ELSEVIER
SCIENCE DIRECT
PHYSICS LETTERS B
Physics Letters B 600 (2004) 83–90
www.elsevier.com/locate/physletb

Estimate of the chiral condensate in quenched lattice QCD

Damir Bećirević [a], Vittorio Lubicz [b]

[a] *Laboratoire de Physique Théorique (Bât 210)* [1], *Université de Paris Sud, Centre d'Orsay, 91405 Orsay-Cedex, France*
[b] *Dipartimento di Fisica, Università di Roma Tre and INFN, Sezione di Roma III, Via della Vasca Navale 84, I-00146 Rome, Italy*

Received 5 March 2004; received in revised form 27 July 2004; accepted 29 July 2004

Editor: G.F. Giudice

Abstract

We determine the value of the quark condensate from quenched QCD simulations on the lattice in two ways: (i) by using the Gell-Mann–Oakes–Renner (GMOR) formula; (ii) by comparing the OPE prediction for the Goldstone pole contribution to the pseudoscalar vertex, at moderately large momenta. In the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV, from the GMOR formula we obtain $\langle \bar{q}q\rangle = -(273 \pm 19\ \mathrm{MeV})^3$. We show that the value extracted from the pseudoscalar vertex, $\langle \bar{q}q\rangle = -(312 \pm 24\ \mathrm{MeV})^3$, although larger, is consistent with the result obtained from the first (standard) method.

1. Motivation

The value of the quark condensate was, and still is, a subject of some controversies. It has been experimentally established that in the theory with the spontaneous symmetry breaking pattern $SU(2)_L \otimes SU(2)_R \to SU(2)_V$, the quark condensate is indeed the order parameter [1]. The extension to the three flavour case still needs to be clarified (for a recent critical discussion see Ref. [2]). Lattice QCD provides, in principle, the method for determining the value of the quark condensate and for studying its dependence on the number of dynamical quark flavours. Up to now, the determination of the quark condensate on the lattice was limited to the quenched QCD (i.e., with $n_F = 0$).[2] Before tackling the theory with $n_F = 2$ and $n_F = 3$ flavors, one would like to learn as much as possible from the quenched theory. For example, one would like to understand if the values of the chiral condensate obtained by using different methods are consistent among themselves.

The standard method relies on the use of the GMOR formula, i.e., on the same set of the background gauge field configurations one computes both the quark masses (m_q) and the corresponding pseudo-

E-mail address: damir.becirevic@roma1.infn.it (D. Bećirević).
[1] Unité mixte de Recherche du CNRS—UMR 8627.

[2] See Ref. [3] for recent results and the exhaustive list of references.

0370-2693 © 2004 Published by Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2004.07.065

scalar meson masses (m_P), and from the slope

$$m_P^2 = 2B_0 m_q, \tag{1}$$

one gets an estimate of the quark condensate as

$$B_0 = -\frac{2}{f^2}\langle \bar{q}q\rangle, \tag{2}$$

where f is the pion decay constant in the chiral limit. The renormalization scale and scheme dependence of the chiral condensate is just the inverse of the one for the quark mass which was discussed in great detail in Ref. [4].

An alternative way for extracting the value of the quark condensate is from the study of the amputated pseudoscalar vertex function, where the $\bar{q}\gamma_5 q$ operator is inserted at momentum zero. At moderately large p^2 (p being the momentum flowing through the legs of the vertex function), one can compare the shape of this function with the corresponding expression derived by means of the operator product expansion (OPE), in which the quark condensate appears in the coefficient of the leading power correction. The lattice estimate based on this strategy, which is the purpose of this Letter, has not been presented so far. We show that its value in the continuum limit is fully consistent with the standard value, obtained by using the GMOR formula, whose value we updated here as well.

2. Pseudoscalar vertex

In this section we discuss the relation between the pseudoscalar vertex and the quark propagator and study the dependence of these functions on the chiral quark condensate, which enters their OPE as a leading power correction.

The starting point is to define the quark propagator and the Green function of the pseudoscalar density with zero momentum insertion,

$$S(p) = \int dx\, e^{-ipx}\langle q(x)\bar{q}(0)\rangle,$$
$$G_P(p) = \int dx\, dy\, e^{-ip(x-y)}\langle q(x)\bar{q}(0)\gamma_5 q(0)\bar{q}(y)\rangle. \tag{3}$$

The amputated vertex function,

$$\Lambda_P(p) = S^{-1}(p)G_P(p)S^{-1}(p), \tag{4}$$

is then conveniently projected onto its tree level value

$$\Gamma_P(p) = \frac{1}{12}\mathrm{Tr}\left[\gamma_5\Lambda_P(p)\right], \tag{5}$$

where the trace goes over Dirac and color indices so that the factor $1/12$ simply provides the normalization to unity.

