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On Evaluating Traffic Lights Performance Sensitivity via Hybrid Systems Models

Angela Di Febbraro[a], Nicola Sacco[a,*]

[a]DIME - University of Genoa, Via Montallegro 1, Genoa 16145, Italy

Abstract

The problem of optimizing traffic light phases dates back to the fifties. Since there, many solutions for different network configurations (isolated intersections, coordinated intersections, and so on) and different modeling and solution approaches (empirical models, queue theory approaches, mathematical programming models, etc.) have been proposed.
In parallel, it has been developed the general theory of hybrid systems, i.e., of those systems characterized by two kinds of states: *normal states* whose variation is governed by a fixed set of equations, and *macro states* whose change is governed by the occurrence of particular conditions or external events.
In this paper, a hybrid model of traffic light dynamics is introduced aiming at providing a modelling framework for evaluating the sensitivity of the performances of different approaches for signal setting optimal design.

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Keywords: Signalized intersections; Hybrid Systems; Sensitivity Analysis

1. Introduction

The problem of optimizing traffic light phases dates back to the fifties (Webster, 1958) and, since there, many solutions for different network configurations (isolated intersections, coordinated intersections, and so on) and different solution approaches (empirical models, queue theory approaches, mathematical programming models, etc.) have been proposed.

In this framework, different solutions and strategies (see Papageorgiou *et al.*, 2003, and the reference therein) have been proposed for large scale networks, such as TRANSYT (Robertson, 1969), and SCOOT (Hunt *et al.*, 1982), which were characterized by limited traffic-responsive capabilities, or OPAC (Gartner, 1983), PRODYN (Farges *et al.*, 1983), and RHODES (Mirchandani and Head, 2001) which, on the contrary, implemented traffic-response strategies.

* Corresponding author. Tel.: +39-010-353-2532; fax: +39-010-353-2555.
E-mail address: nicola.sacco@unige.it

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Nevertheless, in the common practice, the design approaches proposed by Webster (1958), Allsop (1971 and 1976) and (Improta and Cantarella, 1984) for isolated intersections, are still widely used. Despite the clearness and analytical rigorousness of such approaches, they neglect the dynamics of intersection and flows and, in particular, assume the incoming flows to be known and constant for each design reference period.

In parallel, it has been developed the general theory of *hybrid systems*, i.e., of those systems characterized by two kinds of states: *normal states* whose variation is governed by a fixed set of differential equations, and *macro states* whose change is driven by the occurrence of particular conditions or external events (Antsaklis, 1998a and 1998b). For such a class of systems, any macro state transition generally provoke a change of the set of equations that drive the normal state dynamics. Moreover, such systems characterized by two kinds of equilibrium states: those relevant to the normal states and those relevant to the macro states.

In this framework, urban transportation networks and intersections, together with the relevant traffic lights, can be suitably modeled as hybrid systems with macro-states characterized by the different flows, tragic light signals, or even exogenous variables, such as accidents that change the capacity of roads and intersections (Basile et al., 2004; Di Febbraro and Sacco, 2004; Kim et al., 2008; Basile et al., 2012),.

In this paper, a hybrid model of traffic light dynamics is presented with the aim of providing a framework for evaluating the sensitivity of the performances of different common approaches for the optimal signal setting design. In particular, the methodologies for the computation of the optimal signal settings proposed by Allsop (1976) and Improta and Cantarella (1984) are evaluated by (I) determining the optimal signal settings assuming the incoming flows are fixed, and (II) by evaluating the performance of the above methodologies when the incoming flows are considered stochastic variables with given distribution.

To cope with the problem considered in this paper, traffic lights and the relevant queue at the accesses are represented as a hybrid system, being the queues length the normal state variables.

The main result of the paper is a major comprehension of the intersection and traffic light dynamics, with particular attention to the performance sensitivity of different, well-known, traffic light settings optimization approaches.

