6\gamma}{1+6\gamma}} \cos(\omega t) \right] + \mathcal{O}\left(\frac{1}{t^2}\right), \quad (29)$$

where $\omega = \sigma_0 \sqrt{2\mu/(1+6\gamma)}$. On fixing the Planck mass after inflation by $M_{\text{pl}}^2 = \gamma \sigma_0^2$ and requiring curvature perturbations in agreement with observations, the frequency of oscillations ω is $\simeq 2.5 \times 10^{13}$ GeV for $\gamma \ll 1$ (and twice as large for $\gamma \gg 1$). For the regime of small oscillations this study is equivalent to a mass term obtained from the oscillations of σ around a minimum of a potential with $V(\sigma_0) = 0$. Oscillations in the Hubble parameter appear to leading order in Eq. (29) and not to $\mathcal{O}(t^{-2})$ as in the regime of coherent oscillations for a massive field in EG; as a consequence $\dot{H} + 3H^2/2$ is not approximatively zero when averaging through oscillation as in EG [21], i.e. we do not have a standard matter dominated stage during the coherent oscillation regime. Indeed, on considering the IG system in Eqs. (2)–(3) as EG with $M_{\text{pl}}^2 = \gamma \sigma_0^2$, we obtain $w_{\text{eff}} = -3\gamma/(1+9\gamma)$ as equation of state for the effective matter component [8,22].

Eq. (29) can be used to study the evolution of $\delta\sigma$ during the coherent oscillations of σ : on using Eq. (7) the variable $\delta\tilde{\sigma}_k = \sqrt{a^3 Z} \delta\sigma_k$ can be recast in a Mathieu-like form:

$$\frac{d^2 \delta\tilde{\sigma}_k}{d(\omega t)^2} + [A(t) + 2q_1(t) \sin(2\omega t) + 2q_2(t) \sin(\omega t)] \delta\tilde{\sigma}_k = 0 \quad (30)$$

analogously to the EG case [23]. The leading terms in q_1 and q_2 originate from the metric and the potential contributions respectively, and both decay as $1/t$, further $A(t) = k^2/(a^2 \omega^2) + 1 + \mathcal{O}(t^{-2})$. Because of the two oscillating terms the time dependent frequency in Eq. (30) leads to beats. We obtain $q_2 = \sqrt{27\gamma/(2(1+6\gamma))}/(\omega t)$ and $q_1 = 2/(\omega t)$ (the latter is the same as the EG case [23]).

The consequence of coherent oscillations on $\delta\tilde{\sigma}_k$ differs from the resonance for a test scalar field χ coupled to an inflaton through $g^2 \sigma^2 \chi^2$ [24]: for $V(\sigma) = m^2 \sigma^2/2$, $q_2(t) \sim \mathcal{O}(t^{-2})$ decays faster than k^2/a^2 (as a consequence the resonance shuts-off asymptotically for large times although it started in the broad regime), whereas for an inflaton potential as in Eqs. (24), (25), $q_2(t) \sim \mathcal{O}(1/t)$ and χ_k would end in the second resonance band

(this would hold for the inflaton fluctuations in absence of metric perturbations as well). According to Eq. (30), instead, all wavelengths end asymptotically in the first resonance band since $q_1(t) \neq 0$. Thus for short-scale modes with $H \ll k/a \ll \omega$, $\delta\sigma_k$ oscillates with a constant amplitude instead of decaying [22]. Note that such a gravity mediated self-decay of the inflaton in its short-scale fluctuations also exists for $V(\phi) = m^2 \phi^2/2$ or for a potential with a symmetry-breaking term in EG [25]. Although oscillating terms decaying as $1/t$ appear in the equation for gravitational waves – on rewriting Eq. (9) in terms of the variable $a^{3/2} \sigma h_k$ – such terms do not affect the standard behavior of short-scale gravitational waves.

It is also interesting to study a possible connection of this inflationary stage with the problem of dark energy, which can also be modeled in IG through simple potentials [7,8]. Work in this direction is in progress [22].

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