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Fermion mixings vs $d = 6$ proton decay

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Abstract

It is well known, although sometimes ignored, that not only the $d = 5$ but also $d = 6$ proton decay depends on fermion mixings. In general we study carefully the dependence of $d = 6$ decay on fermion mixings using the effective operator approach. We find that without specifying a theory it is impossible to make clear predictions. Even in a given model, it is often not possible to determine all the physical parameters. We point out that it is possible to make a clear test of any grand unified theory with symmetric Yukawa couplings. We discuss in some detail realistic theories based on $SU(5)$ and $SO(10)$ gauge symmetry.

1. Introduction

The decay of the proton is the most dramatic prediction coming from matter unification. Since the paper by Pati and Salam in 1973 [1], proton decay has been the most important constraint for grand unified theories [2–13]. There are different operators contributing to the nucleon decay in GUTs, in supersymmetric scenarios the $d = 5$ contributions are the most important, but quite model dependent. They depend on the whole SUSY spectrum, on the structure of the Higgs sector and on fermion masses. In recent years these contributions have been under discussion, in order to understand if the minimal supersymmetric $SU(5)$ [2,3] is ruled out [14,15]. There are several solutions to this very important issue in the context of the minimal SUSY $SU(5)$ [16,17].

The $d = 6$ contributions for proton decay in general are the second more important, but they are less model dependent. From the non-diagonal part of the gauge field we get the gauge contributions, which basically depend only on fermion masses. The remaining $d = 6$ operators coming from the Higgs sector are less important and they are quite model dependent, since we can have different structures in the Higgs sector. There are several models where due to a specific structure of the Higgs sector, the $d = 5$ operators contributing to the decay of the proton are not present [18].

In general we study in detail the gauge $d = 6$ contributions. Assuming that in the future the decay of the proton will be measured, we analyze all possible information that we could get from these experiments. Using this

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information we will study the possibility to test the realistic grand unified theories based on the $SU(5)$ and $SO(10)$ gauge groups. Our analysis is valid in supersymmetric and non-supersymmetric GUT scenarios.

2. $d = 6$ operators

Using the properties of the Standard Model fields we can write down the possible $d = 6$ operators contributing to the decay of the proton, which are $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant [5–7]:

$$O_{\rm I}^{B-L} = k_1^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{u_{ia}^C}\gamma^\mu Q_{j\alpha a}\overline{e_b^C}\gamma_\mu Q_{k\beta b}, \tag{1}$$

$$O_{\rm II}^{B-L} = k_1^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{u_{ia}^C}\gamma^\mu Q_{j\alpha a}\overline{d_{kb}^C}\gamma_\mu L_{\beta b}, \tag{2}$$

$$O_{\rm III}^{B-L} = k_2^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{d_{ia}^C}\gamma^\mu Q_{j\beta a}\overline{u_{kb}^C}\gamma_\mu L_{\alpha b}, \tag{3}$$

$$O_{\rm IV}^{B-L} = k_2^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{d_{ia}^C}\gamma^\mu Q_{j\beta a}\overline{\nu_b^C}\gamma_\mu Q_{k\alpha b}. \tag{4}$$

In the above expressions $k_1 = g_{\rm GUT} M_{(X,Y)}^{-1}$, and $k_2 = g_{\rm GUT} M_{(X',Y')}^{-1}$, where $M_{(X,Y)}, M_{(X',Y')} \sim M_{\rm GUT} \approx 10^{16}$ GeV and $g_{\rm GUT}$ are the masses of the superheavy gauge bosons and the coupling at the GUT scale. $Q = (u, d)$, $L = (\nu, e)$; i, j and k are the color indices, a and b are the family indices, and $\alpha, \beta = 1, 2$.

The effective operators $O_{\rm I}^{B-L}$ and $O_{\rm II}^{B-L}$ (Eqs. (1) and (2)) appear when we integrate out the superheavy gauge fields $(X, Y) = (3, 2, 5/3)$, where the X and Y fields have electric charge $4/3$ and $1/3$, respectively. This is the case in theories based on the gauge group $SU(5)$. Integrating out $(X', Y') = (3, 2, -1/3)$ we obtain the operators $O_{\rm III}^{B-L}$ and $O_{\rm IV}^{B-L}$ (Eqs. (3) and (4)), the electric charge of Y' is $-2/3$, while X' has the same charge as Y. In $SO(10)$ theories all these superheavy fields are present. Notice that all these operators conserve $B - L$, i.e., the proton always decays into an antilepton. A second selection rule $\Delta S/\Delta B = -1, 0$ is satisfied for those operators [19].