The results of the proposed analysis can help in forecasting and preventing queue instability at intersections, helping also in designing better traffic light control systems. In addition, the proposed model is sufficiently general to be applied to other design approaches, and to allow the statement of a sensitivity minimization problem, as will be discussed in the end of the paper.

The paper is organized as follows: after briefly recalling some basic aspects of hybrid systems, the proposed model is described. Then, by means of a real world case study consisting of an intersection in the Italian city of Benevento, the sensitivity analysis of the considered design approaches is performed. Finally, some considerations about the definition of a sensitivity minimization problem are provided.

2. Basics on Hybrid Systems

In the last decades, the attention of many researchers has been focused on Hybrid Systems (HSs) that can be thought of as the most general systems gathering ordinary Time Driven Systems (TDSs), and Discrete Event Systems (DESs).

In this framework, on one hand, TDSs are the well-known systems whose state variables assume real numeric values and whose dynamics is described by differential equations. The dynamics of flows along road stretches is a valuable example of TDSs.

On the other hand, DESs can be intuitively defined as *discrete-state event-driven systems* where the evolution of the state variables, which are not necessarily numeric, depends entirely on the occurrence of asynchronous, often stochastically predictable, events. In such a definition, an event can be, informally, thought of as *something instantaneous* whose occurrence causes a transition from a state to another state of the system. In DESs, an event can be identified with a specific action taken (for instance, somebody presses a button), with a spontaneous occurrence dictated by nature (for instance, a computer goes down for whatever reason too difficult to figure out) or, finally, with a result of several conditions which are suddenly all met (for instance, the fluid in a tank exceeds a given value).

More formally, a DES is a dynamic system characterized by a set E of feasible events, a discrete (sometimes not numeric) state space X, and an event-driven evolution, described by the equation $x_{k+1} = \delta(x_k, e_k)$, where x_{k+1} is the state after the occurrence of the event e_k, that is the k^{th} occurred event from the initial time instant, and $\delta : X \times E \to X$ is the so-called state transition function.

Traffic lights are valuable examples of DESs. In effects, such systems are characterized by a discrete and non-numeric state space $X = \{green, amber, red\}$, whose elements correspond to the different "lights" at each access of the signalized intersection. Moreover, the transitions among these states is instantaneous and are, in general, asynchronous, especially when traffic lights perform a responsive plan, i.e., when the length of the phases are changed continuously to optimize the traffic behavior.

In conclusion, a HS can be defined as a set of different time-driven dynamics, each represented by a set of differential equations associated with each macro-state: when the macro-state change due to the occurrence of an event, the time-driven dynamics has to be represented by a different set of equations. In the following, the traffic light dynamics will be represented via a HS model.

3. Combined Traffic Light and Queue Model

In this section, the model of the hybrid dynamics of a generic intersection is introduced. In particular, after describing the single access dynamics, a generic intersection is considered. It is worth noting that, for the sake of simplicity, in this framework, only the events that change the traffic lights colors are considered. Nevertheless, the model can be easily extended to consider other kind of events, such as accidents, sudden unavailability of some roads, and in general all the conditions in which the road capacity is reduced.

3.1. Hybrid Model of a Single Access

Let $q_i(t)$ be the *continuous approximation* of the number of vehicles at time t, at a generic access i of an intersection, during the generic phase k of the traffic light. Its dynamics is modeled by the equation

$$\dot{q}_i(t) = \begin{cases} f_i - s_i & \text{if } i \text{ is enabled in the } k^{th} \text{ phase of the traffic light and } q_i(t) > 0 \\ 0 & \text{if } i \text{ is enabled in the } k^{th} \text{ phase of the traffic light and } q_i(t) = 0 \\ f_i & \text{if } i \text{ is not enabled in the } k^{th} \text{ phase of the traffic light} \end{cases} \quad (1)$$

where f_i and s_i are the incoming flow and the saturation flow of the access i, respectively. Note that Eq. (1) can be split into the three macro-states x_1, x_2, and x_3, each characterized by its own peculiar time-driven dynamics. Then, it is possible to draw the state diagram in the left of Figure 1, where the events that drive the state dynamics have the following meanings:

- e_1 is the event representing the reaching of the condition $q_i(t) = 0$, when the access i is enabled;
- e_2 is the green-to-red switch event of the signal for access i, when the queue is empty, that is $q_i(t) = 0$;
- e_3 and e_4 are the green-to-red and red-to-green switching events of the signal for access i, when the queue is not empty, that is $q_i(t) > 0$, respectively.