If we write the bare (lattice regularized) inverse quark propagator as

$$S^{-1}(p) = \Sigma_1(p^2)\not{p} + \Sigma_2(p^2), \tag{6}$$

then the basic RI/MOM renormalization condition for the quark propagator in the chiral limit can be written as[3] [6]

$$\frac{1}{Z_q(\mu^2)}\left[\frac{1}{12}\frac{\mathrm{Tr}(\not{p}S^{-1}(p^2))}{p^2}\right]_{p^2=\mu^2} \equiv \left.\frac{\Sigma_1(p^2)}{Z_q(\mu^2)}\right|_{p^2=\mu^2} = 1, \tag{7}$$

where $Z_q(\mu)$ is the quark field renormalization ($\hat{S}(p,\mu) = Z_q(\mu)S(p)$).

By studying the quark propagator at large momenta, one can get an estimate of the quark mass value, in the RI/MOM scheme, as

$$m_q^{\mathrm{RI/MOM}}(\mu^2) = \frac{1}{12}\mathrm{Tr}\left[\hat{S}^{-1}(p,\mu)\right]_{p^2=\mu^2} = \left.\frac{\Sigma_2(p^2)}{\Sigma_1(p^2)}\right|_{p^2=\mu^2}. \tag{8}$$

This estimate has been already discussed in Ref. [8]. At lower momenta, however, this definition of the quark mass suffers from the presence of the long distance contributions due to the coupling to the Goldstone bosons.

The effect of the Goldstone boson is more clearly seen by considering the quark Ward identity which relates the inverse quark propagator to the amputated pseudoscalar Green function,

$$\gamma_5 S^{-1}(p^2) + S^{-1}(p^2)\gamma_5 = 2Z_A\rho\Lambda_P(p^2), \tag{9}$$

[3] In practice, we are away from the chiral limit, but the renormalization condition applies equally well for $m_q^2/p^2 \ll 1$ [7].

where $Z_A\rho$ is the quark mass obtained from the hadronic axial Ward identity on the lattice.[4] After multiplying Eq. (9) by γ_5 and by taking the trace of both sides, we have

$$\Sigma_2\left(p^2\right) = Z_A \rho \Gamma_P\left(p^2\right). \tag{10}$$

For light quark masses and moderately large momenta, the vertex function $\Gamma_P(p^2)$ is affected by the long distance effects which are due to the presence of the Goldstone boson [5], which by means of the LSZ reduction formula generates the term proportional to the Goldstone boson propagator $1/(q^2 + m_\pi^2)$. Since the operator is inserted at zero momentum, $q^2 = 0$, the vertex function in the chiral limit develops a pole $\propto 1/m_\pi^2 \sim 1/\rho$. To account for that effect, we expand the vertex function in powers of the quark mass,

$$\Gamma_P\left(p^2, \rho\right) = \Gamma_P^{\text{subtr.}}\left(p^2\right) + \frac{B(p^2)}{Z_A\rho} + C\left(p^2\right)\rho, \tag{11}$$

where the first term is the subtracted pseudoscalar vertex, from which the hadronic (Goldstone boson) contribution $\propto 1/m_P^2 \propto 1/\rho$ is subtracted away. The third term is the linear quark mass correction while the higher order terms in the expansion, as well as the logarithmic quark mass dependence, are neglected since we deal with light quark masses varying in a short interval.

The renormalization constant of the pseudoscalar density, $Z_P^{\text{RI/MOM}}(\mu)$, is defined in terms of the subtracted Green function of Eq. (11) through the RI/MOM renormalization condition

$$\frac{Z_P(\mu^2)}{Z_q(\mu^2)} \Gamma_P^{\text{subtr.}}\left(p^2\right)\bigg|_{p^2=\mu^2} = 1. \tag{12}$$

As we already discussed in Ref. [4], the value of $Z_P^{\text{RI/MOM}}(\mu)$ obtained from Eq. (12) is completely consistent with the one obtained by applying the method of Ref. [9], which allows one to circumvent the second term on the r.h.s. of Eq. (11).

