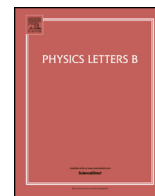Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Test of conformal gravity with astrophysical observations

CrossMark

Rongjia Yang [a,b,*], Bohai Chen [a], Haijun Zhao [c], Jun Li [a], Yuan Liu [d]

[a] *College of Physical Science and Technology, Hebei University, Baoding 071002, China*
[b] *Department of Physics, Tsinghua University, Beijing 100084, China*
[c] *School of Physics and Information Science, Shanxi Normal University, Linfen 041004, China*
[d] *Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-3, Beijing 100049, China*

ARTICLE INFO

ABSTRACT

Since it can describe the rotation curves of galaxies without dark matter and can give rise to accelerated expansion, conformal gravity attracts much attention recently. As a theory of modified gravity, it is important to test conformal gravity with astrophysical observations. Here we constrain conformal gravity with SNIa and Hubble parameter data and investigate whether it suffers from an age problem with the age of APM 08279+5255. We find conformal gravity can accommodate the age of APM 08279+5255 at 3σ deviation, unlike most of dark energy models which suffer from an age problem.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

Many astronomical observations indicate that the Universe is undergoing late-time acceleration. An unknown energy component, dubbed as dark energy, is usually proposed to explain the accelerated expansion. The simplest and most attractive candidate is the cosmological constant model (ΛCDM). This model is consistent with most of current astronomical observations, but suffers from the cosmological constant problem [1], as well as age problem [2]. It is thus natural to pursue alternative possibilities to explain the mystery of the accelerated expansion. Over the past numerous dark energy models have been proposed, such as quintessence, phantom, k-essence, quintom, tachyon, etc. Rather than by introducing a dark energy, modified gravity, such as $f(R)$ theories (for reviews, see e.g. [3–6]), $f(T)$ theories (see e.g. [7,8]), and $f(R, R_{\mu\nu}R^{\mu\nu})$ [9–11] theories, are proposed as ways to obtain a late accelerated expansion by modifying the Lagrangian of general relativity. Conformal gravity (CG, following the original work by Weyl [12], for reviews, see [13–15]), as a special $f(R, R_{\mu\nu}R^{\mu\nu})$ theory, can give rise to accelerated expansion [16]. It was also claimed that CG can describe the rotation curves of galaxies without dark matter [17]. A static solution for a charged black hole in CG was presented in [18]. Perturbations in the cosmology associated with the conformal gravity theory were investigated in [19]. It had been shown that currently available SNIa and GRB samples were accommodated well by CG [21]. Although CG attracts much attention, it is also confronted with some challenges. It was argued in [22] that in the limit of weak fields and slow motions CG

does not agree with the predictions of general relativity, and it is therefore ruled out by Solar System observations (recently, however, it was indicated in [23] that conformal gravity can potentially test well against all astrophysical observations to date). CG cannot describe the phenomenology of gravitational lensing [24] and of clusters of galaxies [25]. CG is not able to explain the properties of X-ray galaxy clusters without resorting to dark matter [26]. CG cannot pass the primordial nucleosynthesis test [27], however, this is an open problem, because all the possible mechanisms producing deuterium are still incomplete.

Besides the dark energy problem, the age problem is another important test for cosmological models. A spatially flat Friedmann–Robertson–Walker (FRW) universe dominated by matter (with age $T = 2/3H_0$), for example, is ruled out unless $h < 0.48$ ($h = H/100 \, \mathrm{km \, s^{-1} \, Mpc^{-1}}$) [28], compared with the 14 Gyr age of the Universe inferred from old globular clusters. Introducing dark energy cannot only explain the accelerated expansion, but also reconcile the age problem. However, the discovery of an old quasar APM 08279+5255 at $z = 3.91$ which was initially estimated to be around 2–3 Gyr [29] and re-evaluated to be 2.1 Gyr [30] has once again led to an age problem for cosmological models, such as ΛCDM [30,31], the creation of cold dark matter models [32], $\Lambda(t)$ model [33], the new agegraphic dark energy [34], parametrized variable Dark Energy Models [35,36], the $f(R) = \sqrt{R^2 - R_0^2}$ model [37], quintessence [38,39], holographic dark energy model [40,41], braneworld modes [42–45], and other models [46–48]. Most of these researches imposed a priori on the Hubble constant H_0, or on the matter density parameter Ω_{m0}, or on other parameters. To a certain extent, the age problem is dependent on the values of H_0 or Ω_{m0} one takes. In [2], the age problem in ΛCDM had been

* Corresponding author.
E-mail address: yangrongjia@tsinghua.org.cn (R. Yang).

investigated in a consistent way, not any special values of Ω_{m0} or H_0 have been taken with prejudice.