Using the operators listed above, we can write the effective operators for each decay channel in the physical basis:

$$O\left(e_\alpha^C, d_\beta\right) = k_1^2 c\left(e_\alpha^C, d_\beta\right)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j \overline{e_\alpha^C}\gamma_\mu d_{k\beta}, \tag{5}$$

$$O\left(e_\alpha, d_\beta^C\right) = c\left(e_\alpha, d_\beta^C\right)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j \overline{d_{k\beta}^C}\gamma_\mu e_\alpha, \tag{6}$$

$$O\left(\nu_l, d_\alpha, d_\beta^C\right) = c\left(\nu_l, d_\alpha, d_\beta^C\right)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu d_{j\alpha}\overline{d_{k\beta}^C}\gamma_\mu \nu_l, \tag{7}$$

$$O\left(\nu_l^C, d_\alpha, d_\beta^C\right) = k_2^2 c\left(\nu_l^C, d_\alpha, d_\beta^C\right)\epsilon_{ijk}\overline{d_{i\beta}^C}\gamma^\mu u_j \overline{\nu_l^C}\gamma_\mu d_{k\alpha}, \tag{8}$$

where:

$$c\left(e_\alpha^C, d_\beta\right) = V_1^{11}V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta}\left(V_2 V_{UD}^\dagger\right)^{\alpha 1}, \tag{9}$$

$$c\left(e_\alpha, d_\beta^C\right) = k_1^2 V_1^{11}V_3^{\beta\alpha} + k_2^2\left(V_4 V_{UD}^\dagger\right)^{\beta 1}\left(V_1 V_{UD} V_4^\dagger V_3\right)^{1\alpha}, \tag{10}$$

$$c\left(\nu_l, d_\alpha, d_\beta^C\right) = k_1^2 (V_1 V_{UD})^{1\alpha}(V_3 V_{EN})^{\beta l} + k_2^2 V_4^{\beta\alpha}\left(V_1 V_{UD} V_4^\dagger V_3 V_{EN}\right)^{1l}, \tag{11}$$

$$c\left(\nu_l^C, d_\alpha, d_\beta^C\right) = \left(V_4 V_{UD}^\dagger\right)^{\beta 1}\left(U_{EN}^\dagger V_2\right)^{l\alpha} + V_4^{\beta\alpha}\left(U_{EN}^\dagger V_2 V_{UD}^\dagger\right)^{l1}, \quad \alpha = \beta \neq 2. \tag{12}$$

The mixing matrices $V_1 = U_C^\dagger U$, $V_2 = E_C^\dagger D$, $V_3 = D_C^\dagger E$, $V_4 = D_C^\dagger D$, $V_{UD} = U^\dagger D$, $V_{EN} = E^\dagger N$ and $U_{EN} = E^{C\dagger} N^C$. The quark mixings are given by $V_{UD} = U^\dagger D = K_1 V_{CKM} K_2$, where K_1 and K_2 are diagonal matrices containing three and two phases, respectively. The leptonic mixing $V_{EN} = K_3 V_l^D K_4$ in case of Dirac neutrino, or $V_{EN} = K_3 V_l^M$ in the Majorana case, V_l^D and V_l^M are the leptonic mixings at low energy in the Dirac and Majorana case, respectively.

Notice that in general to predict the lifetime of the proton due to the presence of $d = 6$ operators we have to know k_1, k_2, V_1^{1b}, V_2, V_3, V_4 and U_{EN}. In addition we have three diagonal matrices containing CP violating phases, K_1, K_2 and K_3, in the case that the neutrino is Majorana. In the Dirac case there is an extra matrix with two more phases.

3. Two body decay channels of the nucleon

As we know the gauge $d = 6$ operators conserve $B - L$, therefore the nucleon decays into a meson and an antilepton. Let us analyze all different channels. Assuming that in the proton decay experiments [20] one cannot distinguish the flavour of the neutrino and the chirality of charged leptons in the exit channel, and using the chiral Lagrangian techniques (see Ref. [21]), the decay rate of the different channels due to the presence of the gauge $d = 6$ operators are given by:

$$\Gamma\big(p \to K^+\bar{\nu}\big) = \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2} A_L^2 |\alpha|^2 \sum_{i=1}^{3} \left| \frac{2m_p}{3m_B} D c\big(\nu_i, d, s^C\big) + \left[1 + \frac{m_p}{3m_B}(D + 3F)\right] c\big(\nu_i, s, d^C\big) \right|^2, \quad (13)$$

$$\Gamma\big(p \to \pi^+\bar{\nu}\big) = \frac{m_p}{8\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad (14)$$

$$\Gamma\big(p \to \eta e_\beta^+\big) = \frac{(m_p^2 - m_\eta^2)^2}{48\pi f_\pi^2 m_p^3} A_L^2 |\alpha|^2 (1 + D - 3F)^2 \Big\{\big|c\big(e_\beta, d^C\big)\big|^2 + k_1^4 \big|c\big(e_\beta^C, d\big)\big|^2\Big\}, \quad (15)$$

