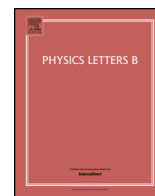


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## Evading Lyth bound in models of quintessential inflation

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## ABSTRACT

Quintessential inflation refers to an attempt to unify inflation and late-time cosmic acceleration using a single scalar field. In this letter we consider two different classes of quintessential inflation, one of which is based upon a Lagrangian with non-canonical kinetic term  $k^2(\phi)\partial^\mu\phi\partial_\mu\phi$  and a steep exponential potential while the second class uses the concept of steep brane world inflation. We show that in both cases the Lyth bound can be evaded, despite the large tensor-to-scalar ratio of perturbations. The post-inflationary dynamics is consistent with nucleosynthesis constraint in these cases.

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## 1. Introduction

It is remarkable that inflationary paradigm may be directly tested using relic gravity waves that are quantum mechanically generated during inflation. The primordial tensor perturbations leave its important imprints on cosmic microwave background radiation. The recent measurements on B-mode polarizations reveal that tensor-to-scalar ratio of perturbations is large and inflation took place around the GUT scale [1]. The large ratio of perturbations, namely,  $r \gtrsim 0.1$  implies that in case of a single field model, the field range over which inflation took place satisfies the Lyth bound  $-\delta\phi \gtrsim 5M_{\text{Pl}}$  [2–15]. This means that inflaton potential should not be very flat allowing a large excursion of field during inflation. It clearly rules out small field models with  $\delta\phi < M_{\text{Pl}}$  [12] in the standard framework. The large field inflation in the super-Planckian region throws a big challenge for model building in effective field theoretic context.

Models of quintessential inflation are of particular interest in cosmology [16–26]. These models aim to unify inflation with late time cosmic acceleration using a single field. Models of quintessential inflation broadly fall into two categories depending upon the behavior of their potentials during inflation: (1) models with shallow potentials at early times and steep thereafter and (2) those in which inflaton potentials are steep throughout the history of universe but turn shallow only at late times. In the first case, one

needs an extra mechanism to invoke late time features in the potential.

The models of class (1): Inflation in this case can be implemented by field Lagrangian with non-canonical kinetic term and a steep exponential potential [26–31]. In second case, inflation can take place by invoking the extra brane damping [17–21,32,33]. As the field rolls down its potential and the high energy brane corrections to Einstein equations on the brane [34] disappear, and graceful exit from inflation takes place.

In both the cases, steep behavior after inflation is required for the commencement of radiative regime. It is possible to find a class of models for non-canonical case which might give rise to large value of  $r$  consistent with BICEP2 [31] (see Refs. [35–73] on the related theme).

In this letter we shall examine the Lyth bound in case of models of quintessential inflation of both the aforesaid categories and check whether the bound can be evaded in compliance with the nucleosynthesis constraint.

## 2. Lyth bound

In the following discussion we shall first review the Lyth bound in the standard case and then we will examine it for non-canonical fields and models of steep braneworld inflation. The bound indeed gets modified in these cases and the influence of post inflationary dynamics on it is crucial.

## 2.1. Canonical scalar field

In case of single canonical scalar field model, the slow-roll parameters are defined as

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$$\epsilon = \frac{M_{\text{Pl}}^2}{2} \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2, \quad (1)$$

$$\eta = \frac{M_{\text{Pl}}^2}{V} \frac{d^2V}{d\phi^2}, \quad (2)$$

where  $\phi$  is the canonical scalar field. In this case, the number of e-folds is given by

$$\mathcal{N} = \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{V(\phi)}{V'(\phi)} d\phi \equiv \frac{1}{M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{d\phi}{\sqrt{2\epsilon}}, \quad (3)$$

which along with the field range of inflation is related to the slow-roll parameter in the following way:

$$\mathcal{N} \lesssim \frac{|\phi_{\text{in}} - \phi_{\text{end}}|}{M_{\text{Pl}} \sqrt{2\epsilon_{\text{min}}}}. \quad (4)$$