After inserting Eq. (11) in (10), multiplying both sides by $Z_q^{-1}(\mu)$, and accounting for the renormalization condition (12), we have

$$\frac{\Sigma_2(p^2)}{\Sigma_1(p^2)}\bigg|_{p^2=\mu^2} = \underbrace{\frac{Z_A\rho}{Z_P(\mu^2)}}_{m_{\text{AWI}}^{\text{RI/MOM}}(\mu)} + \frac{B(p^2)}{Z_q(\mu^2)}, \tag{13}$$

where contributions quadratic in the quark mass have been neglected. The first term on the right-hand side is the usual short distance quark mass, renormalized in the RI/MOM scheme, derived from the axial Ward identity. Eq. (13) differs from Eq. (8) for the presence of the second term on the r.h.s., which represents the power suppressed contribution coming from the Goldstone boson. It has been shown long ago that, at the leading order in the OPE, this term has the form [10]

$$\frac{B(p^2)}{\Sigma_1(p^2)}\bigg|_{\text{OPE}} = c\left(p^2, \mu\right)\frac{\langle \bar{q}q\rangle(\mu)}{p^2} + \mathcal{O}\left(1/p^4\right). \tag{14}$$

From this relation we will derive our first estimate of the quark condensate.

The Wilson coefficient, $c(p^2, \mu)$, has been computed at the next-to-leading order (NLO) in QCD perturbation theory [11]. In the $\overline{\text{MS}}$ scheme, by choosing the Landau gauge (in which the lattice calculations are most easily made), and after setting $p^2 = \mu^2$, one has[5]

$$c^{\overline{\text{MS}}}\left(p^2\right) = -\frac{4\pi}{3}\alpha_S(p)\left[1 + \left(\frac{99}{4} - \frac{10}{9}n_{\text{F}}\right)\frac{\alpha_S(p)}{4\pi}\right]. \tag{16}$$

We notice that the radiative corrections are large so that at moderately large p^2 they must be included in the analysis when extracting the value of the condensate from the lattice data. Besides, the inclusion of the radiative corrections is also necessary for specifying the renormalization scheme (the leading order anomalous dimension of the quark condensate is universal for all renormalization schemes). To eliminate

[4] Recall that

$$2\rho = \frac{\partial_0\langle \sum_{\vec{x}} A_0(x)P(0)\rangle}{\langle \sum_{\vec{x}} P(x)P(0)\rangle},$$

with $P = \bar{q}\gamma_5 q$, $A_0 = \bar{q}\gamma_0\gamma_5 q$, and $Z_A \equiv Z_A(g_0^2)$ is the (known) axial current renormalization constant.

[5] For completeness, we recall the expression for the 2-loop running coupling

$$\alpha_S(p) = \frac{4\pi}{\beta_0 \log(p^2/\Lambda_{\text{QCD}}^2)}\left(1 - \frac{\beta_1 \log\log(p^2/\Lambda_{\text{QCD}}^2)}{\beta_0^2 \log(p^2/\Lambda_{\text{QCD}}^2)}\right);$$

$$\beta_0 = 11 - \frac{2}{3}n_{\text{F}}, \quad \beta_1 = 102 - \frac{38}{3}n_{\text{F}}. \tag{15}$$

the scale dependence of the condensate one defines the renormalization group invariant (RGI) quark condensate, which at NLO in perturbation theory is related to the $\overline{\mathrm{MS}}$ one through

$$\langle\bar{q}q\rangle^{\overline{\mathrm{MS}}}(p) = \left(\alpha_S(p)\right)^{-\frac{\gamma_0}{2\beta_0}} \times\left[1-\frac{\gamma_1\beta_0-\gamma_0\beta_1}{2\beta_0^2}\frac{\alpha_S(p)}{4\pi}\right]\langle\bar{q}q\rangle^{\mathrm{RGI}},$$

$$\gamma_0=8,\quad \gamma_1^{\overline{\mathrm{MS}}}=\frac{4}{3}\left(101-\frac{10}{3}n_{\mathrm{F}}\right), \tag{17}$$

and thus at $n_{\mathrm{F}}=0$, Eq. (14) becomes

$$\frac{B(p^2)}{\Sigma_1(p^2)}\bigg|_{\mathrm{OPE}} = \underbrace{-\frac{4\pi}{3}\left(\alpha_S(p)\right)^{7/11}\left[1+\frac{31945}{1452}\frac{\alpha_S(p)}{4\pi}\right]}_{c^{\mathrm{RGI}}(p)} \times\frac{\langle\bar{q}q\rangle^{\mathrm{RGI}}}{p^2}+\mathcal{O}\left(1/p^4\right). \tag{18}$$