As a theory of modified gravity, it is important to test conformal gravity with astrophysical observations. Here we aim to test CG with observational data including the age of APM 08279+5255. Following [2], we obtain directly observational constraints on parameters from SNIa and $H(z)$ data in the framework of the CG, then investigate whether it suffers from an age problem in the parameter space allowed by these observations.

The structure of this Letter is as follows. In Section 2, we review the cosmology in CG. In Section 3, we consider constraints on the parameters of the cosmology in CG from SNIa and $H(z)$ data, and use the best-fit values to discuss the "age problem". Conclusions and discussions are given in Section 4.

2. Cosmology in conformal gravity

The action of CG with matter is given by

$$\mathcal{I} = -\alpha_g \int C_{\mu\nu\kappa\lambda} C^{\mu\nu\kappa\lambda} \sqrt{-g}\, \mathrm{d}^4 x + \mathcal{I}_m, \tag{1}$$

where α_g is a dimensionless coupling constant and $C_{\mu\nu\kappa\lambda}$ the Weyl tensor. This action is invariant under local conformal transformations: $g_{\mu\nu} \to e^{2\alpha(x)} g_{\mu\nu}$. This symmetry forbids the presence of any $\Lambda\sqrt{-g}\, \mathrm{d}^4 x$ term in the action, so CG does not suffer from the cosmological constant problem. Secondly, since α_g is dimensionless, unlike general relativity CG is renormalizable [15]. Thirdly, though the equations of motion are fourth-order, CG is a ghost-free theory [49]. However, CG is also confronted with some challenges as discussed in the previous section. The matter action can be of the form [27,50]

$$\mathcal{I}_m = -\int \sqrt{-g}\, \mathrm{d}^4 x$$
$$\times \left[\frac{1}{2} S^{;\mu} S_{;\mu} + \lambda S^4 - \frac{1}{12} S^2 R + i\overline{\psi}\gamma^\mu D_\mu \psi - \zeta S\overline{\psi}\psi \right], \tag{2}$$

where scalar field $S(x)$ is introduced to spontaneously break the conformal symmetry and renders the particles massive, ψ is a fermion field representing all matter field, $D_\mu = \partial_\mu + \Gamma_\mu$ is the covariant derivative with Γ_μ the fermion spin connection, λ and ζ are dimensionless coupling constants, γ^μ are the general relativistic Dirac matrices. λS^4 represents the negative minimum of the Ginzburg–Landau potential [16] with $\lambda < 0$. For action (1), variation with respect to the metric generates the field equations

$$4\alpha_g W_{\mu\nu} = T_{\mu\nu}, \tag{3}$$

where

$$W_{\mu\nu} = -\frac{1}{6} g_{\mu\nu} R^{;\lambda}_{;\lambda} + \frac{2}{3} R_{;\mu;\nu} + R^{;\lambda}_{\mu\nu;\lambda} - R^{;\lambda}_{\lambda\nu;\mu} - R^{;\lambda}_{\lambda\mu;\nu}$$
$$+ \frac{2}{3} R R_{\mu\nu} - 2R_{\mu\lambda} R^\lambda_\nu + \frac{1}{2} g_{\mu\nu} R_{\lambda\kappa} R^{\lambda\kappa} - \frac{1}{6} g_{\mu\nu} R^2, \tag{4}$$

and the energy–momentum tensor of matter is

$$T^{\mu\nu} = i\overline{\psi}\gamma^\mu D^\nu \psi + \frac{2}{3} S^{;\mu} S^{;\nu} - \frac{1}{6} g^{\mu\nu} S^{;\kappa} S_{;\kappa} - \frac{1}{3} S S^{;\mu;\nu}$$
$$+ \frac{1}{3} g^{\mu\nu} S S^{;\kappa}_{;\kappa} - g^{\mu\nu} \lambda S^4 - \frac{1}{6} S^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right). \tag{5}$$

By using local conformal invariance, the energy–momentum tensor can be written as

$$T^{\mu\nu} = T^{\mu\nu}_{kin} - \frac{1}{6} S_0^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) - g^{\mu\nu} \lambda S_0^4 \tag{6}$$

where $T^{\mu\nu}_{kin} = i\overline{\psi}\gamma^\mu D^\nu \psi$ and S_0 is a constant. According to Eq. (3) and $W^{\mu\nu} = 0$, we have

$$T^{\mu\nu}_{kin} - g^{\mu\nu} \lambda S_0^4 = \frac{1}{6} S_0^2 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right). \tag{7}$$