Note that not all these events are feasible in all the states: in particular, the event e_1 can occur only when the access i is enabled, whereas there are no events that drive the system directly from the state x_3 to the state x_2.

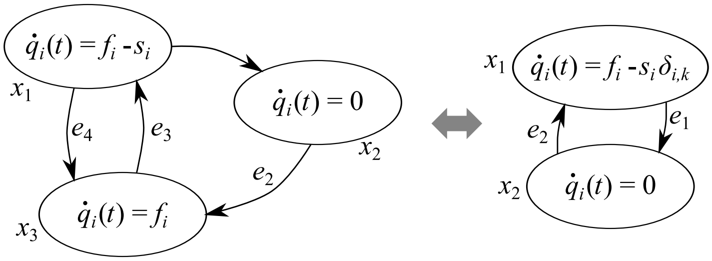

Fig. 1. State diagram with time-driven dynamics of a single access.

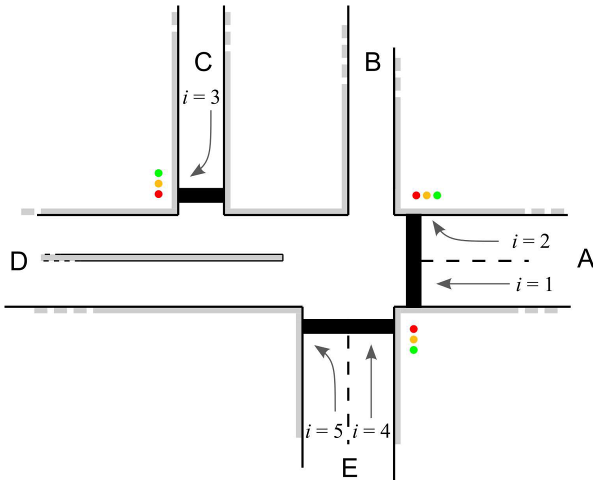

Fig. 2. A real intersection in Benevento (Italy).

In addition, events e_2, e_3 and e_4 occur at fixed instants, determined by the signal plan of the considered intersection, whereas event e_1 occurs at a time depending on the values of $q_i(t)$, f_i and s_i.

Finally, note that with this representation Eq. (1) is divided into three differential equations, each associated with a single macro-state of the diagram in the left side of Figure 1. By the way, for the sake of compactness, it is possible to rewrite Eq. (1) as

$$\dot{q}_i(t) = \begin{cases} f_i - s_i\delta_{i,k} & \text{if } q_i(t) > 0 \\ 0 & \text{if } q_i(t) = 0 \end{cases} \tag{2}$$

where $\delta_{i,k}$ is an element of the so-called phase matrix Δ, and assume the value 1 if the access i is enabled in the phase k, and 0 otherwise. By means of the Eq. (2), the state diagram of the single generic access simply consists of the single state in the right side of Figure 1.

In the following section the dynamics of a complete intersection will be described, generalizing the model above described.

3.2. Hybrid Model of an Intersection

Consider the intersection depicted in Figure 2 and the relevant traffic flows in Table 1. The relevant traffic

Tab. 1. Traffic demand for the intersection in Figure 2.

O/D pair	A/B	A/D	C/D	E/B	E/D
Index	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
f_i (veh/hour)	127	142	13	391	440
s_i (veh/hour)	1200	1200	1200	1200	1200

light plan has been computed via the SIGSET optimization problem (Allsop, 1976), both considering the cycle time as a fixed parameter and as a control variable, and via the Phases Length and Scheduling (PLS) optimization problem described in Improta and Cantarella (1984).

