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LETTER TO THE EDITOR

Newton's *Principia* is a little-known but often quoted work. We should welcome any paper which helps us in deepening our understanding of Newton's masterpiece. Unfortunately, Robert Weinstock's recent paper in *Historia Mathematica* devoted to the treatise's Proposition 15, Section 4, Book 2 is marred by a mistaken translation from the Latin [4].

In Proposition 15, Book 2, Newton stated that—under certain assumptions concerning resistance, density, and inverse-square centripetal force—a body can move along an equiangular spiral. In the statement and in the proof, Newton wrote that given such assumptions "the body can revolve in an [equiangular] spiral." The Latin is "corpus gyrari potest in spirali." Unfortunately, the author translates "potest" as "must or will" [4, 282–283] and so interprets Newton as meaning that the assumptions of Proposition 15 necessarily imply an equiangular spiral trajectory. Of course, this cannot be the case. Once Newton's text is translated according to Weinstock's interpretation, a logical error is easily detected.

Weinstock concludes the paper briefly considering an alternative—and correct translation—according to which Newton wished to prove that an equiangular spiral is a possible trajectory. This might have been the beginning of a more appropriate analysis of Newton's proof, but in these final pages [4, 285–288], Weinstock devotes his efforts to criticizing modern commentators, such as Subrahamyan Chandrasekhar [1] and Herman Erlichson [2].

Newton wanted to show that an equiangular spiral centered at $r = 0$ is a possible trajectory for a body accelerated by a centripetal inverse-square force F with force center at $r = 0$ and by a tangential resistive force R proportional to density times the square of velocity, in a medium whose density varies inversely with the distance from the centripetal force's center. In his demonstration, he began by considering an equiangular spiral and showed that this trajectory is compatible with the above-mentioned dynamic conditions. Newton's analysis implied that the velocity must have a magnitude proportional to $1/\sqrt{r}$ maintained for all times, and that $R/F = (\cos\alpha)/2$ (α is the angle between radius vector and tangent).

If I understand Weinstock correctly Newton's procedure would be invalid—even when Proposition 15 is reconstructed according to a correct translation from Latin—because "one must never, as part of a proof, assume and make use of a statement that one intends to arrive at as a conclusion; introduction and use of such a statement—tantamount to assuming what one seeks to prove—renders a purported proof fallacious" [4, 284]. Since Newton opened by considering an equiangular spiral trajectory, he would have been assuming what he intended to prove. However, Newton's procedure seems to me analogous to the familiar strategy adopted nowadays for checking that a given function y is a solution of a differential equation by direct substitution of y into the differential equation.

But leaving these technical matters aside, I would like to add that we should approach the *Principia* from a more historically motivated point of view. In fact, it is only by taking into

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consideration the mathematical procedures accepted in the late 17th century, as well as other cultural factors, that we can hope to avoid misunderstandings of Newton's demonstrative techniques.

I believe that our historical understanding of the logic of Proposition 15 might improve if we consider the distinction, to which Newton frequently referred, between analytical and synthetic proof methods. In antiquity, the analytical method—or more briefly analysis—was conceived of as a method of discovery or problem solving, which, working backwards step-by-step from what is sought (as if it had already been achieved), eventually arrives at what is known. Many of Newton's proofs have a structure that was quite familiar to 17th-century mathematicians: these proofs are examples of geometrical analysis extended to dynamics. In these problems, Newton began with a geometric construction (in this case, an equiangular spiral) as given and then showed that this construction satisfies certain conditions (in this case, certain assumptions on centripetal force, resistive force, and medium density).

In some problems in dynamics, Newton was able to provide not only a geometric solution, somewhat reminiscent of ancient demonstrative techniques, but also a calculus-based solution that would be much more familiar to 20th-century readers. A close reading of the *Principia* reveals that Newton did, in some cases, employ the calculus. Moreover, he sometimes gave incomplete proofs, or no proofs whatsoever, of his statements. To give a well-known example, in the Scholium to Proposition 34, Book 2, Newton gave the geometric properties of the solid of least resistance, but he did not give the reader any hint as to how he arrived at these properties. The fact that there are such gaps in the demonstrative structure of the *Principia* does not, however, imply that Newton did not have a demonstration (recall that a fluxional proof of the Scholium to Proposition 34 was found in Newton's manuscripts). It should be noted that Newton might have used the calculus in a preliminary analysis to Proposition 15, but then restated his proof in geometric fashion. He hinted at this possibility at the end of Section 4, Book 2, where he stated that "Methodum vero tractandi haec problemata aperui in hujus propositione decima, & lemmate secundo [I have disclosed the method of dealing with these problems in the tenth proposition and in the second lemma of this book]." In Proposition 10, Book 2, Newton used Taylor series in studying motion in resisting media, and in Lemma 2, Book 2, he gave a brief exposition of the rules for differential ("fluxional") calculus. The fact that Newton used the calculus in proving some propositions of the *Principia* becomes evident if his manuscripts and letters, rather than merely his published works, are considered. This private side of Newton's mathematical activity reveals that the geometric style of the *Principia* was often a conscious choice, a choice motivated by a whole spectrum of cultural factors: the audience Newton had in mind, the anti-Cartesian position he wished to hold publicly, his antimodernism rooted in a belief in the *prisca sapientia* of the ancients, his rejection of symbolism devoid of referential content.

I hope that future discussion of Newton's *Principia* will take into consideration the above-mentioned cultural issues (see, for example, [3]).

REFERENCES

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