Assuming then the monotonous behavior of the slow-roll parameters, we have  $\epsilon_{\text{min}} \approx \epsilon_{\text{in}}$ , where  $\epsilon_{\text{in}}$  is the value of  $\epsilon$  where inflation commences. Hence, using the consistency relation  $r_* = 16\epsilon_{\text{in}}$ , where  $r_*$  is the tensor-to-scalar ratio at the commencement of inflation, provides us the Lyth bound,

$$\delta\phi \equiv |\phi_{\text{in}} - \phi_{\text{end}}| \gtrsim \mathcal{N} M_{\text{Pl}} \left( \frac{r_*}{8} \right)^{1/2}, \quad (5)$$

which implies that in the case  $r_* \gtrsim 0.1$  and  $\mathcal{N} = 50$ , the field range of inflation is given by  $\delta\phi \gtrsim 5M_{\text{Pl}}$  [12]. This super-Planckian field range of inflation is of great theoretical concern.

The above quoted result is valid for single-field inflation models with a canonical kinetic term. However, the Lyth bound is modified in case of a single-field inflation with non-canonical kinetic term or canonical kinetic term with brane corrections. In the following subsections we shall examine the Lyth bound in these two backgrounds.

## 2.2. Non-canonical scalar field

Let us consider a non-canonical scalar field  $\phi$ , which can be transformed to canonical form under the transformation

$$\left( \frac{d\sigma}{d\phi} \right)^2 = k^2(\phi), \quad (6)$$

where  $k^2(\phi)$  is the coefficient of the kinetic term of the non-canonical scalar field and  $\sigma$  represents the canonical scalar field. Now using the transformation (6) we have

$$\frac{dV(\sigma)}{d\sigma} = \frac{1}{k(\phi)} \frac{dV(\phi)}{d\phi}. \quad (7)$$

The slow-roll parameters for the non-canonical scalar field are then defined as

$$\epsilon = \frac{M_{\text{Pl}}^2}{2} \left( \frac{1}{V} \frac{dV}{d\sigma} \right)^2 = \frac{M_{\text{Pl}}^2}{2k^2(\phi)} \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2, \quad (8)$$

$$\eta = \frac{M_{\text{Pl}}^2}{V(\sigma)} \frac{d^2V(\sigma)}{d\sigma^2} = 2\epsilon(\phi) - \frac{M_{\text{Pl}}}{\alpha} \frac{d\epsilon(\phi)}{d\phi}. \quad (9)$$

In this case, the number of e-folds ( $\mathcal{N}$ ) is given by

$$\begin{aligned} \mathcal{N} &= \frac{1}{M_{\text{Pl}}^2} \int_{\sigma_{\text{end}}}^{\sigma_{\text{in}}} \frac{V(\sigma)}{dV(\sigma)/d\sigma} d\sigma, \\ &= \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} k^2(\phi) \frac{V(\phi)}{V'(\phi)} d\phi, \end{aligned} \quad (10)$$

which using Eq. (8) gives us the bound

$$\mathcal{N} \lesssim \frac{\delta\phi}{M_{\text{Pl}}^2} \left| k^2(\phi) \frac{V(\phi)}{V'(\phi)} \right|_{\text{max}} = \frac{\delta\phi}{M_{\text{Pl}}} \frac{k_{\text{max}}}{\sqrt{2\epsilon_{\text{min}}}}. \quad (11)$$

Similarly to the previous subsection we can consider the case where  $\epsilon_{\text{min}} \approx \epsilon_{\text{in}}$  and the tensor-to-scalar ratio is  $r_* = 16\epsilon_{\text{in}}$ , which after using relation (11) gives us the following expression

$$\delta\phi \gtrsim \left( \mathcal{N} M_{\text{Pl}} \sqrt{\frac{r_*}{8}} \right) \frac{1}{k_{\text{max}}}. \quad (12)$$

From Eq. (12) we deduce that the Lyth bound depends upon the value of the coefficient of the kinetic term at the beginning of inflation. Note that for large values of  $k_{\text{max}}$ , we can have sub-Planckian Lyth bound even if we have large value of the tensor-to-scalar ratio  $r_*$ .