Considering a perfect fluid, $T^{\mu\nu}_{kin} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$, in the Friedmann–Robertson–Walker–Lemaître (FRWL) spacetime with the scale factor $a(t)$

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right], \tag{8}$$

where the spatial curvature constant $K = +1$, 0, and -1 corresponds to a closed, flat and open Universe, respectively, Eq. (7) takes the form

$$H^2 + \frac{K}{a^2} = -\frac{2\rho_m}{S_0^2} - 2\lambda S_0^2, \tag{9}$$

where $H = \dot{a}/a$ and ρ_m represents energy density of matter which can be separated into a relativistic and a non-relativistic component: $\rho_m = \rho_{nr} + \rho_r = \rho_{nr0}a^3 + \rho_{r0}a^4$. Taking substitutions $G = -3/(4\pi S_0^2)$ and $\Lambda = -6\lambda S_0^2$, Eq. (9) is identical to the standard Friedmann equation: $H^2 + K/a^2 = 8\pi G\rho/3 + \Lambda/3$. Both G and Λ, however, are negative and depend on the same parameter S_0^2 in the opposite way in CG. Constrained from the rotation curves of spiral galaxies, K must be negative: $K < 0$ [17].

If we define $\Theta_m \equiv 2\rho_m/(H^2 S_0^2) = \Theta_{nr} + \Theta_r$, $\Theta_\Lambda \equiv -2\lambda S_0^2/H^2$, and $\Theta_K \equiv -K/(a^2 H^2)$ (because $K < 0$ and $\lambda < 0$, all these parameters are positive), Eq. (9) yields: $\Theta_\Lambda + \Theta_K - \Theta_{nr} - \Theta_r = 1$. By taking the derivative of Eq. (9), we obtain

$$\frac{\ddot{a}}{a} = H^2 \left(\Theta_\Lambda + \Theta_r + \frac{\Theta_{nr}}{2} \right), \tag{10}$$

which is always positive, whereas the deceleration parameter

$$q \equiv -\frac{\ddot{a}a}{\dot{a}} = -\Theta_\Lambda - \Theta_r - \frac{\Theta_{nr}}{2}, \tag{11}$$

is always negative. So the expansion of the universe in CG accelerates at all times, unlike the standard cosmology. With the present values of Θ parameters, Eq. (9) can be re-expressed as

$$H^2 = H_0^2 \left(\Theta_{\Lambda 0} + \Theta_{K0}a^{-2} - \Theta_{nr0}a^{-3} - \Theta_{r0}a^{-4} \right), \tag{12}$$

which is analogous to the standard Friedmann equation, but has negative signs in front of the matter parameters Θ_{nr0} and Θ_{r0}. This equation implies that in CG a will reach a minimum value $a_{min} > 0$, rather than the singularity $a = 0$. By adjusting parameters, we can obtain smaller and smaller a_{min} (lager and lager z_{max}). Because there is no need to introduce dark matter in CG, terms Θ_{nr0} and Θ_{r0} can be neglect for large scale factor a, comparing with terms $\Theta_{\Lambda 0}$ and Θ_{K0}. The present age of the universe is found to be

$$H_0 t_0 = \frac{1}{\sqrt{-q_0}} \mathrm{atanh}(\sqrt{-q_0}). \tag{13}$$

3. Observational constraints on conformal gravity

In this section, we use the Union2.1 SNIa data and the observational Hubble parameter data to consider observational bounds on CG, and test it with the age of an old quasar by using the best-fit values constrained from SNIa and $H(t)$ data.

3.1. Constrain conformal gravity with SNIa and $H(t)$ data

The SNIa data provide the main evidence for the existence of dark energy in the framework of standard cosmology. The Union2.1 compilation, consisting of 580 SNIa data [51–53], is the largest published and spectroscopically confirmed SNIa sample to date. Each SNIa data point at redshift z_i includes the Hubble-parameter free distance modulus $\mu_{obs}(z_i)$ ($\equiv m_{obs} - M$, where M is the absolute magnitude) which is derived from the direct observables provided by the SNIa data and is not a directly observable quantity, and the corresponding error σ_i^2. The resulting theoretical distance modulus $\mu_{th}(z)$ is defined as

$$\mu_{th}(z) \equiv 5 \log_{10} d_L(z) + 25, \qquad (14)$$

where the luminosity distance in units of Mpc is expressed as [20, 21]

$$d_L = \frac{(1+z)^2}{q_0 H_0} \left[\left(1 + q_0 - \frac{q_0}{(1+z)^2} \right)^{1/2} - 1 \right]. \qquad (15)$$

This equation is obtained approximatively from Eq. (12) and therefore is dependent on the cosmological model considered here, but it can be safely applied to the real Universe at sufficiently late times [21].

