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Z-valued t-norm and t-conorm operators-based aggregation of partially reliable information

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Abstract

T-norm and T-conorm operators are successfully used for processing uncertainty in system analysis, decision analysis, control, modeling and forecasting applications. The complexity of real-world applications is that relevant information is partially reliable. In this regard, Prof. L.A. Zadeh suggested the concept of a Z-number as an ordered pair $Z_1 + Z_2$ of fuzzy numbers used to describe a value of a random variable X , where A is a fuzzy constraint on values of X and B is a fuzzy reliability of A, and is considered as a value of probability measure of R_2 . However, processing of Z-number based information is at its initial stage of crystallization. In this paper we consider Z-number valued T-norm and T-conorm operators of Z-numbers. A real-world application is used to illustrate validity and efficiency of Z-number valued T-norm and T-conorm operators based processing of partially reliable information.

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1. Introduction

Solving a wide class of real-world applications requires aggregation of imperfect information obtained from a set of sources. Formally, aggregation is the process of combining numbers, linguistic terms etc into resulting overall. Aggregation problems are investigated within several fields of science such as decision making under uncertainty, multicriteria decision making, data mining etc. The nature of information is one of the important issues. In the case of linguistic quantities, aggregation can be provided in the framework of fuzzy sets[1]. In the probability theory, aggregation occurs in calculus of functions of random variables[2].

Operators of aggregation form three main groups [3]. Conjunctive operators implement 'pessimistic' aggregation, because mainly the lowest value(s) dictates the result. Typical conjunctive operators are triangular norms (t-norms). In contrast, disjunctive operators are based on the utilization the highest value(s). Triangular co-norms (t-conorm) are typical disjunctive operators. The third group is the averaging operators. The results of such operations are located between the results of conjunctive and disjunctive operators.

Any aggregation operator exhibits a set of mathematical properties, which depends on imposed axiomatic assumptions. Main mathematical properties which can be requested for aggregation are the following [3]: transitivity; idempotence; continuity; monotonicity; compensativeness; associativity. The behavioral properties are also important for modeling behavior of a decision maker: decisional behavior; interpretability of the parameters; weights on the arguments; interaction of arguments [4].

Aggregation of information in real-world applications is complicated by the fact that information itself is imperfect. On the one side, this means that information is often of linguistic description carrying imprecision and vagueness. On the other side, knowledge of a human being is partially reliable. This partial reliability is also imprecise and can be formalized as a fuzzy value of probability measure. In order to ground the formal basis for dealing with real-world information, L.A. Zadeh suggested the concept of a Z-number [15] as an ordered pair $Z=(A,B)$ of fuzzy numbers used to describe a value of a random variable X, where A is a fuzzy constraint on values of X and B is a fuzzy reliability of A, and is considered as a value of probability measure of A .Nowadays a series of works devoted to Z-numbers and their application in decision making, control and other fields [16-21] exists. A general and computationally effective approach to computation with discrete Z-numbers is suggested in [21-23]. The authors provide motivation of the use of discrete Z-numbers mainly based on the fact that NL-based information is of a discrete framework.

Unfortunately, nowadays there is no research on aggregation operators for Z-number valued information. In this paper we develop T-norm and t-conorm based aggregation operators for information described by discrete Z-numbers [22]. The paper is structured as follows. In Section II we present some prerequisite material. In Section III we describe the suggested T-norm and T-conorm operators of Z-number valued information. In Section IV we illustrate application of the suggested aggregators to real-world problem. Section V concludes.

2. Preliminaries

Definition 1. A discrete fuzzy number [24]. A fuzzy subset A of the real line R with membership function $\mu_A:\mathrm{R}\to[0,1]$ is a discrete fuzzy number if its support is finite, i.e. there exist $x_1,...,x_n\in\mathrm{R}$ with $x_1<x_2<...<x_n$, such that $\mathrm{supp}(A)=\{x_1,...,x_n\}$ and there exist natural numbers s,t with $1\le s\le t\le n$ satisfying the following conditions:

1. p_2 for any natural number $\mu_{p_j}(p_j)$ with $j=1,2$
2. $\mu_A(x_i)\le\mu_A(x_j)$ for each natural numbers i,j with $1\le i\le j\le s$
3. $\mu_A(x_i)\ge\mu_A(x_j)$ for each natural numbers i,j with $t\le i\le j\le n$