Let us now consider a particular case with the following action in the standard FRW cosmology [26–30]<sup>1</sup>:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{\text{Pl}}^2}{2} R + \frac{k^2(\phi)}{2} \partial^\mu \phi \partial_\mu \phi + V(\phi) \right], \quad (13)$$

$$k^2(\phi) = \left( \frac{\alpha^2 - \tilde{\alpha}^2}{\tilde{\alpha}^2} \right) \frac{1}{1 + \beta^2 e^{\alpha\phi/M_{\text{Pl}}}} + 1, \quad (14)$$

where  $V(\phi) = M_{\text{Pl}}^4 e^{-\alpha\phi/M_{\text{Pl}}}$ ; the parameter  $\tilde{\alpha}$  controls the slow roll such that  $\tilde{\alpha} \ll 1$  and  $\beta$  is related to the scale of inflation. In the region where  $\phi$  is large, the kinetic function  $k(\phi) \rightarrow 1$  reduces the action to the scaling form. Nucleosynthesis constraints [77] imply that  $\alpha$  should be large, thereby the potential is steep in the post inflationary regime [26,27,31]. Let us also mention that in the small-field approximation, the potential in terms of the canonical field  $\sigma \simeq \alpha\phi/\tilde{\alpha}$  has the form of a shallow exponential potential, namely  $V(\sigma) \sim e^{-\tilde{\alpha}\sigma/M_{\text{Pl}}}$  as  $\tilde{\alpha} \ll 1$  [26]. Thus, a model is suitable for quintessential inflation provided that we invoke late-time features in the potential.

In this case, the slow-roll parameter in terms of the non-canonical field  $\phi$  is given by [27,31]

$$\epsilon = \frac{M_{\text{Pl}}^2}{2k^2(\phi)} \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2 = \frac{\alpha^2}{2k^2(\phi)} \rightarrow \frac{\alpha^2}{2} (1 + X) \quad (15)$$

where  $X \equiv \beta^2 e^{\alpha\phi/M_{\text{Pl}}}$ . The number of e-folds and the slow-roll parameter is related as [31]

$$\epsilon = \frac{\tilde{\alpha}^2}{2} \frac{1}{1 - e^{-\tilde{\alpha}^2 \mathcal{N}}}. \quad (16)$$

The large field region  $X \gg 1$  corresponds to  $\tilde{\alpha}^2 \ll 1/\mathcal{N} = 1$  implying that  $X_{\text{end}} \simeq 2/\tilde{\alpha}^2 \gg 1$  which means that inflation ends in the region of large values of the field. The boundary of small and large field limits is given by  $\tilde{\alpha}^2 = 1/\mathcal{N}$  [27,31]. Note that Eq. (16) implies that  $\epsilon$  is a monotonously increasing function of  $\tilde{\alpha}$ . Therefore, if inflation begins in the region around the boundary, we might improve upon the values of  $\tilde{\alpha}$  and hence the ratio  $r_*$ , allowing for larger range of inflation. Finally, using Eq. (12) we obtain the following bound for the action (13):

$$\delta\phi \gtrsim \left( \mathcal{N} M_{\text{Pl}} \sqrt{\frac{r_*}{8}} \right) \frac{\tilde{\alpha}}{\alpha}. \quad (17)$$

Let us now explicitly check whether we can really get the required range consistent with Lyth bound. We consider the following ratio [31]

<sup>1</sup> Non-canonical field is also considered in Refs. [74–76] for unification of phantom inflation and late-time cosmic acceleration.

$$\frac{V_{\text{end}}}{V_{\text{in}}} = \frac{\tilde{\alpha}^2}{2(e^{\tilde{\alpha}^2 \mathcal{N}} - 1)} = \frac{r_*}{16} e^{-\tilde{\alpha}^2 \mathcal{N}}, \quad (18)$$

which gives

$$\begin{aligned} \frac{\alpha}{M_{\text{pl}}} |\phi_{\text{in}} - \phi_{\text{end}}| &= \frac{\alpha \delta \phi}{M_{\text{pl}}} = \left| \ln \left[ \frac{\tilde{\alpha}^2}{2(e^{\tilde{\alpha}^2 \mathcal{N}} - 1)} \right] \right| \\ &= \left| \ln \left( \frac{r_*}{16} \right) - \tilde{\alpha}^2 \mathcal{N} \right|. \end{aligned} \quad (19)$$

