

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)**SciVerse ScienceDirect**

Procedia - Social and Behavioral Sciences 40 (2012) 215 – 219

---

---

**Procedia**  
Social and Behavioral Sciences

---

---

APBITM 2012 International (Spring) Conference

## Optimal Policy Combinations of Abatement Subsidy and Pollution Tax in Vertical Oligopolies

Chul-Hi Park<sup>a</sup>, Seung-Leul Kim<sup>b</sup>, Sang-Ho Lee<sup>c</sup>, \*<sup>a,b</sup>Graduate Student of Economics, Chonnam National University, Kwangju, 500-757, Korea<sup>c</sup> Professor of Economics, Chonnam National University, Kwangju, 500-757, Korea,

---

### Abstract

This article investigates environmental regulations on eco-industry in vertical oligopolies, in which the upstream industry produces abatement goods reducing pollutants and the downstream industry produces consumption goods emitting pollutants. We devise the optimal combination of appropriate policy instruments and show that an optimal pollution tax should be used for the negative externality and output restrictions in final production, and an optimal abatement subsidy should incorporate the effect of upstream market restrictions on abatement activity. We also examine the welfare effect of the subsidy policy on the abatement technology in tax/subsidy combination.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of the Asia Pacific Business Innovation and Technology Management Society Open access under [CC BY-NC-ND license](http://creativecommons.org/licenses/by-nc-nd/4.0/).

Keywords: eco-industry; vertical oligopoly; abatement subsidy; pollution tax; endogenous entry;

---

### 1. Introduction

Recent concern over environmental policy in eco-industry has been increasing in part due to the importance of environmental technology innovation on pollution abatement. However, imperfect competition among environmental firms can restrict the production of abatement goods and thus have a direct negative impact on the environment [9]. Appropriate industrial regulation on the eco-industry has become an important topic for environmental policy in lessening gross emission.

The basic framework for environmental taxation on environmental firms in eco-industry was first introduced by [1] David and Sinclair-Desgagne. They showed that the market power of the eco-industry would bring about a higher pollution tax than the marginal social cost of damage. [2] Canton, et al. extended the analysis to the vertical Cournot oligopolies. They showed that if the pollution tax is the only available instrument used to regulate these distortions, the second-best optimal tax level depends on the market power between the eco-industry and the polluting industry in the vertical structure.

This article considers the eco-industry and investigates its effect of environmental regulations in vertical oligopolies, in which the upstream industry produces abatement goods reducing pollutants and the downstream industry produces consumption goods emitting pollutants. We devise the optimal

---

\* Corresponding author. Tel.: +82-62-530-1553; fax: +82-62-530-1559. [sangho@chonnam.ac.kr](mailto:sangho@chonnam.ac.kr).

combination of appropriate policy instruments and show that an optimal pollution tax should be used for the negative externality and output restrictions in final production, and an optimal abatement subsidy should incorporate the effect of upstream market restrictions on abatement activity. We also examine the welfare effect of subsidy policy on abatement technology in tax/subsidy combination.

**2. The Basic Model**

*2.1. Downstream Industry*

There are  $n$  symmetric downstream firms, indexed by  $i$ , where the amount of production of the firm is  $q_i$ . Each firm’s cost function is given by  $C_d(q_i)$ , where  $C'_d(q_i) > 0$  and  $C''_d(q_i) \geq 0$ . The inverse market demand function of the consumption good is given by  $P(Q)$  where  $Q = \sum q_i$  and  $P'(Q) < 0$ . However, production activity generates some pollution, which is denoted by an emission function,  $e(q_i)$ . This is identical for all firms, and it is assumed that  $e'(q_i) > 0$  and  $e''(q_i) > 0$ .

The downstream firms are regulated by environmental tax,  $t$ , levied on the amount of emissions. Thus, each firm has an incentive to reduce the environmental tax by using a cleanup activity that requires purchase of some specific abatement goods  $a_i$ , sold by upstream firms at a market price of  $r$ . We assume that the market-clearing price of abatement goods is determined by demand and supply in the eco-industry. That is, we eliminate the strategic interactions of the downstream firms and thus, they behave as price takers at the market equilibrium. The effectiveness of the abatement goods is given by a function,  $w(a_i)$  which measures the amount of pollution reduced by the purchase of  $a_i$ . We assume that this pollution abatement technology is characterized by decreasing marginal productivity, i.e.,  $w'(a_i) > 0$  and  $w''(a_i) \leq 0$ ; that is, more abatement goods consumed decrease the net amount of pollution with a decreasing rate. Then, the net amount of pollution can be defined as  $y_i(q_i, a_i) = e(q_i) - w(a_i)$ . Notice that we focus on the end-of-pipe pollution abatement, in which abatement activities are additively separable from the production process.