Then, the relevant phase matrices are

$$\Delta_{\text{fix}} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \Delta_{\text{opt}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}. \tag{3}$$

In particular, Δ_{fix} is the fixed matrix assumed in the implementation of the SIGSET optimization problem, and Δ_{opt} is the optimal one determined by means of the PLS problem.

Then, for the sake of simplicity and without losing generality, consider the two-phase traffic light dynamics determined by the phase matrix Δ_{fix} . In this case, it is possible to generalize Eq. (2) and write the set of equations describing the whole intersection dynamics as

$$\begin{cases} \dot{q}_1(t) = \begin{cases} f_1 - s_1\delta_{1,k} & \text{if } q_1(t) > 0 \\ 0 & \text{if } q_1(t) = 0 \end{cases} \\ \dot{q}_2(t) = \begin{cases} f_2 - s_2\delta_{2,k} & \text{if } q_2(t) > 0 \\ 0 & \text{if } q_2(t) = 0 \end{cases} \\ \dot{q}_3(t) = \begin{cases} f_3 - s_3\delta_{3,k} & \text{if } q_3(t) > 0 \\ 0 & \text{if } q_3(t) = 0 \end{cases} \\ \dot{q}_4(t) = \begin{cases} f_4 - s_4\delta_{4,k} & \text{if } q_4(t) > 0 \\ 0 & \text{if } q_4(t) = 0 \end{cases} \\ \dot{q}_5(t) = \begin{cases} f_5 - s_5\delta_{5,k} & \text{if } q_5(t) > 0 \\ 0 & \text{if } q_5(t) = 0 \end{cases} \end{cases} \tag{4}$$

Although each of the equations in Eq. (4) corresponds to the two macro-state diagram in Figure 1 (right side), the complete state diagram results to be much more complex, since two or more enabled access can reach the condition $q_i(t) = 0$, if together enabled. Nevertheless, since a queue can empty only if it is enabled, with reference to the phase matrix Δ_{fix} it descends that:

1. only the queue at the accesses 1 and 2 can empty during the first phase;

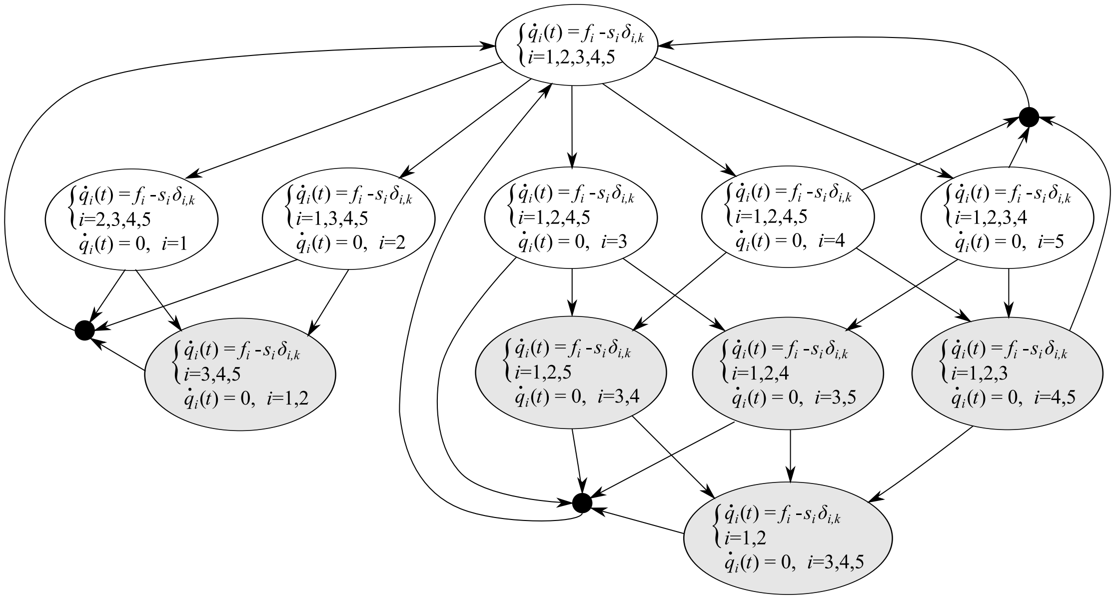

Fig. 3. Complete State Diagram of the intersection in Figure 2 with the phase matrix Δ_{fix} .