In order to discuss the age problem in CG in a consistent way, we treat H_0 as a parameter rather than marginalize it over in data-fitting. Assuming the measurement errors are Gaussian, the likelihood function is $\mathcal{L} \propto e^{-\chi^2/2}$. The model parameters yield a minimal χ^2 and a maximal $\mathcal{L}$ will be favored by the observations. The χ^2 function for SNIa data is

$$\chi^2_{SNIa}(q_0, H_0) = \sum_{i=1}^{580} \frac{[\mu_L^{obs}(z_i) - \mu_L^{th}(z_i)]^2}{\sigma_i^2}. \qquad (16)$$

We also use 28 Hubble parameter data to constrain CG. Based on the work in [54], 9 $H(z)$ data were obtained by using the age of evolving galaxies [55]. These data were revised at 11 redshifts from the differential ages of red-envelope galaxies [56]. 2 $H(z)$ data were obtained by taking the BAO scale as a standard ruler in the radial direction [57]. 3 $H(z)$ data were acquired by combining measurements of the baryon acoustic peak and Alcock–Paczynski distortion from galaxy clustering in the WiggleZ Dark Energy Survey [58]. Recently, 8 $H(z)$ data were obtained from the differential spectroscopic evolution of early-type galaxies as a function of redshift [59]. Other 4 $H(z)$ data were presented in [60]. Observed values of the Hubble parameter can be used to constrain the parameters of CG. The χ^2 function of the $H(z)$ data is given by

$$\chi^2_H(q_0, H_0) = \sum_{i=1}^{28} \frac{[H_{obs}(z_i) - H_{th}(z_i)]^2}{\sigma_{H_i}^2}. \qquad (17)$$

Since the SNIa and $H(z)$ data are effectively independent measurements, we can minimize their total χ^2 value given by

$$\chi^2(q_0, H_0) = \chi^2_{SNIa} + \chi^2_H, \qquad (18)$$

to find the best-fit values of the parameters of CG.

Constraining CG only with SNIa data, we find the best-fit values of the parameters at 68.3% confidence as: $q_0 = -0.29 \pm 0.07$ and $H_0 = 69.15 \pm 0.57$ km s^{-1} Mpc^{-1} with $\chi^2_{min} = 575.41$ ($\chi^2_{min}/$dof $= 0.99$, dof is the logogram of degree of freedom), as shown in Table 1. If 28 $H(z)$ data points are also included in fitting, we find the best-fit values of the parameters at 68.3% confidence as: $q_0 = -0.33 \pm 0.06$ and $H_0 = 69.3 \pm 0.5$ km s^{-1} Mpc^{-1} with $\chi^2_{min} = 599.9$ ($\chi^2_{min}/$dof $= 0.99$), also as shown in Table 1.

Table 1
The best-fit values of the parameters (q_0, H_0) of CG with the corresponding formation redshift z_f and $\chi^2_{min}/$dof fitting from SNIa and SNIa + $H(z)$ observations with 1σ confidence level, here H_0 with dimension km s^{-1} Mpc^{-1}.

Observations	q_0	H_0	$\chi^2_{min}/$dof
SNIa	-0.29 ± 0.07	69.15 ± 0.57	0.99
SNIa + $H(z)$	-0.33 ± 0.06	69.3 ± 0.5	0.99

In [20], Mannheim obtained $q_0 = -0.37$ by constraining CG from 54 SNIa data. In [21], Diaferio et al. obtained the best-fit value as: $q_0 = -0.12^{+0.08}_{-0.16}$ constrained from 115 GRBs; $q_0 = -0.225^{+0.068}_{-0.066}$ constrained from 397 SNIa data; and $q_0 = -0.164^{+0.015}_{-0.022}$ constrained from 115 GRBs and 397 SNIa data. In [62], constraints on parameters at 1σ level in power–law cosmology were obtained as: $q_0 = -0.18 \pm 0.12$ and $H_0 = 68.4 \pm 2.8$ km s^{-1} Mpc^{-1} from $H(z)$ data, $q_0 = -0.38 \pm 0.05$ and $H_0 = 69.18 \pm 0.55$ km s^{-1} Mpc^{-1} from SNIa data, and $q_0 = -0.34 \pm 0.05$ and $H_0 = 68.93 \pm 0.53$ km s^{-1} Mpc^{-1} from the joint test using $H(z)$ and SNe Ia data. Recently, Planck 2013 results found a low value of the Hubble constant in the framework of ΛCDM model: $H_0 = 67.3 \pm 1.2$ km s^{-1} Mpc^{-1} [63]. Our constraints on parameters of CG are consistent with all these results.

