Reliability Evaluation for Shuffle Exchange Interconnection Network

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Abstract

Shuffle-exchange networks (SENs) have been generally considered as a convenient interconnection systems due to the size of their switching elements (SEs) and simple configuration. Evaluation of reliability performance has been attempted by researchers in the past. This paper is a depth study of reliability evaluation in shuffle exchange network. We propose a SEN with minus one stage (SEN-) and compared with three other types of SEN. The measurement includes three parameters; terminal, broadcast and network reliability.

1. Introduction

Interconnection networks offer an attractive and cost-effective solution to the communication and interconnection among system components \cite{1}. Advances in the parallel computing area have made Multistage Interconnection Networks (MINs) a practical networking preference to meet the increasing demands of high performance computing \cite{2}. Multistage interconnection networks are commonly recommended as connections in multiprocessor system or network switches \cite{3}. MINs are acknowledged as being an economical solution offering programmable data paths among functional modules in multiprocessor systems \cite{4}. These networks are typically employed with simple modular switches, consist of two input and two output switching elements \cite{5}.

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MINs with the Omega property are designed to connect a large number of processors to establish a multiprocessor system. An omega network has the qualities of better scalability and simple routing techniques. MINs represents a very significant class of interconnecting systems used for constructing interconnections for multiprocessor systems and communication networks. Reliability and performance of interconnection network systems largely depend on the interconnection of their components, as well as certain other factors. Reliability of the network is concerned with the capability of the network to carry out its preferred network operation successfully. A critical set of components is defined as a set of switching components, each from different groups, such that a network failure will occur if all the components become faulty simultaneously. There are three main types of reliability measure, that are important to MINs, namely terminal, broadcast and network reliability which are measured in this paper. These three parameters are determined to measure the reliability performance in the networks.

2. Literature Review

Advancement in the processor technology with higher processing capability, make an interconnection networks highly demand in computer networking area. Interconnection network provide the capability of connecting multiple processors allowing sharing of resources such as memory and processing time. Much research has been performed in investigating the communication process, particularly in area of interconnection networks. Methods to evaluate the performance include the simulation of shuffle exchange networks in MINs. Some research groups which deal with interconnection networks also apply shuffle exchange networks as a simulation method. However, many systems are too difficult to be determining by simulation method if detailed and accurate results are required. Then, measurement often turns out to be the only feasible solution. The performance evaluation of interconnection network is broadly based on the reliability performance in the networks. Furthermore, measurement also helps authenticate any results achieved by simulation methods. Therefore, this research additionally focusing on measurement related to the investigated performance evaluation methods called reliability performance. In this research the topology are focusing on multistage interconnection network. MINs consist of layers of switching element connected together in predefined topology providing the connectivity between input and output. MIN commonly used in circuit switching and packet switching networks. The interconnection design, number of stages, and the type of switching element used in the network configuration differentiate each MIN. MINs refer to networks where a unique path from an input to an input exists. For MINs size of $N \times N$ input/output consists of $c \times c$ switching elements with $n = \log_2 N$ stages. Fig. 1 shows the multistage interconnection network connection via number of stages between $p$ inputs and $b$ outputs, and connection within number of stages.

![Fig. 1. Multistage Interconnection Network.](image)

The network architecture illustrates the network topology and its physical realization by determining the parameter and kinds of the network elements in detail. The network topological only gives the structure of the connection between the nodes related to graph theory. The local parts of such a network are usually connected by switches. MINs are essentially involving a crossbar and shared bus networks with a various types of multiprocessor interconnections networks. MINs attempt to reduce cost and decrease the path length.
several types of MINs were proposed from the previous researchers. Table 1 shows the related works of MINs from the previous research.

<table>
<thead>
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<td>No</td>
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</tr>
</tbody>
</table>

2.1. Shuffle Exchange Network

Shuffle Exchange Network (SEN) is one network in topologically large class MINs that include the omega, indirect binary, cube, baseline, and generalized cube. The stage and number of switching elements may vary from each network. SEN consist of unique path MINs with only a single path between a particular input and output. The switching element in the network can transmit the inputs either straight or cross connections. The breakdown of any component in the interconnection network can influence the entire system to stop working. In Omega network the routing of a message from a given source to a given destination is based on the destination address. Normally MINs connects \( N \times N \) from source to the destination address. An eight input and eight output SEN with three stages, 12 switches (SEs), and 32 links is shown in Fig. 2.