3. Lattice data and extraction of the quark condensate

We work with the $\mathcal{O}(a)$ improved Wilson quark action and use the data-sets consisting of $\mathcal{O}(1000)$ independent gauge field configurations, obtained at four different lattice spacings, corresponding to $\beta=6.0$, 6.2, 6.4, and 6.45. More complete information about the data-sets, as well as the improvement coefficients with the appropriate list of references can be found in Refs. [4,12]. Since we work at four different lattice spacings, we are able to extrapolate to the continuum limit. To eliminate the lattice spacing from the results obtained at each lattice coupling, we use the ratio a/r_0 computed in Ref. [13],

$$\left(\frac{a}{r_0}\right)_\beta = \{0.1863_{6.0},\ 0.1354_{6.2},\ 0.1027_{6.4},\ 0.0962_{6.45}\}, \tag{19}$$

so that all our results will be expressed in units of the scale r_0. To convert into physical units we will use $r_0=0.530(25)$ fm, which corresponds to $a^{-1}_{\beta=6.0}=2.0(1)$ GeV. We will also need the quenched value of Λ_{QCD}, for which we take $r_0\Lambda^{n_{\mathrm{F}}=0}_{\overline{\mathrm{MS}}}=0.602(48)$ [14].[6]

[6] In physical units, $\Lambda^{n_{\mathrm{F}}=0}_{\overline{\mathrm{MS}}}=0.225(20)$ MeV.

3.1. $\langle\bar{q}q\rangle$ from the pseudoscalar vertex

In order to determine the chiral condensate from the long distance behavior of the pseudoscalar vertex, we first need to extract the function $B(p^2)$. That is made by using 10 different vertex functions, 4 of which are computed with the external legs degenerate in the quark mass, and 6 nondegenerate. With these 10 points, for each p^2, we fit the data to the form (11), which we rewrite as

$$\Gamma_P\left(p^2,\rho_i,\rho_j\right) = \Gamma_P^{\mathrm{subtr.}}\left(p^2\right)+\frac{2B(p^2)}{Z_A(\rho_i+\rho_j)} + C\left(p^2\right)(\rho_i+\rho_j). \tag{20}$$

The illustration of this fit is provided in Fig. 1 for four values of p^2. We see that the presence of the Goldstone pole is indeed pronounced at moderately large values of p^2.

Once we identify the Goldstone contribution to the pseudoscalar vertex, we perform a number of fits to the form

$$\frac{B(p^2)}{\Sigma_1(p^2)} = c^{\mathrm{RGI}}(p)\frac{\langle\bar{q}q\rangle^{\mathrm{RGI}}}{p^2}+\frac{\gamma}{p^4}+\delta+\lambda p^2, \tag{21}$$

where the first term on the r.h.s. is the one that we are interested in (the coefficient c^{RGI} is defined in Eq. (18)), the second term is the subleading power correction, while the last two terms take into account possible contributions of lattice artifacts. To make use

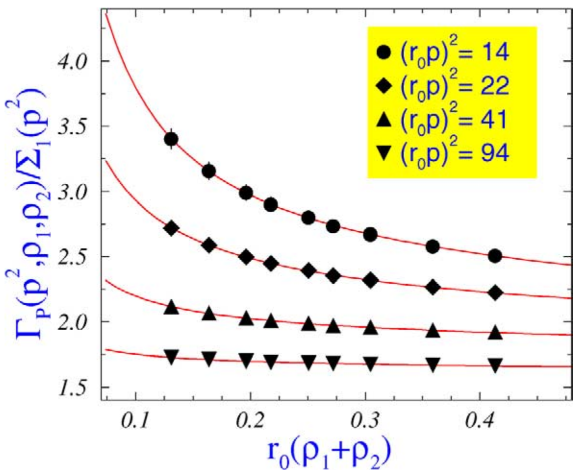

Fig. 1. Illustration of the fit to the form (20) at $\beta=6.2$, from which we could extract the function $B(p^2)$, needed for the determination of the chiral condensate.