In [21], a Bayesian approach has been used to infer the cosmological parameters, while a χ^2 procedure based on the distance modulus equation (14) is used here and in [20]. The distance modulus equation (14) is not a directly observable quantity but derives from assumptions on the cosmological model, in other words, the χ^2 procedure is model-dependent, while the Bayesian approach does not. Strictly speaking, a Bayesian approach is required to constrain CG with observations, as done in [21]. The difference between the Bayesian approach and the χ^2 procedure may can explain the two to three σ difference between the values of q_0 derived in [21] and the values derived in [20] or in our work. The degeneracies between different parameters may be another factor resulted to the different values of q_0 obtained from Bayesian approach or from χ^2 procedure.

3.2. Test conformal gravity with the age of old high-redshift objects

Old high-redshift objects are usually used to constrain parameters or test cosmological models [61]. Recently, many dark energy models have been tested with the age of APM 08279+5255, such as ΛCDM [30,31], the creation of cold dark matter models [32], $\Lambda(t)$ model [33], the new agegraphic dark energy [34], parametrized variable Dark Energy Models [35,36], the $f(R) = \sqrt{R^2 - R_0^2}$ model [37], quintessence [38,39], holographic dark energy model [40,41], braneworld modes [42–45], and other models [46–48], and it has been shown that none of these dark energy models can accommodate the age of quasar APM 08279+5255.

In order to investigate whether CG can accommodate the quasar APM 08279+5255, we must first understand the possible range of the age of APM 08279+5255. From *XMM-Newton* observations of APM 08279+5255, an iron overabundance of Fe/O of 3.3 ± 0.9 (the abundance ratio has been normalized to the solar value) has been derived for the broad absorption line system [64], and the age of APM 08279+5255 was estimated to lie within the interval 2–3 Gyr by using an Fe/O = 3 abundance ratio derived from X-ray observations [29]. In [30], the age of APM 08279+5255 was re-evaluated by using a chemodynamical model for the evolution of spheroids: an age of 2.1 Gyr was obtained when the Fe/O abundance ratio of the model reaches 3.3 which is the best-fit value acquired in [64]; an age of 1.8 Gyr was obtained when the Fe/O abundance ratio reaches 2.4 which is 1σ deviation from the best-fitting value; an age of 1.5 Gyr was set when the Fe/O abundance

ratio reaches 2 which is a highly improbable value of Fe/O, as it would require N_H in excess of 1.2×10^{23} cm^{-2} (see Fig. 3 in [64]), which seems to be ruled out from the determinations of $N_H = (5.3–9.1) \times 10^{22}$ cm^{-2} by other *Chandra* and *XMM* observations; even considering only the *XMM*2 data set, the lowest value of Fe/O is 2.4 at $N_H = 1.28 \times 10^{22}$ cm^{-2} (see Fig. 3 in [64]). So the age of APM 08279+5255 since the initial star formation has been estimated as [2]: (1) the best estimated value is 2.1 Gyr; (2) 1σ lower limit is 1.8 Gyr; (3) the lowest limit is 1.5 Gyr.

The age of a cosmic object (for example, a quasar, a galaxy, or a galaxy cluster) is defined as the difference between the age of the Universe at redshift z and the one when the object was born (at its formation redshift z_f)

$$T(z) = \int_{z_f}^{z} \frac{dz'}{(1+z')H(z')}. \tag{19}$$

To test CG with the age of APM 08279+5255, we must also know the formation redshift z_f of APM 08279+5255. This redshift, however, can only be inferred from available observational results. In many literatures, the z_f of APM 08279+5255 have been taken as infinity, which is incorrect. Only in [2], a finite z_f has been taken into account. Following [2], we will take a finite z_f to discuss the age problem in CG. Unlike in [2] the z_f of APM 08279+5255 in the frame of ΛCDM can be inferred from WMAP data: the peak epoch of reionizing was found to be at $z_{reion} = 17 \pm 10$ from WMAP1 data [65] and at $z_{reion} = 10.8 \pm 1.4$ from WMAP5 data [66], and the star formation processes could be inferred as early as $z = 15$–17 in high density peaks. The z_f of APM 08279+5255 in CG, however, should be inferred from observations which are independent on cosmological models. Results based on the new Hubble WFC3/IR imaging in the Ultradeep Field implied that the global star formation rate density might start from a very high value at $z \approx 10$ [67]. Taking into account this result and taking the results from WMAP data for reference, we take $z_f = 15$ as the formation redshift of APM 08279+5255 in the discussions.