$$\Gamma\big(p \to K^0 e_\beta^+\big) = \frac{(m_p^2 - m_K^2)^2}{8\pi f_\pi^2 m_p^3} A_L^2 |\alpha|^2 \left[1 + \frac{m_p}{m_B}(D - F)\right]^2 \Big\{\big|c\big(e_\beta, s^C\big)\big|^2 + k_1^4 \big|c\big(e_\beta^C, s\big)\big|^2\Big\}, \quad (16)$$

$$\Gamma\big(p \to \pi^0 e_\beta^+\big) = \frac{m_p}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \Big\{\big|c\big(e_\beta, d^C\big)\big|^2 + k_1^4 \big|c\big(e_\beta^C, d\big)\big|^2\Big\}, \quad (17)$$

$$\Gamma\big(n \to K^0\bar{\nu}\big) = \frac{(m_n^2 - m_K^2)^2}{8\pi m_n^3 f_\pi^2} A_L^2 |\alpha|^2 \times \sum_{i=1}^{3} \left| c\big(\nu_i, d, s^C\big)\left[1 + \frac{m_n}{3m_B}(D - 3F)\right] - c\big(\nu_i, s, d^C\big)\left[1 + \frac{m_n}{3m_B}(D + 3F)\right] \right|^2, \quad (18)$$

$$\Gamma\big(n \to \pi^0\bar{\nu}\big) = \frac{m_n}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad (19)$$

$$\Gamma(n \to \eta\bar{\nu}) = \frac{(m_n^2 - m_\eta^2)^2}{48\pi m_n^3 f_\pi^2} A_L^2 |\alpha|^2 (1 + D - 3F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad (20)$$

$$\Gamma\big(n \to \pi^- e_\beta^+\big) = \frac{m_n}{8\pi f_\pi^2} A_L^2 |\alpha|^2 (1 + D + F)^2 \Big\{\big|c\big(e_\beta, d^C\big)\big|^2 + k_1^4 \big|c\big(e_\beta^C, d\big)\big|^2\Big\}. \quad (21)$$

In the above equations m_B is an average baryon mass satisfying $m_B \approx m_\Sigma \approx m_\Lambda$, D, F and α are the parameters of the chiral Lagrangian, and all other notation follows [21]. Here all coefficients of four-fermion operators are evaluated at M_Z scale. A_L takes into account renormalization from M_Z to 1 GeV. $\nu_i = \nu_e, \nu_\mu, \nu_\tau$ and $e_\beta = e, \mu$.

Let us analyze all different channels. When the nucleon decays into a strange meson plus an antineutrino the amplitudes (Eqs. (13) and (18)) of these channels are proportional to a linear combination of the coefficients $c(\nu_i, s, d^C)$ and $c(\nu_i, d, s^C)$. In the case of the nucleon decays into a light unflavored meson plus an antineutrino, the amplitudes (Eqs. (14), (19) and (20)) are proportional to $\sum_{i=1}^{3} c(\nu_i, d, d^C)$. Looking at the channels with a charged antilepton, we see that the amplitudes (Eqs. (15), (17) and (21)) of the channels with a light meson are proportional to a linear combination of the coefficients $c(e_\alpha, d^C)$ and $c(e_\alpha^C, d)$, while in the case that we have a strange meson they are proportional to a linear combination of $c(e_\alpha, s^C)$ and $c(e_\alpha^C, s)$ (Eq. (16)). If the neutrinos are Dirac-like we have extra channels to the decay of the nucleon, where we have the decays into ν_i^C and a meson. The amplitudes in this case are proportional to $c(\nu_i^C, d, d^C)$, $c(\nu_i^C, s, d^C)$ and $c(\nu_i^C, d, s^C)$, respectively. Notice that from the radiative decays [22] we get the same information as in the case of the decays into a charged antilepton.

Note that from Eqs. (13)–(21) we can get only seven relations for all coefficients of the gauge $d = 6$ operators contributing to nucleon decay. Therefore, if we want to test a grand unified theory the number of physical quantities entering in the proton decay amplitude must be less than seven. This is an important result which helps us to know when it is possible to test a GUT scenario. However, as we will see in the next section looking only at the antineutrino channels we can get interesting predictions.

4. Testing GUT models

Let us analyze the possibility to test the realistic grand unified models, the $SU(5)$ and $SO(10)$ theories, respectively. Let us make an analysis of the operators in each theory, and study the physical parameters entering in the predictions for proton decay. Here we do not assume any particular model for fermion masses, in order to be sure that we can test the grand unification idea.