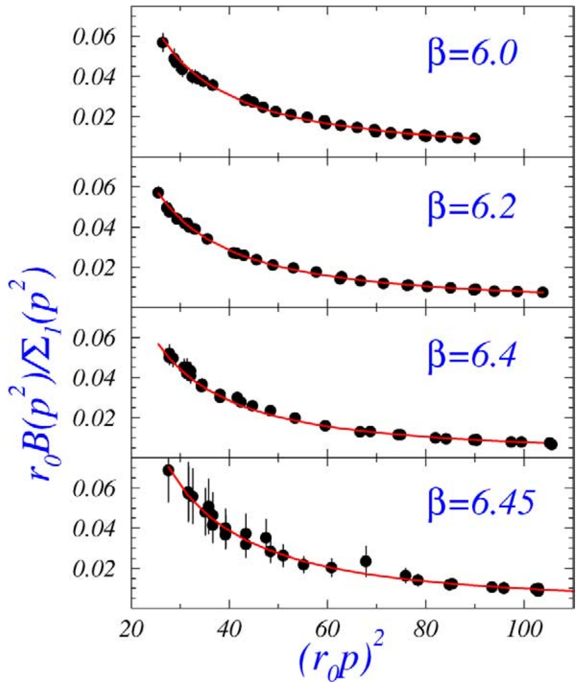

Fig. 2. Fit of the lattice data to the form (21) for all four lattice spacings considered in this Letter.

of the OPE formula we should aim at working at sufficiently large p^2 so that higher powers in $1/p^2$ are sufficiently suppressed. To do so, we fit the lattice data starting from $p_{\text{cut}} \approx 2$ GeV, which corresponds to $(r_0 p_{\text{cut}})^2 \approx 25$, for which the radiative correction term in $c^{\text{RGI}}(p)$ is below 35%. If we set $\gamma = \delta = \lambda = 0$ in (21), then for all our lattices we have $\chi^2/\text{d.o.f.} > 2$. Therefore, one has to let free at least one more parameter. The result of such a fit with $\gamma \neq 0$ is presented in Table 1 and denoted as fit I (see also Fig. 2 for illustration). At fixed lattice spacing, however, the lattice artifacts may be significant. To examine their impact on the value of the quark condensate, we repeat the fits by including either the term with p^2 ($\propto \lambda$) or the constant one ($\propto \delta$). Both sets of results are reported in Table 1, labelled as fit II and fit III, respectively. Finally, if we set $p_{\text{cut}} \gtrsim 3$ GeV, the fit with $\gamma = \delta = \lambda = 0$ gives a satisfactory $\chi^2/\text{d.o.f.}$ The corresponding results are denoted as fit IV in Table 1. We also tried to fit with all the parameters in Eq. (21) free (fit V in Table 1).

The remaining step towards the determination of the quark condensate is the extrapolation to the continuum limit. Since our action and the renormalization constants are $\mathcal{O}(a)$-improved, we may attempt extrap-

Table 1
Details of the fit of the lattice data to the form (21). Various fit forms (labelled as I, II, III, IV and V) are discussed in the text

Fit	β	$-r_0[\langle\bar{q}q\rangle^{\text{RGI}}]^{1/3}$	$r_0^5\gamma$	$-r_0\delta \times 10^3$	$-r_0^{-1}\lambda \times 10^6$
I	6.0	0.71 ± 0.01	23 ± 3	–	–
	6.2	0.72 ± 0.01	18 ± 1	–	–
	6.4	0.70 ± 0.02	22 ± 3	–	–
	6.45	0.75 ± 0.04	31 ± 9	–	–
II	6.0	0.82 ± 0.03	11 ± 5	–	29 ± 6
	6.2	0.74 ± 0.02	17 ± 2	–	3.4 ± 3.7
	6.4	0.73 ± 0.02	20 ± 3	–	4 ± 2
	6.45	0.73 ± 0.04	34 ± 12	–	-4 ± 4
III	6.0	0.90 ± 0.04	5 ± 6	5 ± 1	–
	6.2	0.75 ± 0.03	16 ± 3	1 ± 1	–
	6.4	0.75 ± 0.02	18 ± 3	1 ± 1	–
	6.45	0.71 ± 0.05	35 ± 14	-1 ± 1	–
IV	6.0	0.80 ± 0.02	–	–	–
	6.2	0.80 ± 0.01	–	–	–
	6.4	0.79 ± 0.02	–	–	–
	6.45	0.85 ± 0.06	–	–	–
V	6.0	1.14 ± 0.10	-22 ± 15	27 ± 11	112 ± 73
	6.2	0.90 ± 0.07	5 ± 6	9 ± 4	53 ± 27
	6.4	0.85 ± 0.07	10 ± 4	6 ± 4	24 ± 18
	6.45	0.87 ± 0.06	24 ± 11	5 ± 4	27 ± 19

olating quadratically in the lattice spacing, i.e.,

$$r_0\big[\langle\bar{q}q\rangle^{\rm RGI}\big]^{1/3}_{\beta} = C_0 + C_1(a/r_0)^2_{\beta}. \quad (22)$$