Taking $q_0 = -0.29$ and $H_0 = 69.15$ km s^{-1} Mpc^{-1} fitted from SNIa data only, and according to Eq. (13), we find the present age of the Universe in CG is $t_0 = 15.82$ Gyr, larger than 14 Gyr estimated from old globular clusters [28]. CG accommodates the age of APM 08279+5255 at 3σ deviation, as shown in Fig. 1. Taking $q_0 = -0.33$ and $H_0 = 69.3$ km s^{-1} Mpc^{-1} fitted from SNIa and $H(t)$ data, we find the present age of the Universe in CG is $t_0 = 16.07$ Gyr, also larger than 14 Gyr estimated from old globular clusters [28]. In this case, CG also accommodates the age of APM 08279+5255 at 3σ deviation, as shown in Fig. 2.

4. Conclusions and discussions

We have constrained the conformal gravity with 580 SNIa data and 28 $H(t)$ data. We obtained the best-fit values of the parameter at 68% confidence level: $q_0 = -0.29 \pm 0.07$ and $H_0 = 69.15 \pm 0.57$ km s^{-1} Mpc^{-1} constrained from 580 SNIa data; and $q_0 = -0.33 \pm 0.06$ and $H_0 = 69.3 \pm 0.5$ km s^{-1} Mpc^{-1} constrained from 580 SNIa data and 28 $H(t)$ data. With these best-fit values of the parameters of CG, we test CG with the age of APM 08279+5255. We have found that CG can accommodate the best estimated value of the age of APM 08279+5255 at 3σ deviation.

In the discussions, we have ignored the contributions of radiation and non-relativistic matter. If their contributions are take into account, the age of APM 08279+5255 will be a little lager. In this cases, CG can more easily accommodate the best estimated value of the age of APM 08279+5255. We can conclude that unlike most of dark energy models CG does not suffer from a problem with
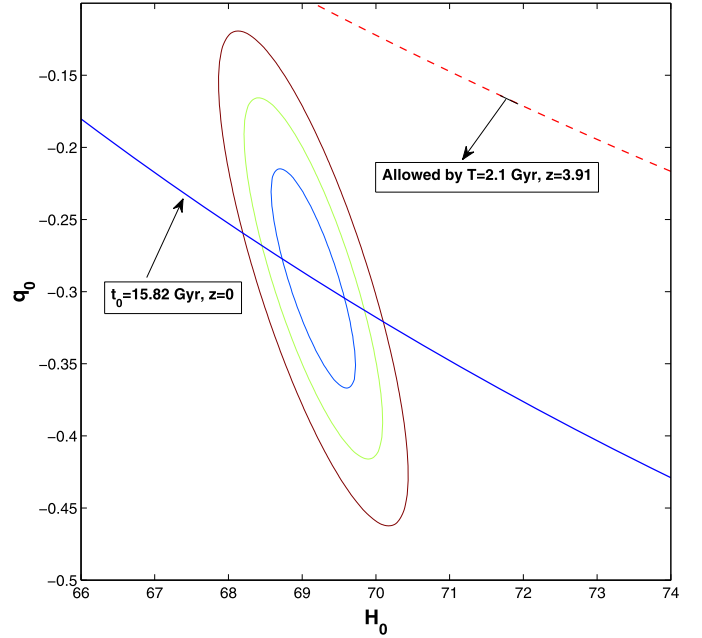

Fig. 1. The 68.3%, 95.4% and 99.7% confidence regions in the H_0-q_0 (H_0 with dimension km s^{-1} Mpc^{-1}) plane fitting from SNIa data. The line with $t_0 = 15.82$ Gyr at $z = 0$ and the region allowed by the age of APM 08279+5255 are also shown.
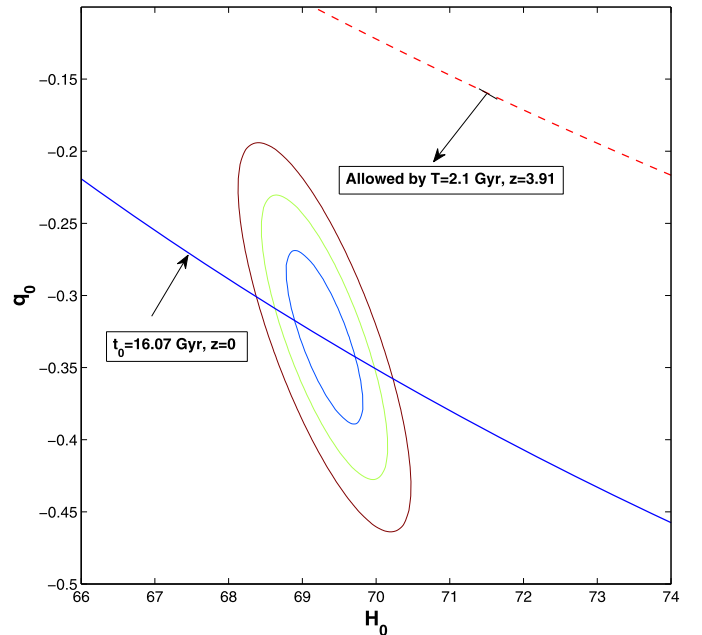