Each downstream firm compete with Cournot manner and wants to maximize its profit function over the two variables,  $q_i$  and  $a_i$ , the individual level of the production and the amount of purchased abatement goods, respectively.

$$\max_{q_i, a_i} \Pi_i = P(Q)q_i - C_d(q_i) - r \cdot a_i - t \cdot y_i(q_i, a_i) \tag{1}$$

The first-order necessary conditions for Cournot-Nash equilibrium output of consumption goods and consumption of abatement goods are as follows:

$$\frac{\partial \Pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - C'_d(q_i) - te'(q_i) \tag{2}$$

$$\frac{\partial \Pi_i}{\partial a_i} = -r - t[-w'(a_i)] = 0 \Rightarrow r = t \cdot w'(a_i) \tag{3}$$

*2.2 Upstream Industry*

There are  $m$  symmetric upstream firms, indexed by  $j$ , where the amount of abatement goods produced by the firm is  $a_j$ . Each firm’s cost function is given by  $C_u(a_j)$ , where  $C'_u > 0$  and  $C''_u > 0$ . We assume that upstream firms are supported by an abatement subsidy,  $s$ , based on the sales of abatement goods.

Then, given the market price of  $r$ , each upstream firm competes with Cournot manner and wants to maximize its profit function over the variable  $a_j$ , the individual level of production.

$$\max \Pi_j = r \cdot a_j - C_u(a_j) + s \cdot a_j \tag{4}$$

Since all firms in the upstream industry are able to anticipate the behaviors of downstream firms in (2) and (3), which are separable decisions, the upstream firms can anticipate the demand of the abatement goods. So, the profit function of the upstream firm can be changed to

$$\max \Pi_j = t \cdot w'(a_i) \cdot a_j - C_u(a_j) + s \cdot a_j$$

Furthermore, from the assumption that downstream firms are price-takers in the trade, the eco-industry market-clearing price for the abatement goods will be set at  $\sum a_i = \sum a_j$ ; that is,  $a_i = \frac{1}{n} \sum a_j$  at symmetric equilibrium in the downstream market. Then, the upstream firm's profit function can be changed as follows:

$$\max \Pi = t \cdot w' \left( \frac{1}{n} \sum_{j=1}^m a_j \right) \cdot a_j - C_u(a_j) + s \cdot a_j$$

The first-order necessary condition for Cournot-Nash equilibrium output of abatement goods can be written as

$$\frac{\partial \Pi_j}{\partial a_j} = t \cdot w' \left( \frac{1}{n} \sum_{j=1}^m a_j \right) + t \cdot w'' \left( \frac{1}{n} \sum_{j=1}^m a_j \right) \cdot \frac{a_j}{n} - C'_u(a_j) + s = 0 \tag{5}$$

Then, the symmetric equilibrium for identical upstream firms, in which  $n \cdot a_i = m \cdot a_j$ , yields the following condition:

$$t \left( w' \left( \frac{1}{n} \sum_{j=1}^m a_j \right) + w'' \left( \frac{1}{n} \sum_{j=1}^m a_j \right) \cdot \frac{a_j}{n} \right) + s = C'_u(a_j)$$

From equation (5), the decision of abatement production is determined by the shape of the pollution abatement technology, the marginal cost of producing abatement goods, the number of firms in both upstream and downstream industries, and the regulator's two instruments—abatement subsidy,  $s$ , and environmental tax,  $t$ .

### 3. Optimal Environmental Regulations

Let  $D(Y)$  denote environmental damages from pollution, where  $D'(Y) > 0, D''(Y) > 0$  and  $Y = ny_i$ . Then, social welfare is defined as the sum of consumers' and producers' surplus less the environmental damages in (6).

The regulator's problem is to choose the levels of output of consumption and abatement goods, maximizing the following social welfare function:

$$\max W = \int_0^{nq_i} P(u) du - n \cdot C_d(q_i) - m \cdot C_u(a_j) - D(Y) \tag{6}$$

The first-order necessary conditions for interior solutions can be written for the optimal allocation as follows:

$$\frac{\partial W}{\partial q_i} = n \left( P(Q) - C'_d(q_i) - D'(Y) \cdot e'(q_i) \right) = 0 \tag{7}$$

$$\frac{\partial W}{\partial a_j} = m \left( -C'_u(a_j) + D'(Y) \cdot w' \left( \frac{m}{n} a_j \right) \right) = 0 \tag{8}$$

Notice that where  $Q = nq_i$  and  $A = na_j = ma_j$  at the symmetric equilibrium. The solutions give the principle of marginal optimality, i.e., (7) says that market price of the consumption goods should be equal to marginal production cost of consumption goods plus marginal damage of production, and (8) says that marginal benefit of abatement goods on the environmental damage should be equal to the marginal cost of production of abatement goods.