Definition 2. Arithmetic operations over discrete random variables[10]. Let X_1 and X_2 be two independent discrete random variables with the corresponding outcome spaces $\mathrm{X}_1=\{x_{11},...,x_{1i},...,x_{1n_1}\}$ and $\mathrm{X}_2=\{x_{21},...,x_{2i},...,x_{2n_2}\}$ and the corresponding discrete probability distributions p_1 and p_2. The probability distribution of $X_{12}=X_1*X_2$, where $*$ is a two-place operation, is the convolution $p_{12}=p_1\circ p_2$ defined as follows:

$$p_{12}(x)=\sum_{x=x_1*x_2} p_1(x_1)p_2(x_2),\ \ x\in\{x_1*x_2\,|\,x_1\in \mathrm{X}_1, x_2\in \mathrm{X}_2\},\ x_1\in \mathrm{X}_1,\ x_2\in \mathrm{X}_2$$

Definition 3. A discrete Z-number [22,23,25]. A discrete Z-number is an ordered pair $Z=(A,B)$ of discrete fuzzy numbers A and B. A plays a role of a fuzzy constraint on values that a random variable X may take. B is a discrete fuzzy number with a membership function $\mu_B:\{b_1,...,b_n\}\to[0,1]$, $\{b_1,...,b_n\}\subset[0,1]$, playing a role of a fuzzy constraint on the probability measure of A, $P(A)$.

3. Z-valued T-norm and T-conorm Operators

Let Z_1 and Z_2 be discrete Z-numbers describing imperfect information about values of random variables X_1 and X_2. Let us provide an algorithm of computation of the value of Z-valued T-norm operator $Z_{12}=T(Z_1,Z_2)$. Computation of Z-valued T-conorm $Z_{12}=S(Z_1,Z_2)$ is analogous.

Step 1. Compute the value $A_{12}=T(A_1,A_2)$ of the T-norm of fuzzy numbers.

Step 2. Given fuzzy restrictions $\sum_{i=1}^{n_j}\mu_{A_j}(x_{ji})p_j(x_{ji})\ is\ B_j$, extract probability distributions $p_j, j=1,2$ by solving the following goal linear programming problem:

$$c_1v_1+c_2v_2+...+c_nv_n\to b_j \tag{1}$$

subject to

$$\left.\begin{aligned} v_1+v_2+...+v_n&=1\\ v_1,v_2,...,v_n&\geq 0\end{aligned}\right\}. \tag{2}$$

where $c_k=\mu_{A_j}(x_{jk})$ and $v_k=p_j(x_{jk}),\ k=1,..,n_j,\ k=1,..,n_j$.

Thus, to probability distributions $p_j, j=1,2$, we need to solve m simple problems (1)-(2). Let us mention that, in general, problem (1)-(2) does not have a unique solution. In order to guarantee existence of a unique solution, the compatibility conditions can be used[14].

Step 3. Given $p_j, k=1,..,n_j$, construct the convolutions $p_{12}=p_1\circ p_2$, as the result of T-norm operation over random variables $X_{12}=T(X_1,X_2)$ by using Definition 2:

$$p_{12s}(x)=\sum_{x=T(x_1,x_2)} p_1(x_1)p_2(x_2),\forall x_{12}\in X_{12}; x_1\in X_1, x_2\in X_2.$$

Step 4. Construct the fuzzy set of convolutions p_{12}, which is naturally induced by the fuzzy sets of probability distributions p_j, as

$$\mu_{p_{12}}(p_{12})=\max_{p_{12}=p_1\circ p_2}\ \min\{\mu_{p_1}(p_1),\mu_{p_2}(p_1)\} \tag{3}$$

subject to

$$\mu_{p_j}(p_j)=\mu_{B_j}\left(\sum_{k=1}^{n_j}\mu_{A_j}(x_{jk})p_j(x_{jk})\right),\ j=1,2 \tag{4}$$

Step 5. As the fuzziness of information on p_{12s} described by Z_{12} induces fuzziness of the value of $P(A_{12})=\sum_{k=1}^{n}\mu_{A_{12}}(x_{12k})p_{12}(x_{12k})$, construct a discrete fuzzy number $Z_{12}=(A_{12},B_{12})$:

$$\mu_{B_{12}}(b_{12})=\max(\mu_{p_{12}}(p_{12})) \qquad (5)$$

subject to

$$b_{12}=\sum_{k=1}^{n}\mu_{A_{12}}(x_{12k})p_{12}(x_{12k}) \qquad (6)$$

As a result, a value of Z-valued T-norm operator $Z_{12}=T(Z_1,Z_2)$ is obtained as $Z_{12}=(A_{12},B_{12})$.