Our result for C_0, the chiral condensate in the continuum limit, for all four fit forms discussed above, are

$$r_0\big[\langle\bar{q}q\rangle^{\rm RGI}\big]^{1/3}_{\rm cont.} = \big\{-0.721(23)_{\rm I},\ -0.681(28)_{\rm II},\ -0.672(34)_{\rm III},\ -0.792(24)_{\rm IV},\ -0.54(15)_{\rm V}\big\}. \quad (23)$$

One may argue that terms of $\mathcal{O}(a)$ may still be present, since the function $B(p^2)$ is obtained from the off-shell vertex functions $\Gamma_P(p^2,\rho_i,\rho_j)$, for which the on-shell $\mathcal{O}(a)$ improvement does not apply. However, the function $B(p^2)$ refers to the chiral limit, and terms in $\Gamma_P(p^2,\rho)$ proportional to the quark mass are already taken care of in the fit to the form (11). In addition, it has been shown in appendix of Ref. [4] that, for these correlation functions, the $\mathcal{O}(a)$ contribution of operators which are either non gauge-invariant or vanish on-shell by the equation of motion vanish in the chiral limit. Therefore, while the $\mathcal{O}(a)$ effects may affect the functions $\Gamma^{\rm subtr.}(p^2)$ and $C(p^2)$ in Eq. (20) when away from the chiral limit, the function $B(p^2)$ is polluted by the artefacts $\mathcal{O}(a^2)$ and higher. This brings us back to the continuum extrapolation form (22).

What do we learn from the results (23) in the continuum limit? As it can be seen from Table 1, the corrections $\propto 1/p^4$ are large and positive for every β. Their neglect in the fit IV then expectedly lead to an overestimate of the value for the chiral condensate, as confirmed by the last number in Eq. (23). Fits II and III give quite consistent values for the condensate (in the $a\to 0$ limit). In other words, the quark condensate in the continuum limit is very weakly sensitive to the form of the artifacts that we include in our fits (constant or $\propto p^2$). The tendency of the artifacts, upon their inclusion in the fit, is to lower the value of the condensate. The same tendency is observed also in the fit form V, although with larger error bars.

As our final value we will quote the result of the fit I. The difference between the central value of that and the fits obtained by including the artifacts (II and III) is included in the systematic uncertainty. The result of the fit V has larger errors and is consistent with the results obtained by other fits. As we already pointed out, the radiative corrections are large and we take them into account when fitting the lattice data to Eq. (21). To account for the systematics induced by the omission of higher order corrections in $\alpha_S(p)$, we will add ±10% of uncertainty (which represents the square of the 30% effect of the known radiative corrections at $p=3$ GeV). Finally we have[7]

$$\begin{aligned}\langle\bar{q}q\rangle^{\rm RGI} &= -\big(269\pm 9^{+00}_{-18}\pm 12\ {\rm MeV}\big)^3 \pm 10\%\\ \Leftrightarrow \langle\bar{q}q\rangle^{\rm RGI} &= -(260\pm 9\pm 9\pm 12\ {\rm MeV})^3\pm 10\%\\ \Rightarrow \langle\bar{q}q\rangle^{\overline{\rm MS}}(2\ {\rm GeV}) &= -(312\pm 11\pm 11\pm 15\pm 10\ {\rm MeV})^3, \end{aligned} \quad (24)$$

where the errors are, respectively, statistical, systematics due to the continuum extrapolation, to the uncertainty in r_0 and to the uncertainty due to N^{2}LO corrections in the Wilson coefficient $c^{\rm RGI}(p)$ (see Eq. (18)). Notice that in the second line we symmetrised the systematic error bars.