Using market equilibrium conditions in equation (2) and the optimality conditions in (7), we have the optimal environmental tax:

$$t = D'(Y) + P'(Q)q_i / e'(q_i) = D'(Y) + P'(Q)Q / ne'(q_i) \tag{9}$$

If the regulator imposes an emission tax in (9) to downstream polluting firms, each firm produces the social optimum production in (7). Then, the optimal environmental tax in (9) is the sum of the distortion from environmental damages and the distortion from the downstream firm's market power per marginal emission. As we can see, the first term of environmental distortion is positive and the second term of market distortion is negative. Therefore, the environmental tax could be either positive or negative, depending on the relative size of the distortions from environmental damages and downstream firm's

market power, where a negative value for the environmental tax would correspond to a subsidy.<sup>b</sup> Notice that when competition is perfect, i.e.,  $n \rightarrow \infty$ , where the market power is insignificant, the optimal tax is exactly the same as the social marginal damage.

Similarly, using market equilibrium conditions in equation (5) and the optimality conditions in (8), we have the optimal environmental abatement subsidy:

$$s = D'(Y)w'(a_i) - t(w'(a_i) + w''(a_i)a_i / m) \tag{10}$$

If the regulator imposes a production subsidy in (10) to upstream firms in eco-industry, each firm produces the social optimum production in (8). Using the optimal tax in (9), we then have the optimal abatement subsidy:  $s = D'(Y)w''(a_i)a_i / m - P(Q)q_i / e'(q_i)w'(a_i)[m - \varepsilon_w] / m$  where  $\varepsilon_w = -w''(a_i) / w'(a_i)a_i > 0$ , which indicates the relative concavity of the abatement function.

A few remarks are in order. First, the optimal abatement subsidy in (10<sup>2</sup>) is also the combination of two distortions—environmental damages and downstream firm’s market power—with some weights on each distortion. This implies that the optimal abatement subsidy is closely related to the optimal environmental tax in (9). However, notice that  $t$  is solely determined irrespective of the size of  $s$ , while  $s$  should be adjusted according to the relative size of  $t$ . Second, when the downstream market is in a perfect competition, the optimal tax is positive,  $t = D'$  and the optimal subsidy is also positive,  $s = -D'w''a_i / m$ . Notice that this subsidy decreases as the number of environmental firms increases. In particular, when the upstream market is in perfect competition, i.e.  $m \rightarrow \infty$ , the optimal subsidy is zero.

#### 4. Example: Monopolistic Innovator

As an extension, we assume eco-industry consists of one monopolist who innovates the clean technology in upstream industry and licenses the patent to downstream firms and charge a usage fee. The innovator licenses  $k$  out of  $n$  firms. Let the licensee firms be denoted by  $L$  and un-licensee firms be  $N$ . For simplicity, we assume that the production cost of downstream firm and upstream<sub>k</sub> are zero and the inverse demand function for the final goods is linear,  $P = P(Q) = A - Q$  where  $Q = \sum q_i^L + \sum q_i^N$  and  $S$  is the set of licensees. Finally, followed by [8] Canton, J., David, M. and B. Sinclair<sup>2</sup> Desgagné, the emission function is assumed as  $y_i(q_i, a_i) = 1/2(q_i^L - a_i)^2$ .

We use a four stage-game to explain the licensing behaviors in the vertical model. At the 1st stage, the regulator announces the emission tax  $t$  and abatement subsidy  $s$  to the downstream market. At the 2nd stage, the innovator decides the price of clean technology  $r$  and the number of licensed firms  $k$ . At 3rd stage, after the announcement of emission tax/subsidy rates and the price of abatement technology device, polluting firms make a decision whether obtaining a license from innovating firm or not. Finally, at the 4th stage, given emission tax and the price of clean technology, polluting firms chooses their optimal level of  $q_i^L(q_i^N)$  and  $a_i$  (the quantity of abatement device) and compete in Cournot fashion.

In the followings, we compare the welfare effect of environmental regulation. In particular, we analyze whether tax/subsidy combination improves the social welfare, compared than the tax-alone regulation under the monopolistic innovator.