2. only the queue at the accesses 3, 4 and 5 can empty during the second phase;

Then, the complete state diagram is depicted in Figure 3, where the grey states represent the *new* necessary states representing the conditions in which two queues are emptied at the same time, and the black circles are simple connectors added with the aim of simplifying the diagram layout.

It is worth noting that the state diagram change significantly when the phase matrix Δ_{opt} is considered, since there are five phases and several queues are enabled in more than a phase. Although drawing such a diagrams is easy, for the sake of compactness such a diagram will be not reported in this paper.

In the following sections, the Matlab® implementation of the proposed intersection model will be used for a sensitivity evaluation of the performances of some of the most common traffic light optimization procedures.

4. Sensitivity Analysis

In this section, an interesting application of the above model will be described. In particular, the sensitivity analysis of the performances provided by different solution approaches for the traffic light phase plan optimization will be performed.

In effects, most of the well-known optimization procedures assume that the O/D flows between the incoming and outgoing road stretches are fixed and known in the reference design period. Nevertheless, in practice, traffic flows are not perfectly known and vary, determining significant differences between the optimized expected performances and the real ones. Therefore, in this section, after a brief discussion on the considered input data, the following cases will be analyzed:

1. SIGCAP (hereafter indicated with the index $j = 1$), considering fixed cycle time, thus optimizing only the phase lengths;
2. SIGCAP (hereafter indicated with the index $j = 2$), considering also the cycle time as a variable to optimize;
3. PLS (hereafter indicated with the index $j = 3$), considering the phase matrix, the phase lengths, and the cycle time as variables to optimize.

In particular, the considered procedure consists of the following steps:

1. given the reference values reported in Table 1, the optimal sequence and lengths of the phases are computed;
2. a set of H incoming flow samples are stochastically generated, as described in the following section 4.1;
3. for each sample, the relevant real intersection capacity ξ_h^j is computed applying the optimal plan obtained by means of the j^{th} methodology to the HS model Introduced in Sec. 3. In this framework, the capacity sample is defined as the ratio

$$\xi_h^j = \frac{N_{out,h}^j}{N_{in,h}^j}, \quad h = 1,2,\cdots,H, \quad j = 1,2,3, \tag{5}$$

   where $N_{out,h}^j$ (resp., $N_{in,h}^j$) is the number of vehicles that cross (resp., arrive at) the intersection, during one simulated hour, considering the h^{th} flow sample and the j^{th} design methodology;
4. the sample mean capacities and the sample variances

$$\hat{\xi}^j = \frac{1}{H}\sum_{h=1}^{H}\xi_h^j \quad \text{and} \quad S_j^2 = \frac{1}{H-1}\sum_{h=1}^{H}(\xi_h^j - \hat{\xi}^j)^2, \quad j = 1,2,3 \tag{6}$$

   are computed and the resulting output data are compared and discussed.

To conclude, it is worth saying that, although the statement of the capacity optimization problem considering the stochastic nature of the incoming flows is beyond the aim of the present paper, all the above design methodologies problem should be modified by introducing the cost function

$$\max E\left[\xi^j\right] \tag{7}$$

where $E\left[\xi^j\right]$ is the expectation of the capacity ξ^j, $j = 1,2,3,$ that results to be a stochastic variable. Work is in progress to tackle with this problem.