Finally, we repeated the entire exercise by using the alternative quark mass definition, namely the one derived from the vector Ward identity, $m_q=\frac{1}{2}(1/\kappa_q-1/\kappa_{\rm crit})$, instead of the quark mass $Z_A\rho$, used above. The value we obtain in this way is barely distinguishable from the one we quoted in Eq. (24).[8]

3.2. $\langle\bar{q}q\rangle$ from the GMOR formula

We now repeat the standard exercise of extracting the value of the quark condensate by employing the GMOR formula. The values of the pseudoscalar meson and the quark masses are all listed in Table 2 of Ref. [12]. In Table 2 of the present Letter, we give the results obtained by using Eqs. (1) and (2), where we use for the quark mass the one defined via the axial Ward identity (ρ). The needed renormalization constants, Z_A and $Z_P^{\rm RI/MOM}(1/a)$, are given in Ref. [4]. For completeness, we also present the values of the (improved) pseudoscalar meson decay constant in the chiral limit, f, which is obtained by linearly extrapolating in the quark masses ($f_P = f + {\rm const}\cdot\rho$). To convert the quark condensate from the RI/MOM scheme to the RGI form, we use the anomalous dimension known up to 4-loops [15]. These latter results

[7] We remind the reader that $r_0=0.530(25)$ fm, is equivalent to $r_0=2.68(13)$ GeV^{-1}.

[8] More specifically, with m_q instead of ρ, we get $\langle\bar{q}q\rangle^{\overline{\rm MS}}(2\ {\rm GeV}) = -(313\pm 11\pm 13\pm 15\pm 10\ {\rm MeV})^3$.

Table 2
Results of the pseudoscalar decay constant in the chiral limit (f) and the chiral condensate, obtained by means of the GMOR formula (see Eqs. (1) and (2)) for all four lattice spacings. We also present the results of the linear extrapolation in a^2 to the continuum limit ($a \to 0$)

β	$r_0 f$	$-r_0[\langle\bar{q}q\rangle^{\text{RI/MOM}}(\mu a=1)]^{1/3}$	$-r_0[\langle\bar{q}q\rangle^{\text{RGI}}]^{1/3}$
6.0	0.360(7)	0.709(10)	0.611(9)
6.2	0.365(10)	0.727(14)	0.612(12)
6.4	0.358(11)	0.730(16)	0.605(13)
6.45	0.365(39)	0.743(50)	0.613(41)
∞	0.362(13)	–	0.601(25)

are then extrapolated to the continuum limit linearly in a^2 (see Eq. (22)). In physical units, our results read

$$\langle\bar{q}q\rangle^{\text{RGI}} = -(224 \pm 9 \pm 10\ \text{MeV})^3 \Rightarrow \langle\bar{q}q\rangle^{\overline{\text{MS}}}(2\ \text{GeV}) = -(273 \pm 11 \pm 15\ \text{MeV})^3. \tag{25}$$

We checked that this value is completely consistent with the alternative definition of the quark mass, namely, with $m_q = \frac{1}{2}(1/\kappa_q - 1/\kappa_{\text{crit}})$, and with $Z_S^{\text{RI/MOM}}(1/a)$ also given in Ref. [4].[9] Finally we also note that the above result agrees very well with the QCD sum rule estimate of Ref. [16], where $\langle\bar{q}q\rangle^{\overline{\text{MS}}}(2\ \text{GeV}) = -(267 \pm 16\ \text{MeV})^3$ has been quoted.

4. Summary and conclusion

We now briefly summarize our findings and comment on some results reported in the literature.

(1) We update the value of the chiral condensate obtained from quenched QCD on the lattice, by using the GMOR formula. After combining the errors given in Eq. (25) in the quadrature, we have

$$\langle\bar{q}q\rangle^{\overline{\text{MS}}}_{\text{GMOR}}(2\ \text{GeV}) = -(273 \pm 19\ \text{MeV})^3. \tag{26}$$

This result is obtained from simulations performed at four lattice spacings, by employing non-perturbative renormalization and $\mathcal{O}(a)$-improvement, followed by an extrapolation to the continuum limit.

(2) We compute the quark condensate by using an alternative strategy, namely, by studying the long distance (Goldstone) part of the pseudoscalar vertex function. In terms of the OPE, the chiral condensate appears in the coefficient of the leading power correction in $1/p^2$. From the calculations at four lattice spacings and after extrapolating to the continuum limit we obtain

$$\langle\bar{q}q\rangle^{\overline{\text{MS}}}_{\text{OPE}}(2\ \text{GeV}) = -(312 \pm 24\ \text{MeV})^3. \tag{27}$$

(3) From the above results, it seems that the two completely different strategies lead to quite a consistent value of the quark condensate. To better appreciate this point we rewrite the RGI results in units of r_0, i.e.,

$$-r_0\left[\langle\bar{q}q\rangle^{\text{RGI}}\right]^{1/3} = \left\{0.701(23)(20)(23)_{\text{OPE}},\ 0.601(25)_{\text{GMOR}}\right\}.$$

The method based on using OPE is less reliable since radiative and further power corrections are large. Even if we combine the errors in quadrature the agreement would be at the 2σ-level, which is far from what has been claimed in Ref. [5], where the OPE and GMOR results were argued to differ by a factor of 3.