Then using Eq. (16) and the relation  $r_* = 16\epsilon_{\text{in}}$ , we find that  $r_* \approx 0.15$  for  $\tilde{\alpha} = 0.06$  and  $\mathcal{N} = 60$ . Considering these values and using Eq. (19), we arrive at the estimate  $\delta\phi/M_{\text{pl}} \approx 5/\alpha$ . The parameter  $\alpha$  can be fixed from the post inflationary dynamics using nucleosynthesis constraints, obtaining  $\alpha \geq 20$  [26]. For  $\alpha = 20$ ,  $\delta\phi = 0.25M_{\text{pl}}$  which is the maximum value of  $\delta\phi$ . However, using Eq. (17) and taking  $\mathcal{N} = 60$ ,  $r_* = 0.15$ ,  $\alpha = 20$  and  $\tilde{\alpha} = 0.06$ , we obtain the bound  $\delta\phi \geq 0.0246M_{\text{pl}}$ . Thus, we conclude that the model under consideration obeys Lyth bound.

Now, concerning the parameter  $\beta$  in (14), that is related with the scale of inflation, we can fix it from COBE normalization, which gives us the relation between the parameters  $\tilde{\alpha}$ ,  $\beta$  and e-foldings  $\mathcal{N}$  [31] as

$$\frac{\beta^2 \sinh^2(\tilde{\alpha}^2 \mathcal{N}/2)}{\tilde{\alpha}^2} = 6.36 \times 10^{-8}. \quad (20)$$

We can use  $\beta$  from (20) in order to acquire the scale of inflation as [31]

$$V_{\text{in}}^{1/4} = \left( \frac{2.5 \times 10^{-7} \tilde{\alpha}^2}{1 - e^{-\tilde{\alpha}^2 \mathcal{N}}} \right)^{1/4} M_{\text{pl}} = 3.2 \times 10^{16} r_*^{1/4} \text{ GeV}, \quad (21)$$

which for  $r_* = 0.15$  gives us  $V_{\text{in}}^{1/4} = 2 \times 10^{16}$  GeV. For this scale of inflation using Eq. (18) and  $\mathcal{N} = 60$ ,  $r_* = 0.15$  and  $\tilde{\alpha} = 0.06$ , we find that

$$\left| \frac{\phi_{\text{end}}}{M_{\text{pl}}} \right| \approx \frac{24}{\alpha}, \quad (22)$$

which implies that the field value at the end of inflation is super-Planckian for  $20 \lesssim \alpha \lesssim 24$  and sub-Planckian for  $\alpha > 24$ .

We can also estimate the field value at the beginning of inflation considering the above calculated scale of inflation and the fact that for small field approximation the canonical field  $\sigma$  is related to the non-canonical field  $\phi$  through the relation  $\sigma \approx (\alpha/\tilde{\alpha})\phi$  [26], which makes the potential of the same form but with different slope  $\tilde{\alpha}$ . We obtain the following estimate

$$\left| \frac{\phi_{\text{in}}}{M_{\text{pl}}} \right| = \frac{19.2}{\alpha}, \quad (23)$$

from which we deduce that the non-canonical field  $\phi$  is always sub-Planckian for  $\alpha \gtrsim 20$  but the canonical field  $\sigma$  is super-Planckian for  $\tilde{\alpha} \ll 1$ . Since in this case the inflaton rolls from smaller to larger values, the field value of the canonical field  $\sigma$  at the end of inflation will also be super-Planckian.

### 2.3. Canonical field with brane correction

It is interesting to examine the Lyth bound in the case of steep braneworld inflation. In this case, the Friedmann equation is modified due to high-energy corrections to Einstein equations on the brane as [32–34]

$$H^2 = \frac{\rho}{3M_{\text{pl}}^2} \left( 1 + \frac{\rho}{2\lambda_{\text{B}}} \right), \quad (24)$$

and the slow-roll parameters read [32]

$$\epsilon = \epsilon_0 \frac{1 + V/\lambda_{\text{B}}}{(1 + V/2\lambda_{\text{B}})^2} \quad (25)$$

$$\eta = \eta_0 (1 + V/2\lambda_{\text{B}})^{-1}, \quad (26)$$

where  $\lambda_{\text{B}}$  is the brane tension and  $\epsilon_0$  and  $\eta_0$  are standard slow-roll parameters.