4.1. Case Study and Input data

The considered intersection, depicted in Figure 2, consists of a real world intersection in the Italian city of Benevento. The relevant nominal incoming flows have been determined by means of measures and of an assignment procedure.

As regards the input data, traffic flows arriving at the incoming direction i, $i = 1,2,3,4,5,$ (and entering the relevant queue) are assumed to be samples of the a normal stochastic variable with expectations equal to the reference values f_i used for the phases optimization, and variances σ_i corresponding to the 5% of such reference values. The histograms of the $H = 1000$ generated samples are reported in Figure 4 for each incoming flow.

4.2. Output Data Analysis

For what concerns the output data, consider the diagrams reported in Figure 5, where the samples and the relevant histograms are depicted. In these diagrams it is possible to observe that the PLS approach provides better results than both the SIGCAP methods. Such a result indicates that the considered phase matrix is far from the optimality.

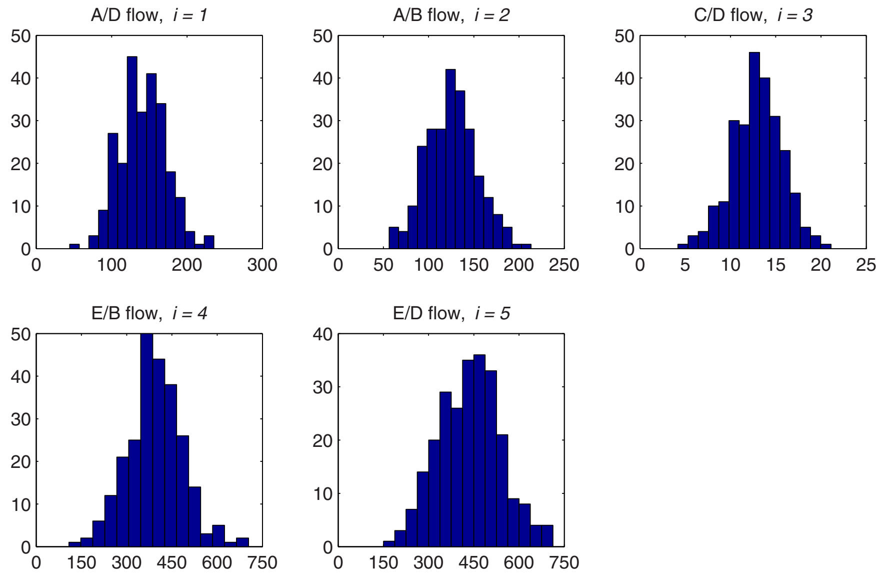

Fig. 4. Incoming flows distribution.

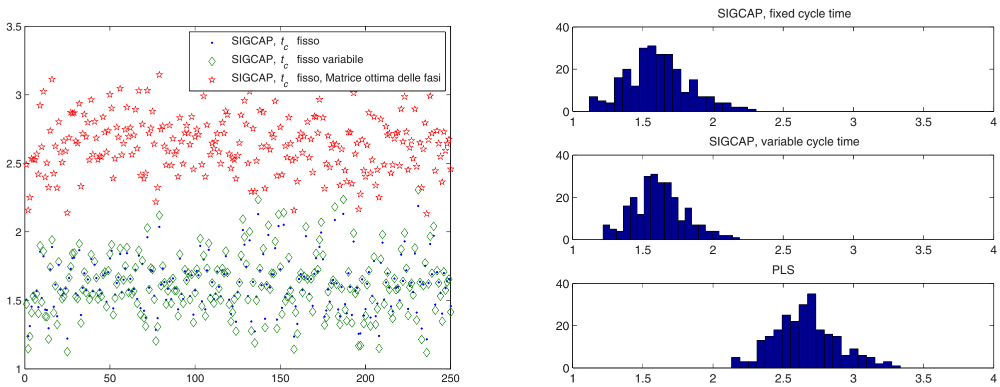

Fig. 5. Sample plot and histograms of the intersection capacity, computed for each of the generated sample.