Fig. 2. The 68.3%, 95.4% and 99.7% confidence regions in the H_0–q_0 (H_0 with dimension km s^{-1} Mpc^{-1}) plane fitting from SNIa and $H(t)$ data. The line with $t_0 = 16.07$ Gyr at $z = 0$ and the region allowed by the age of APM 08279+5255 is also shown.

the estimated age of APM 08279+5255 at redshift $z = 3.91$, based on the values of Hubble constant H_0 and the deceleration parameter q_0 constrained from the currently available SNIa and $H(z)$ data. The results we obtained here can be tested with future cosmological observations.

References

[1] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1;
S.M. Carroll, Living Rev. Relativ. 4 (2001) 1.
[2] R.-J. Yang, S.N. Zhang, Mon. Not. R. Astron. Soc. 407 (2010) 1835.
[3] A.D. Felice, S. Tsujikawa, Living Rev. Relativ. 13 (2010) 3.
[4] T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. 82 (2010) 451.
[5] K. Bamba, S. Capozziello, S. Nojiri, S.D. Odintsov, Astrophys. Space Sci. 342 (2012) 155.
[6] T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Phys. Rep. 513 (2012) 1.
[7] G.R. Bengochea, R. Ferraro, Phys. Rev. D 79 (2009) 124019.
[8] R. Yang, Eur. Phys. J. C 71 (2011) 1797.
[9] S.M. Carroll, A.D. Felice, V. Duvvuri, D.A. Easson, M. Trodden, M.S. Turner, Phys. Rev. D 71 (2005) 063513.
[10] T. Clifton, J.D. Barrow, Phys. Rev. D 72 (2005) 123003.
[11] I. Navarro, K. Van Acoleyen, J. Cosmol. Astropart. Phys. 03 (2006) 008.
[12] H. Weyl, Reine Infinitesimalgeometrie, Math. Z. 2 (1918) 384.
[13] R.K. Nesbet, Entropy 15 (2013) 162.
[14] E. Scholz, arXiv:1111.3220.
[15] P.D. Mannheim, arXiv:1101.2186.
[16] P.D. Mannheim, Prog. Part. Nucl. Phys. 56 (2006) 340;
P.D. Mannheim, Astrophys. J. 561 (2001) 1.
[17] P.D. Mannheim, Astrophys. J. 479 (1997) 659;
J.G. O'Brien, P.D. Mannheim, Mon. Not. R. Astron. Soc. 421 (2012) 18351273.
[18] J.L. Said, J. Sultana, K.Z. Adami, Phys. Rev. D 86 (2012) 104009.
[19] P.D. Mannheim, Phys. Rev. D 85 (2012) 124008.
[20] P.D. Mannheim, Int. J. Mod. Phys. D 12 (2003) 893.
[21] A. Diaferio, L. Ostorero, V.F. Cardone, JCAP 10 (2011) 008.
[22] E.E. Flanagan, Phys. Rev. D 74 (2006) 023002.
[23] C. Cattani, M. Scalia, E. Laserra, I. Bochicchio, K.K. Nandi, Phys. Rev. D 87 (2013) 047503.
[24] S. Pireaux, Class. Quantum Gravity 21 (2004) 1897.
[25] K. Horne Mon, Not. R. Astron. Soc. 369 (2006) 1667.
[26] A. Diaferio, L. Ostorero, Mon. Not. R. Astron. Soc. 393 (2009) 215.
[27] D. Elizondo, G. Yepes, Astrophys. J. 428 (1994) 17.
[28] F. Pont, M. Mayor, C. Turon, D.A. Vandenberg, A&A 329 (1998) 87.
[29] S. Komossa, G. Hasinger, in: G. Hasinger, Th. Baller, A.N. Parmar (Eds.), XEUS Studying the Evolution of the Universe, MPE, Garching, 2003, p. 281, MPE Report.