![Fig. 2. Shuffle Exchange Network.](image)

2.2. Shuffle Exchange Network with Additional Stage

Shuffle Exchange Network with an additional stage (SEN+) is derived by additional of extra one stage in the SEN. It has \( n = \log_2 N + 1 \) stages and each stage has \( N/2 \) switching element. The switch complexity for the \( N \times N \) in SEN+ is equal to \( N/2 (\log_2 N + 1) \). The reason for the additional stage to the SEN is to allow two connection paths for communication among each source and destination. The \( 8 \times 8 \) SEN+ is shown in Fig. 3.
2.3. Shuffle Exchange Network with Two Additional Stage

Shuffle Exchange Network with two additional stages (SEN+2) is derived by addition of two extra stage in the network. The SEN+2 consist of \( \log_2 N + 2 \) stages with \( N \) inputs and outputs, \( N/2 \) switching element per stage, and \( N \log_2 N + 3 \) links. The network complexity can be defined as the total number of SEs in the MIN equivalents to, \( N/2 (\log_2 N + 2) \) for \( N \times N \) sizes in SEN+2. The 8 x 8 size for SEN+2 is shown in Fig. 4.

3. Proposed Shuffle Exchange Network with Minus One Stage

Interconnection networks offer an attractive and cost effective solution to communication and interconnection between system components. In this paper, a Shuffle Exchange Network with minus one stage (SEN-) that provide more reliability is proposed. SEN- has \( n = \log_2 N - 1 \) equal to two stages in the network. The reason for reducing the SEN by one stage is to allow communication along the paths between each source and destination with less conflict. By reducing the stages, the links complexity will be decreased and lead to a lower failure. Since it has a lesser SEs compared to other SEN, it can decrease a link complexity in the network and be able to avoid a system from failure. It also helps to reduce the number of links failure in the network. SEN- is more reliable than SEN, SEN+ and SEN+ based on the switching element increase the results shows that the reliability performance increase compared to SEN and Gamma network. SEN- with eight inputs and eight outputs consists of 8 switches (SEs) and 24 links is shown in Fig. 5.
4. Terminal Reliability Measurement

The terminal reliability of a MIN is described as the possibility of existence of at least one fault free path between a designated pair of input (s) and output (t)\(^17\). All the switches in a MIN are assumed to be of size 2 x 2 which is only has two possible states, straight and cross connection\(^10\). SEN has \(N\) input switches and \(N\) output switches and \(n\) stages, where \(n = \log_2 N\). Each stage consists of \(N/2\) switching elements. The probability of a switch being operational can be defined as \(r\). The terminal reliability for SEN can be calculated\(^{15}\) as follows:

\[
TR(SEN for N) = r^{\log_2 N}
\]  

(1)

SEN+ consists of two paths MIN. It has 4 stages 16 SEs and 40 links. SEN+ provides two connection paths for communication between each source and destination. Therefore, the terminal reliability for 8 x 8 SEN+ is higher than SEN. The terminal reliability for size 8 x 8 SEN+ can be formulated\(^{15}\) as follows:

\[
TR(SEN + for 8 x 8) = 2r^4 - r^6
\]

(2)

SEN+2 have one additional stage compared to SEN+, it has 5 stages in the network. It has 20 SEs and 48 links for eight inputs and eight outputs as shown in Fig. 4. It has four terminal paths for any pair of inputs and outputs. The terminal reliability for size 8 x 8 SEN+2 can be calculated\(^{15}\).

\[
TR(SEN + 2 for 8 x 8) = r^{10} - 2r^9(1 - r) + 8r^8 + (1 - r)^2 + 8r^7(1 - r)^3 + 2r^7(1 - r)^2
\]

\[
+ 4r^6(1 - r)^3 + 4r^6(1 - r)^2 + 4r^5(1 - r)^2
\]

(3)

The proposed SEN- consists of two stages for transmitting input from the source to the destination. It has \(N\) inputs and outputs. By reducing one stage from SEN it will result the decreasing number of links in the network. Therefore, the terminal reliability for SEN- is higher compared to other SENs. The terminal reliability for SEN- can be derived as follows:

\[
TR(SEN - for N) = r^{\log_2 N - 1}
\]

(4)
5. Broadcast Reliability Measurement

The broadcast reliability of a MIN is described as the possibility that a single input terminal is capable to broadcast data to entire output terminals in the network. A network is supposed to have failed when a connection cannot be made from the given input terminal to at least one of the output terminals. There is only one broadcast path in the SEN, if one failure occurs in the switches it will contribute to system failure. The broadcast reliability for SEN can be calculated as follows:

\[
BR(SEN\; for\; N) = r^{(N-1)}
\]