In the high-energy limit, that is when  $V \gg \lambda_{\text{B}}$ , we have  $\epsilon, \eta \ll 1$  despite the fact that  $\epsilon_0, \eta_0$  are large. The number of e-folds in this case is related to  $\delta\phi$  and the slow-roll parameter, given by the following relation:

$$\mathcal{N} = \frac{1}{M_{\text{pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{V}{V'} \left( 1 + \frac{V}{2\lambda_{\text{B}}} \right) d\phi. \quad (27)$$

Therefore, in the high-energy limit ( $V \gg \lambda_{\text{B}}$ ), using Eq. (25) we can re-write Eq. (27) as

$$\mathcal{N} \approx \frac{1}{M_{\text{pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{V}{V'} \frac{V}{2\lambda_{\text{B}}} d\phi = \frac{1}{M_{\text{pl}}} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{1}{\sqrt{\epsilon}} \sqrt{\frac{V}{2\lambda_{\text{B}}}} d\phi. \quad (28)$$

From Eq. (28) we obtain the relation

$$\mathcal{N} \lesssim \frac{\delta\phi}{M_{\text{pl}}} \sqrt{\frac{1}{\epsilon_{\text{min}}} \frac{V_{\text{max}}}{2\lambda_{\text{B}}}} = \frac{\delta\phi}{M_{\text{pl}}} \sqrt{\frac{1}{\epsilon_{\text{min}}} \frac{V_{\text{in}}}{2\lambda_{\text{B}}}}, \quad (29)$$

where  $\delta\phi = |\phi_{\text{in}} - \phi_{\text{end}}|$ . Hence, an interesting expression for the Lyth bound on the brane follows from Eq. (29) as

$$\delta\phi \gtrsim \left( \mathcal{N} M_{\text{pl}} \sqrt{\frac{r_*}{8}} \right) \sqrt{\frac{2\lambda_{\text{B}}}{3V_{\text{in}}}}, \quad (30)$$

where we have considered the fact that for brane inflation  $r_* = 24\epsilon_{\text{in}}$ . Since during steep braneworld inflation  $V_{\text{in}} \gg \lambda_{\text{B}}$ , expression (30) tells us that  $\delta\phi$  is suppressed by high energy corrections and can still be sub-Planckian despite  $r_*$  being large. This is related to the fact that the slow roll is not realized due to the potential slope and the smallness of the curvature, but it is facilitated by the high-energy corrections to the Einstein equations on the brane. We shall give concrete numerical estimates for a particular case in the following discussion.

In the case of braneworld models of quintessential inflation the inflaton potential is a steep exponential, and only at late times there is a feature in the potential which allows for an exit from the scaling regime [17,18]. For instance a potential of the form

$$V = V_0 \left[ \cosh \left( \frac{\gamma\phi}{M_{\text{pl}}} \right) - 1 \right]^p, \quad \text{where } p > 0, \quad (31)$$

behaves like steep exponential for large  $\phi$  provided that  $\alpha \equiv \gamma p$  is large, while around the origin  $V \sim \phi^{2p}$  such that the average equation of state is  $\langle \omega_\phi \rangle = (p-1)/(p+1)$  and can give rise to the desired negative value at late times. This scenario can describe late-time acceleration provided that  $V_0 \sim 3H_0^2 M_{\text{pl}}^2$ .

At early times, brane damping allows the field to derive inflation. We can specialize to steep exponential potential, in which case (27) gives a simple relation, namely  $V_{\text{in}} = (\mathcal{N} + 1)V_{\text{end}}$  and  $V_{\text{in}}/2\lambda_{\text{B}} = (\mathcal{N} + 1)\alpha^2$  [17]. Additionally, the Nucleosynthesis constraint [77] leads to [17]

$$\frac{\rho_\phi}{\rho_{\text{B}} + \rho_\phi} = \frac{3(1 + w_{\text{B}})}{\alpha^2} \lesssim 0.01, \quad (32)$$

where  $\rho_B$  and  $w_B$  are respectively the background density and equation-of-state parameter. For radiation,  $w_B = 1/3$  and from Eq. (32) we acquire  $\alpha \gtrsim 20$ .