4. An application. Z-T-norm and T-conorm aggregation of experts' opinions

Suppose that 3 experts need to generate a single opinion on a suggested commercial decision. Due to uncertainty and imprecision, each expert assigns his opinion Q_i in terms of Z-numbers:

$$A_{Q_1}=0/1+0.3/1+0.4/2+0.7/3+1/4+0.8/5+0.6/6+0/7,$$

$$B_{Q1}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9;$$

$$A_{Q_2}=0.2/0+0.4/1+1/2+0.4/3+0.2/4+0/5,$$

$$B_{Q2}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9;$$

$$A_{Q_3}=0/1+0.5/2+0.6/3+0.7/4+1/5+0.7/6+0/7,$$

$$B_{Q_3}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9.$$

Moreover, degrees of knowledge of experts are different. The degrees of knowledge of experts are also described by discrete Z-numbers $Z_{w_1}=(A_{w_1},B_{w_1})$:

$$A_{w_1}=0/0+0.6/1+0.8/2+1/3+0.7/4+0/5,\ B_{w_1}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9;$$

$$A_{w_2}=0/1+0.4/2+0.6/3+1/4+0.8/5+0/6,\ B_{w_2}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9;$$

$$A_{w_3}=0/2+0.4/3+0.6/4+1/5+0.8/6+0/7,\ B_{w_3}=0/0.4+0.01/0.5+0.14/0.6+0.6/0.7+1/0.8+0.6/0.9.$$

The problem is to determine a final expert group opinion on the considered decision as the Z-T-norm and the Z-T-conorm operators based aggregation:

$Aggreg(Z_1,Z_1,...,Z_1)=Z_1$.

The problem is solved as follows.

Step 1. An expert's weighted evaluation of Q denoted $Z_{Qw_i}=(A_{Qw_i},B_{Qw_i})$ is determined on the base of T-norm operation (Section 3). Let us consider computation of Z_1 for the third expert. As a T-norm, the *min* function $T(Z_Q,Z_w)=\min(Z_Q,Z_w)$ will be used. The results obtained are as follows:

$$A_{Qw_1}=0.2/0+0.4/1+1/2+0.4/3+0.2/4+0/0,$$

$$B_{Qw_1}=0/0.80+0.01/0.82+0.14/0.85+0.6/0.87+1/0.89+0.6/0.9;$$

$$A_{Qw_2}=0/0+0.6/1+0.8/2+1/3+0.7/4+0/1,$$

$$B_{Qw_2}=0/0.64+0.01/0.68+0.14/0.73+0.6/0.77+1/0.82+0.6/0.90;$$

$$A_{Qw_3}=0/1+0.5/2+0.6/3+0.7/4+1/5+0.7/6+0/1,$$

$$B_{Qw_3}=0/0.77+0.01/0.79+0.14/0.81+0.6/0.84+1/0.86+0.6/0.9.$$

Step 2. An aggregation of weighted evaluations is conducted on the base of T-conorm (12)-(13). At first, we need to compute T-conorm of $Z_{Qw_1}=(A_{Qw_1},B_{Qw_1})$ and $Z_{Qw_2}=(A_{Qw_2},B_{Qw_2})$ (related to the first and the second experts). As a T-conorm, the *max* function $S(Z_{Qw_1},Z_{Qw_2})=\max(Z_{Qw_1},Z_{Qw_2})$ will be used. The results obtained are as follows:

$$A_{12}=0/1+0.5/2+0.6/3+0.7/4+1/5+0.7/6+0/1,$$

$$B_{12}=0/0.46+0.01/0.55+0.14/0.63+0.61/0.72+1/0.81+0.6/0.9.$$

Finally, we computed the result of T-conorm operation $S(Z_{12},Z_{Qw_3})=(A,B)$ to obtain the final group evaluation based on (22)-(23):

$$A=0/0+0.5/2+0.6/3+0.7/4+1/5+0.7/6+0/0,$$

$$B=0/0.73+0.01/0.76+0.14/0.78+0.61/0.82+1/0.85+0.6/0.91.$$

Therefore, the final expert group evaluation as an aggregation of individual expert opinions on the base of T-norm and T-conorm operations is obtained: $Z^{agg}=(A,B)$.

5. Conclusion

Real-world problems in economics, modeling, forecasting etc are characterized by fuzzy and partially reliable information. The concept of Z-number provides a formal basis to deal with such kind of information. In view of this, a systematic computational framework is needed as a basis of adequate decision analysis and reasoning under Z-valued information. In this paper we consider Z-valued T-norm and T-conorm operators for processing fuzzy and partially reliable information. The considered operators are applied to solving a real-world problem of decision making on the basis of aggregation of experts opinions expressed in form of Z-numbers.

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