Before closing this Letter, we should explain why our conclusion is qualitatively different from the one reported in Ref. [5]. The first difference is that in Eq. (11), besides the Goldstone term ($\propto 1/\rho$) we also allow for the presence of the term linear in quark mass. Such a term could not be studied in Ref. [5] since only three quark masses were considered. The net effect of this modification is that the function $B(p^2)$ becomes smaller. Secondly, in the OPE, we allow for the presence of the term $\propto 1/p^4$, and we find that, for moderately large momenta, this (subleading) power

[9] When using the quark mass defined via the vector Ward identity instead of the axial one, we get $\langle\bar{q}q\rangle^{\overline{\text{MS}}}(2\ \text{GeV}) = -(268 \pm 13 \pm 15\ \text{MeV})^3$.

correction is not negligible, while in Ref. [5] such a term was not found. Finally, we also accounted for the terms that are due to the lattice artifacts (see Eq. (22)), which further reduce the value of the condensate in the continuum limit. Such effects were not studied in Ref. [5] (where they only considered the data produced at $\beta = 6.0$). Notice also that the reference value of the chiral condensate considered in Ref. [5], was 20% smaller than the one we obtain here by using the GMOR formula.

Acknowledgement

We wish to thank our collaborators of Ref. [4], and especially Alain Le Yaouanc, for many useful discussions.

References

[1] G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rev. Lett. 86 (2001) 5008, hep-ph/0103063;
S. Pislak, et al., BNL-E865 Collaboration, Phys. Rev. Lett. 87 (2001) 221801, hep-ex/0106071.

[2] S. Descotes-Genon, N.H. Fuchs, L. Girlanda, J. Stern, hep-ph/0311120.

[3] H. Wittig, Nucl. Phys. B (Proc. Suppl.) 119 (2003) 59, hep-lat/0210025;
L. Giusti, F. Rapuano, M. Talevi, A. Vladikas, Nucl. Phys. B 538 (1999) 249, hep-lat/9807014;
T.W. Chiu, T.H. Hsieh, Nucl. Phys. B 673 (2003) 217, hep-lat/0305016.

[4] D. Becirevic, V. Gimenez, V. Lubicz, G. Martinelli, M. Papinutto, J. Reyes, hep-lat/0401033.

[5] J.R. Cudell, A. Le Yaouanc, C. Pittori, Phys. Lett. B 454 (1999) 105, hep-lat/9810058.

[6] G. Martinelli, C. Pittori, C.T. Sachrajda, M. Testa, A. Vladikas, Nucl. Phys. B 445 (1995) 81, hep-lat/9411010.

[7] S. Weinberg, Phys. Rev. D 8 (1973) 3497.

[8] D. Becirevic, V. Gimenez, V. Lubicz, G. Martinelli, Phys. Rev. D 61 (2000) 114507, hep-lat/9909082;
F.D.R. Bonnet, P.O. Bowman, D.B. Leinweber, A.G. Williams, J.B. Zhang, Phys. Rev. D 65 (2002) 114503, hep-lat/0202003.

[9] L. Giusti, A. Vladikas, Phys. Lett. B 488 (2000) 303, hep-lat/0005026.

[10] K.D. Lane, Phys. Rev. D 10 (1974) 2605;
H. Pagels, Phys. Rev. D 19 (1979) 3080.

[11] P. Pascual, E. de Rafael, Z. Phys. C 12 (1982) 127.

[12] D. Becirevic, V. Lubicz, C. Tarantino, SPQcdR Collaboration, Phys. Lett. B 558 (2003) 69, hep-lat/0208003.

[13] S. Necco, R. Sommer, Nucl. Phys. B 622 (2002) 328, hep-lat/0108008.

[14] S. Capitani, M. Luscher, R. Sommer, H. Wittig, ALPHA Collaboration, Nucl. Phys. B 544 (1999) 669, hep-lat/9810063.

[15] K.G. Chetyrkin, A. Retey, Nucl. Phys. B 583 (2000) 3, hep-ph/9910332.

[16] M. Jamin, Phys. Lett. B 538 (2002) 71, hep-ph/0201174.