Finally, taking  $\alpha = 20$ ,  $\mathcal{N} = 60$  and  $r_* = 0.1$  and using Eq. (30), we get the bound on the range of inflation on the brane, namely

$$\delta\phi \gtrsim \sqrt{\frac{r_*}{24(\mathcal{N}+1)}} \frac{\mathcal{N}}{\alpha} M_{\text{Pl}} \simeq 0.0248 M_{\text{Pl}}. \quad (33)$$

We can also independently estimate  $\delta\phi$  for the exponential potential, using the relation between  $V_{\text{end}}$  and  $V_{\text{in}}$  and the brane tension, as

$$\delta\phi = \frac{M_{\text{Pl}}}{\alpha} \ln(\mathcal{N}+1), \quad (34)$$

which for the quoted values of parameters gives  $\delta\phi = 0.2 M_{\text{Pl}}$ , consistently with Eq. (33).

Proceeding further, we can use relation  $r_* = 24\epsilon_{\text{in}}$ , in the high-energy limit ( $V \gg \lambda_B$ ), in order to estimate the scale of inflation in terms of the tensor-to-scalar ratio as

$$V_{\text{in}}^{1/4} = \left( \frac{96\alpha^2 \lambda_B}{r_*} \right)^{1/4}. \quad (35)$$

The value of  $\lambda_B$  is fixed from the COBE normalization and is given by [17]

$$\lambda_B = \frac{2.6 \times 10^{-10}}{\alpha^6} \left( \frac{8\pi M_{\text{Pl}}}{\mathcal{N}+1} \right)^4. \quad (36)$$

Thus, for  $\alpha = 20$ ,  $\mathcal{N} = 60$  and  $r_* = 0.4$  from (35) we obtain

$$V_{\text{in}}^{1/4} = 7.9 \times 10^{14} \text{ GeV}. \quad (37)$$

Let us note that for  $\mathcal{N} = 60$  and considering scale of inflation  $\sim 10^{15}$  GeV we acquire

$$\left| \frac{\alpha\phi_{\text{in}}}{M_{\text{Pl}}} \right| \simeq \ln \left( \frac{10^{60}}{3H_0^2 M_{\text{Pl}}^2} \right) \simeq 244 \quad (38)$$

$$\left| \frac{\alpha\phi_{\text{end}}}{M_{\text{Pl}}} \right| \simeq \ln \left( \frac{10^{60}}{(\mathcal{N}+1)3H_0^2 M_{\text{Pl}}^2} \right) \simeq 240, \quad (39)$$

which clearly show that inflation takes place in the range of large  $\phi$  for viable values of  $\alpha$ . The inflationary potential contains a free parameter  $\alpha$  to be fixed by post-inflationary requirements. The requirement of scaling behavior implies that  $\alpha > \sqrt{3}$ , but the nucleosynthesis constraint [77] demands larger numerical values, namely  $\alpha \gtrsim 20$ , such that  $\delta\phi < M_{\text{Pl}}$ . In this case the Lyth bound does not impose any restriction on the post-inflationary dynamics. On the other hand, it is the post-inflationary dynamics which makes the range of inflation sub-Planckian and the Lyth bound is evaded, despite the field values being super-Planckian.<sup>2</sup>

### 3. Conclusion and discussion

In this letter we have examined the structure of the Lyth bound in two classes of models of quintessential inflation. The quintessential inflation with non-canonical kinetic term can give rise to large tensor-to-scalar ratio of perturbations, consistent with BICEP2 measurements [1], such that the scale of inflation is around

<sup>2</sup> However, the tensor-to-scalar ratio  $r$  is slightly higher in the case of steep braneworld inflation. Replacing the exponential potential by an inverse power-law  $\phi^{-p}$  makes the situation worse. In this case the minimum of the ratio is reached for large  $p$ , as the inverse power law potential approaches the exponential form.