[30] A. Friaca, J. Alcaniz, J.A.S. Lima, Mon. Not. R. Astron. Soc. 362 (2005) 1295.
[31] J.S. Alcaniz, J.A.S. Lima, J.V. Cunha, Mon. Not. R. Astron. Soc. 340 (2003) L39.
[32] J. Chen, P. Wu, H. Yu, Z. Li, Eur. Phys. J. C 72 (2012) 1861.
[33] J.V. Cunha, R.C. Santos, Int. J. Mod. Phys. D 13 (2004) 1321.
[34] Y.-H. Li, J.-Z. Ma, J.-L. Cui, Z. Wang, X. Zhang, Sci. China, Phys. Mech. Astron. 54 (2011) 1367.
[35] E.M. Barboza Jr., J.S. Alcaniz, Phys. Lett. B 666 (2008) 415.
[36] M.A. Dantas, J.S. Alcaniz, D. Jain, A. Dev, Astron. Astrophys. 467 (2007) 421.
[37] M.S. Movahed, S. Baghram, S. Rahvar, Phys. Rev. D 76 (2007) 044008.
[38] S. Capozziello, P.K.S. Dunsby, E. Piedipalumbo, C. Rubano, Astron. Astrophys. 472 (2007) 51.
[39] J.F. Jesus, R.C. Santos, J.S. Alcaniz, J.A.S. Lima, Phys. Rev. D 78 (2008) 063514.
[40] L.N. Granda, A. Oliveros, W. Cardona, arXiv:0905.1976.
[41] H. Wei, S.N. Zhang, Phys. Rev. D 76 (2007) 063003.
[42] M.S. Movahed, S. Ghassemi, Phys. Rev. D 76 (2007) 084037.
[43] M.S. Movahed, A. Sheykhi, Mon. Not. R. Astron. Soc. 388 (2008) 197.
[44] U. Alam, V. Sahni, Phys. Rev. D 73 (2006) 084024.
[45] N. Pires, Z.H. Zhu, J.S. Alcaniz, Phys. Rev. D 73 (2006) 123530.
[46] R.C. Santos, J.V. Cunha, J.A.S. Lima, Phys. Rev. D 77 (2008) 023519.
[47] G. Sethi, A. Dev, D. Jain, Phys. Lett. B 624 (2005) 135.
[48] E.M.C. Abreu, L.P.G. De Assis, C.M.L. dos Reis, Int. J. Mod. Phys. A 24 (2009) 5427.
[49] C.M. Bender, P.D. Mannheim, Phys. Rev. Lett. 100 (2008) 110402;
C.M. Bender, P.D. Mannheim, Phys. Rev. D 78 (2008) 025022.
[50] P.D. Mannheim, Astrophys. J. 391 (1992) 429.
[51] N. Suzuki, et al., Astrophys. J. 746 (2012) 85.
[52] R. Amanullah, et al., Astrophys. J. 716 (2010) 712.
[53] M. Kowalski, Astrophys. J. 686 (2008) 749.
[54] R. Jimenez, L. Verde, T. Treu, D. Stern, Astrophys. J. 593 (2003) 622.
[55] J. Simon, et al., Phys. Rev. D 71 (2005) 123001.
[56] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski, S.A. Stanford, J. Cosmol. Astropart. Phys. 02 (2010) 008.
[57] E. Gaztanaga, A. Cabré, L. Hui Mon, Not. R. Astron. Soc. 399 (2009) 1663.
[58] C. Blake, et al., Mon. Not. R. Astron. Soc. 425 (2012) 405.
[59] M. Moresco, et al., J. Cosmol. Astropart. Phys. 08 (2012) 006.
[60] C. Zhang, H. Zhang, S. Yuan, T.-J. Zhang, Y.-C. Sun, arXiv:1207.4541.
[61] J.A.S. Lima, J.F. Jesus, J.V. Cunha, Astrophys. J. Lett. 690 (2009) L85.
[62] S. Kumar, Mon. Not. R. Astron. Soc. 422 (2012) 2532–2538.
[63] Planck Collaboration, P.A.R. Ade, et al., arXiv:1303.5076.
[64] G. Hasinger, N. Schartel, S. Komossa, Astrophys. J. 573 (2002) L77.
[65] C.L. Bennett, et al., Astrophys. J. Suppl. Ser. 148 (2003) 1.
[66] J. Dunkley, et al., Astrophys. J. Suppl. Ser. 180 (2009) 306.
[67] H.-J. Yan, et al., Res. Astron. Astrophys. 10 (2010) 867.