##### 4.1 Downstream Industry

Licensed firm maximizes the following profit function:

$$\max \pi_i^L = P(Q)q_i^L - ra_i - \frac{t}{2}(q_i^L - a_i)^2 \text{ where } i = 1, 2, \dots, n. \tag{11}$$

Non-licensed firm maximizes the following profit function:

$$\max \pi_i^N = P(Q)q_i^N - \frac{t}{2}(q_i^N)^2 \text{ where } i = 1, 2, \dots, n. \tag{12}$$

---

<sup>b</sup> On this point, see [3] Buchanan (1969) and [4] Barnett (1980) regarding regulating monopolist, and [5] Shaffer (1995) and [6] Lee (1999) regarding regulating oligopolies. [7] Requate synthesized important works on pollution tax under imperfect competition. All these research provided the rationale for the second-best solution of a higher/lower optimal tax level, depending upon the relative effects of distortions, such as market power, excessive entry, and externality

From the first-order conditions for (11) and (12), we get the following results:

$$q_i^L = [A(1+t) - ka_i t + a_i t(1+n+t)] / (1+t)(1+n+t) \quad \text{and} \quad q_i^N = [A(1+t) - ka_i t] / (1+t)(1+n+t).$$

Therefore,  $a_i = [(1+t)\{tA - r(1+n+t)\}] / t(1+n+t+tk)$ .

#### 4.2 Upstream Industry

First, with the abatement subsidy on clean technology, the innovator's profit maximization problem is  $\max \pi^M = rka_i + ska_i$  s.t.  $a_i = [(1+t)\{tA - r(1+n+t)\}] / t(1+n+t+tk)$ . Then, we have  $r^* = tA / (1+n+t)$ ,  $k^{r,k} = n$ , and  $a_i^*(r^*, k^*) = [(1+t)tA + s(1+n+t)] / 2t(1+n+t+tk^*)$ .

Next, without abatement subsidy, the innovator's profit maximization problem is  $\max \pi^M = rka_i$  s.t.  $a_i = [(1+t)\{tA - r(1+n+t)\}] / t(1+n+t+tk)$ . Then, we have  $\hat{r} = tA / 2(1+n+t)$ ,  $\hat{k}^{r,k} = n$ , and  $\hat{a}_i(\hat{r}, \hat{k}) = A / 2(1+n)$ . We considered that the number of patents and the price of abatement device are determined by the upstream firm. In this condition, monopolist firm sells the patent to all polluting firms to maximize profit.

Therefore, we know that  $\pi_i^{*L} > \hat{\pi}_i^L$ ,  $q_i^{*L} > \hat{q}_i^L$ ,  $p^* < \hat{p}$ ,  $a_i^* > \hat{a}_i$ , and  $\pi_M^* < \hat{\pi}_M$ .

#### 4.3 Regulator

The regulator maximizes the following social welfare function :

$$\max W = \int_0^Q P(u) du - \frac{n}{2} (q_i - a_i)^2 \quad \text{where} \quad Q = nq_i^{*L} \quad \text{or} \quad Q = n\hat{q}_i^L.$$

As a result, we get  $W^2 > \hat{W}$  under a positive subsidy to upstream firm. Therefore, a tax-subsidy scheme is more efficient than a tax alone.

### 5. Concluding Remarks

This article analyzed the relationship between pollution abatement technology and environmental policy in a vertical oligopoly structure, in which imperfect competitions among upstream environmental firms and downstream polluting firms are taken into consideration. In particular, we employed the appropriate combination of policy instruments, such as emission tax and abatement subsidy, to correct simultaneously for the pollution externality and output distortion. We show that a policy combination of tax-subsidy scheme is more efficient than a tax alone.

### References

- [1] David, M. and B. Sinclair-Desgagne., "Environmental regulation and the eco-industry," *Journal of Regulatory Economics* 2005, 28(2), 141-155.
- [2] Canton, J., A. Soubeyran. and H. Stahn, "Environmental taxation and vertical Cournot oligopolies: How eco-industries matter.," *Environmental and Resource Economics* 2008, 40, 369–382.
- [3] Buchanan, J. M., "External diseconomies, corrective taxes and market structures." *American Economic Review* 1969, 59, 174–177.
- [4] Barnett, A. H., "The Pigouvian tax rule under monopoly," *American Economic Review* 1980, 70, 1037–1041.
- [5] Shaffer, S., "Optimal linear taxation of polluting oligopolists," *Journal of Regulatory Economics* 1995, 7, 85–100.
- [6] Lee, S. H., "Optimal taxation for polluting oligopolists with endogenous market structure," *Journal of Regulatory Economics* 1999, 15, 293–308.
- [7] Requate, T., "Environmental policy under imperfect competition," In *The International Yearbook of Environmental and Resource Economics* 2006/2007, ed. T. Tietenberg and H. Folmer. Northampton, MA: Edward Elgar; 2007.
- [8] Canton, J., David, M. and B. Sinclair-Desgagne., "Environmental regulation and Horizontal Mergers in the Eco-industry," Working papers, Fondazione Eni Enrico Mattei 2008.
- [9] Hazlina Abdul Kadir, Reza Masinaei, Nasim Rahmani (2011), *Long-Term Effects of Bank Consolidation Program in a Developing Economy*, Journal of Asia Pacific Business Innovation and Technology Management .Volume 1, No. 1, P20-30