$10^{16}$  GeV. The coefficient of the kinetic term ( $k^2(\phi)$ ) is given by relation (14), which has a maximum value  $(\alpha/\tilde{\alpha})^2$  for small field values [26] and approaches unity in large-field approximations. The parameter  $\tilde{\alpha}$  controls the inflationary dynamics such that  $\tilde{\alpha} \ll 1$ , while  $\alpha$  is related to post inflationary evolution and is constrained by the nucleosynthesis bound [77]. It should be noted that the parameter  $\alpha$  does not appear in any physical quantity related to inflation. However, the presence of a non-canonical term in the action (13) modifies the Lyth bound to  $\delta\phi \gtrsim \frac{\mathcal{N} M_{\text{Pl}}}{k_{\text{max}}} \sqrt{r_* / 8}$  with  $k_{\text{max}} = \alpha/\tilde{\alpha}$ . Since  $\tilde{\alpha} \ll 1$  and  $\alpha \geq 20$ , we find that  $k_{\text{max}}$  can be a large number leading to suppression of  $\delta\phi$ . For instance, for a viable choice of parameters,  $\mathcal{N} = 60$ ,  $r_* = 0.15$ ,  $\alpha = 20$  and  $\tilde{\alpha} = 0.06$ , we find that  $\delta\phi \simeq 0.0246 M_{\text{Pl}}$ , which implies that the Lyth bound is clearly evaded in this case. It is interesting to note that we have not imposed any extra restriction other than the viability of inflation and post-inflationary evolution.

In case of steep braneworld inflation, for  $\mathcal{N} = 60$ ,  $\alpha = 20$  and  $r_* = 0.1$ , we obtained  $\delta\phi = 0.25 M_{\text{Pl}}$ , which obeys the Lyth bound and has sub-Planckian value. We have also calculated the field value at the beginning and at the end of inflation. We found that the non-canonical field, from the beginning of inflation to its end, remains sub-Planckian provided  $\alpha \geq 24$ , which is consistent with the nucleosynthesis constraint  $\alpha \geq 20$ . It is important to note that though the parameter  $\alpha$  does not play any role during inflation, it appears in the Lyth bound and plays a crucial role. Nucleosynthesis constraint makes  $\alpha$  large which helps to evade the Lyth bound by making  $\delta\phi$  sub-Planckian.

In the steep braneworld inflation the slow roll takes place along a steep potential due to the brane damping. The ratio  $r$  is enhanced by a factor of  $V/\lambda_B$  such that the Lyth bound is suppressed by its inverse, in high-energy approximation valid during inflation. Similar to the non-canonical case, nucleosynthesis constraint makes the range of inflation  $\delta\phi$  sub-Planckian on the brane, despite  $r$  being large. The scale of steep braneworld inflation is about  $10^{15}$  GeV. We should however emphasize that inflation is realized for super-Planckian values of the field in this case. Secondly, the induced numerical value of  $r$  is slightly higher than the observed values in the BICEP2 experiment [1].

It is indeed interesting that the Lyth bound imposes severe restrictions on the single canonical scalar field models of inflation in the standard FRW cosmology. However, as we demonstrated, the bound can be evaded in case of models of quintessential inflation considered here. Unfortunately, the steep braneworld inflation cannot meet the BICEP2 requirement.

It is remarkable that the class of models corresponding to action (13) not only evades the Lyth bound but also meet the BICEP2 requirements. We have therefore demonstrated that  $\delta\phi$  is sub-Planckian despite the tensor-to-scalar ratio of perturbations  $r$  being large.

An important comment relating to the effective field theoretic description of inflation is in order. As we mentioned earlier, the canonical field  $\sigma$  associated with  $\phi$  is super-Planckian. We might take the orthodox view and abandon the transformation to venture into super-Planckian region and focus on the non-canonical-field ( $\phi$ ) description. But in that case, unlike the canonical Lagrangian where one computes quantum correction to potential, the corrections should be calculated to the total Lagrangian [7,78,79]. As noticed in the case of polynomial type of Lagrangian, these correction might get large for large values of the kinetic function  $k(\phi)$ . This could possibly be the manifestation of the problem shifted from canonical Lagrangian to its non-canonical description, and it might be worthwhile to check it for the Lagrangian considered here. The full investigation of the quantum behavior lies beyond the scope of the present letter, and is left for a future project.



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