Moreover, it is easy to note that the sample dispersion around the mean value is almost the same in the three cases. Such results are confirmed by the computation of the relevant variances, which results to be:

- SIGCAP, considering fixed cycle time → $\hat{\xi}^1 = 1.55$, $S_1^2 = 0.24$;
- SIGCAP, considering the cycle time as a variable to optimize → $\hat{\xi}^2 = 1.57$, $S_2^2 = 0.21$;
- PLS → $\hat{\xi}^3 = 2.53$, $S_3^2 = 0.22$.

Nevertheless, the relative variance, defined as the ratio of between the variance and the mean, respectively

0.15, 0.13, and 0.087, shows that the PLS optimization approach provide more robust and stable results.

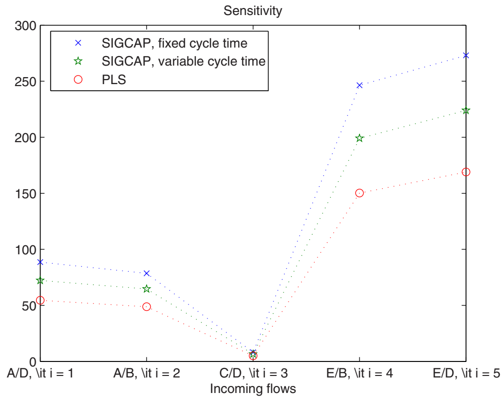

Fig. 6. Sensitivity indexes for all the accesses and for all the considered methodologies.

A second analysis consists of the computation of the amplification (>1) or attenuation (<1) factors of the relative variability of the intersection capacity determined by all the considered methodologies, with respect to the relative variability of each single incoming flow i , $i = 1,2,3,4,5,$ by means of the simple sensitivity indexes

$$\eta_i^j = \frac{S_j / \hat{\xi}^j}{\sigma_i / f_i} = \frac{S_j}{\sigma_i} \frac{f_i}{\hat{\xi}^j}, \qquad j = 1,2,3, \qquad i = 1,2,3,4,5, \tag{8}$$

where S_j is the standard deviation of the capacity determined by applying the timing computed via the j^{th} methodology, and defined in Eq. (6), whereas σ_i represents the standard deviation of the incoming flow at the access i .,

Finally, Figure 6 shows the plots of the sensitivity indexes η_i^j for the different considered methodologies. In such a graph, it is easy to observe that:

- all the sensitivity indexes η_i^j are greater than one, that is, the relative variability of all the incoming flows is amplified with all the considered methodologies;
- the shapes of the graphs are similar for all the optimization approaches, although the PLS approach results to be the best. Such a result was expected, since all the methodologies show the almost the same sample variance whereas PLS is characterized by the highest sample mean;
- the access $i = 6$, has the greatest sensitivity index in all the considered configurations;
- the access $i = 3$, has the smallest sensitivity index in all the considered configurations;
- the accesses $i = 1$ and $i = 2$ have almost the same sensitivity indexes in all the considered configurations.

Such results confirm those above discussed, and demonstrate that, for the considered intersection, the PLS optimization approach results to be not only the most performing, but is also characterized by the high robustness of the results.

Conclusions

In this paper a traffic light model, able to represent both the time-driven dynamics of queues and the event-driven dynamics of traffic light has been introduced. Then, the proposed model has used for evaluating, via simulation, the performances and the robustness of the results of three well-known optimization approaches. To this aim, a real world intersection of the city of Benevento (Italy) has been considered.

The results of the described sensitivity analysis, which shows that the PLS optimization approach provides the best results in terms of both performance and robustness.

Therefore, it is worth underlining that the proposed model can be used also for simulating the intersection behavior when particular kinds of events occurs (such as, for instance, the incoming flow changes that cannot be represented as normal variability, the saturation flow reduction due to an accident, and so on).

Work is in progress to extend the model capabilities, to make the sensitivity analysis more reliable by means of the introduction of extended indexes, and to state suitable stochastic optimization problems.

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