SEN+ has one additional stage in the network. By adding a stage in the network the broadcast path in SEN+ also increases. In SEN+ there are two broadcast paths from every input to the output. The broadcast reliability for size 8 x 8 SEN+ can be formulated as follows:

\[
BR(SEN\; +\; for\; 8\; x\; 8) = 2r^8 - 2r^9 - 4r^{10} + r^{11}
\]

Since SEN+2 have two extra stages adding in the network, the broadcast path in this network increases to four broadcast paths within the network. The broadcast reliability for size 8 x 8 SEN+2 can be calculated as follows:

\[
BR(SEN\; +\; 2\; for\; 8\; x\; 8) = r^{15} + 4r^{14} + 20r^{13} + (1 - r)^2 + 32r^{12}(1 - r)^3 + 16r^{11}(1 - r)^4 + 10r^{11}(1 - r)^2 + 12r^{10}(1 - r)^3 + 4r^9(1 - r)^2
\]

SEN- has reduced the SEs in the network by eliminating one stage in the network. It has only one broadcast path in the SEN- from every input and output in this network. For the N=8 network sizes, the broadcast reliability for SEN- can be derived as follows:

\[
BR(SEN\; -\; for\; N) = r^{(N^2-1)}
\]

6. Network Reliability Measurement

The network reliability of MINs is described as the possibility that all input terminals are connected to all the output terminals. By assuming the products of all possible permutations of SE states, the exact network reliability can be determined. SEN is a single path MINs, the failure of one switches will cause the system to fail. SEN consist of N/2 (log₂ N) switching element in series. The network reliability for SEN can be calculated as follows:

\[
NR(SEN\; for\; N) = r^{\frac{N^2}{2} \log_2 N}
\]

SEN+ has N inputs and N outputs with four stages in the network. Since the stages in the network increase, the SEs and links also increase. In these types of network there is 16 SEs and 40 links for the 8 x 8 size of network. The network reliability for size 8 x 8 SEN+ can be formulated as follows:

\[
NR(SEN\; +\; for\; 8\; x\; 8) = 2r^{12} + 4r^{14} - 8r^{15} + 3r^{16}
\]

Since SEN+2 have two extra stages adding in the network, the broadcast path in this network increases to four broadcast paths within the network. The broadcast reliability for size 8 x 8 SEN+2 can be calculated as follows:

\[
SEN+2\; have\; 20\; SEs\; in\; the\; network\; with\; 48\; links\; provided.\; It\; also\; has\; \log_2 N + 2\; stages\; equivalent\; of\; 5\; stages.\; Each\; stage\; consists\; of\; N/2\; switching\; element\; per\; stages.\; The\; network\; reliability\; for\; size\; 8\; x\; 8\; SEN+2\; can\; be
\]
calculated as follows:

\[
NR(SEN + 2 \text{ for } 8 \times 8) = r^{20} + 4r^{19}(1-r) + 36r^{18}(1-r)^2 + 120r^{17}(1-r)^3 + 168r^{16}(1-r)^4 + 2r^{16}(1-r)^2 + 168r^{16}(1-r)^4 + 20r^{15}(1-r)^3 + 16r^{14}(1-r)^6 + 14r^{14}(1-r)^4 + 4r^{13}(1-r)^3
\]  

(11)

SEN- consists of two stages with 24 links in the network. It also consists of \(N/2 \log_2 N\) switching elements equivalent 8 SEs in 8 x 8 size of network. The network reliability for SEN- can be derived as follows:

\[
NR(SEN - \text{ for } N) = r \left( \frac{N}{2} \right)^{(\log_2 N - 1)}
\]  

(12)

7. Results and Discussion

The comparison for the four types of SEN with 8 x 8 network size, in terms of terminal reliability is shown in Table 2. From the table it shows that the terminal reliability of SEN- is the highest approximately 11%, 3%, and 55%, as compared to SEN, SEN+ and SEN+2, respectively. The SEN+2 have the lowest reliability among other SENs. The additional paths increase the links complexity of the network, which will lead to higher failure. From the Fig. 6, it can be conclude that reducing one stage in the SEN is capable of improving the terminal reliability rather than adding a stage in the SEN.

Table 2. Terminal Reliability Comparison

<table>
<thead>
<tr>
<th>SEs</th>
<th>0.9</th>
<th>0.92</th>
<th>0.94</th>
<th>0.95</th>
<th>0.96</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEN-</td>
<td>0.8099</td>
<td>0.8464</td>
<td>0.8836</td>
<td>0.9025</td>
<td>0.9216</td>
<td>0.9604</td>
<td>0.9801</td>
</tr>
<tr>
<td>SEN</td>
<td>0.7209</td>
<td>0.7786</td>
<td>0.8305</td>
<td>0.8573</td>
<td>0.8847</td>
<td>0.9411</td>
<td>0.9702</td>
</tr>
<tr>
<td>SEN+</td>
<td>0.7807</td>
<td>0.8264</td>
<td>0.8716</td>
<td>0.8939</td>
<td>0.9159</td>
<td>0.9588</td>
<td>0.9797</td>
</tr>
<tr>
<td>SEN+2</td>
<td>0.5209</td>
<td>0.5792</td>
<td>0.6518</td>
<td>0.6946</td>
<td>0.7426</td>
<td>0.8567</td>
<td>0.9243</td>
</tr>
</tbody>
</table>

Fig. 6. Terminal Reliability Measurement.

Based on the Table 3 shown below, SEN- has the highest broadcast reliability approximately 52%, 31%, and 78%, compared to SEN, SEN+ and SEN+2, respectively. SEN- only has one path to broadcast the input to the output, consequently the broadcast reliability will increase since it has lower links complexity. As can observed from the Fig. 7, SEN+2 has the lowest broadcast reliability since it has four paths to broadcast the input to the output it leads to higher links failure in the network.
Table 3. Broadcast Reliability Comparison

<table>
<thead>
<tr>
<th>SEs</th>
<th>0.9</th>
<th>0.92</th>
<th>0.94</th>
<th>0.95</th>
<th>0.96</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEN-</td>
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<td>0.7786</td>
<td>0.8055</td>
<td>0.8573</td>
<td>0.8847</td>
<td>0.9411</td>
<td>0.9702</td>
</tr>
<tr>
<td>SEN</td>
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<td>0.5578</td>
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<td>0.6983</td>
<td>0.7514</td>
<td>0.8681</td>
<td>0.9321</td>
</tr>
<tr>
<td>SEN+</td>
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<td>0.6386</td>
<td>0.7188</td>
<td>0.7631</td>
<td>0.8079</td>
<td>0.9019</td>
<td>0.9504</td>
</tr>
<tr>
<td>SEN+2</td>
<td>0.4087</td>
<td>0.4757</td>
<td>0.5596</td>
<td>0.6101</td>
<td>0.6679</td>
<td>0.8098</td>
<td>0.8978</td>
</tr>
</tbody>
</table>

Fig. 7. Broadcast Reliability Measurement.

The comparisons of the network reliability for four types of network are shown in Table 4. The results shown that the SEN- has the highest network reliability, approximately 52%, 10%, and 63%, compared to SEN, SEN+ and SEN+2, respectively. From Fig. 8, it can be concluded that by reducing one stage, the network reliability performance will increase in the network.

Table 4. Network Reliability Comparison

<table>
<thead>
<tr>
<th>SEs</th>
<th>0.9</th>
<th>0.92</th>
<th>0.94</th>
<th>0.95</th>
<th>0.96</th>
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<tr>
<td>SEN-</td>
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<td>0.5132</td>
<td>0.6095</td>
<td>0.6634</td>
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<td>0.6487</td>
<td>0.7161</td>
<td>0.8476</td>
<td>0.9218</td>
</tr>
<tr>
<td>SEN+2</td>
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<td>0.3296</td>
<td>0.4214</td>
<td>0.4801</td>
<td>0.5498</td>
<td>0.7335</td>
<td>0.8542</td>
</tr>
</tbody>
</table>

Fig. 8. Network Reliability Measurement.
8. Conclusion

In this paper, SEN- is derived based on some modification from SEN topology. As an illustration, the comparisons between the others topology including SEN, SEN+ and SEN+2 with three types of reliability performance are evaluated. It can be seen from the results that the SEN- topology provides the highest reliability for the performance of all three parameters compared to others topology. SEN- is designed to increase the reliability performance and utilize the links in the network. Since it has a fewer SEs compared to the others topology, it can decrease the link complexity in the network and can prevent failure of the system. It can be concluding that SEN- is a good choice for high performance computing and communication applications. SEN- is more reliable than others topology based on the network size and also shows that the reliability performance gained in comparison to SEs. Some future work can be considered to extend this work by increasing the size of network to evaluate the reliability performance and investigate the links failure in the interconnection network.

Acknowledgements

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References