In these models the diagonalization of the Yukawa matrices is given by:

$$U_C^T Y_U U = Y_U^{\text{diag}}, \tag{22}$$

$$D_C^T Y_D D = Y_D^{\text{diag}}, \tag{23}$$

$$E_C^T Y_E E = Y_E^{\text{diag}}. \tag{24}$$

4.1. A GUT based on $SU(5)$

Let us start with the simplest grand unified theories, which are based on the gauge group $SU(5)$. In these theories the unification of quark and leptons is realized in two irreducible representations, 10 and $\bar{5}$. The minimal Higgs sector is composed by the adjoint representation Σ, and two Higgses 5_H and $\bar{5}_H$ in the fundamental and anti-fundamental representations, respectively [2,3], if we want to keep the minimal Higgs sector and write down a realistic $SU(5)$ theory, we need to introduce non-renormalizable operators, Planck suppressed operators, to get the correct quark–lepton mass relations. A second possibility is introduce a Higgs in the 45_H representation.

In this case we have only the operators O_{I}^{B-L} (Eq. (1)), and O_{II}^{B-L} (Eq. (2)) contributing to the decay of the proton. Using Eqs. (9)–(12), and taking $k_2 \equiv 0$ the coefficients for the proton decay predictions are given by:

$$c\big(e_\alpha^C, d_\beta\big)_{SU(5)} = V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta} \big(V_2 V_{UD}^\dagger\big)^{\alpha 1}, \tag{25}$$

$$c\big(e_\alpha, d_\beta^C\big)_{SU(5)} = k_1^2 V_1^{11} V_3^{\beta\alpha}, \tag{26}$$

$$c\big(\nu_l, d_\alpha, d_\beta^C\big)_{SU(5)} = k_1^2 (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l}, \quad \alpha = \beta \neq 2, \tag{27}$$

$$O\big(\nu_b^C, d_\alpha, d_\beta^C\big)_{SU(5)} = 0. \tag{28}$$

We see from these expressions that in order to make predictions in any theory based on the $SU(5)$ gauge group using proton decay, we have to know k_1, V_1^{1i} and the matrices V_2, and V_3. Note that in a $SU(5)$ theory there are not decays into a ν^C, even if the neutrino is a Dirac-like particle (see Eq. (28)), it could be a possibility to distinguish a $SU(5)$ theory of the rest of GUTs.

In order to compute the decay rate into antineutrinos we must use the following relation:

$$\sum_{l=1}^{3} c(\nu_l, d_\alpha, d_\beta)^*_{SU(5)} c(\nu_l, d_\gamma, d_\delta)_{SU(5)} = k_1^4 \left(V_1^* K_1^* V_{CKM}^*\right)^{1\alpha} \left(K_2^*\right)^{\alpha\alpha} \left(V_1 K_1 V_{CKM}\right)^{1\gamma} K_2^{\gamma\gamma} \delta^{\beta\delta}. \tag{29}$$

Using this expression we can see that the antineutrino channel depends on the matrices V_1 and K_1. Since we have only three independent equations (Eqs. (13), (14) and (18)) for these channels, it is clear that we cannot test a GUT model based on $SU(5)$. Notice that from the channels with charged leptons is even more difficult to get some information, due to the presence of the matrices V_2, V_3 and the elements V_1^{1i}. In the naive case without all CP violation sources beyond V_{CKM} we could get the information about V_1 from the nucleon decays into antineutrinos.

Let us analyze a particular case, the unrealistic minimal $SU(5)$ model, where $Y_U = Y_U^T$ and $Y_D = Y_E^T$ (see Ref. [23]), in this case we have the following relations:

$$c\left(e_\alpha^C, d_\beta\right)_{SU(5)}^{\text{unreal-min}} = \left(K_u^*\right)^{11} \left[\delta^{\alpha\beta} + V_{CKM}^{1\beta} K_2^{\beta\beta} \left(K_2^*\right)^{\alpha\alpha} \left(V_{CKM}^\dagger\right)^{\alpha 1}\right], \tag{30}$$

$$c\left(e_\alpha, d_\beta^C\right)_{SU(5)}^{\text{unreal-min}} = k_1^2 \left(K_u^*\right)^{11} \delta^{\beta\alpha}, \tag{31}$$

$$c\left(\nu_l, d_\alpha, d_\beta^C\right)_{SU(5)}^{\text{unreal-min}} = k_1^2 \left(K_u^*\right)^{11} K_1^{11} V_{CKM}^{1\alpha} K_2^{\alpha\alpha} V_{EN}^{\beta l}, \quad \alpha = \beta \neq 2. \tag{32}$$

Notice that in this naive GUT model, all the channels are determined by V_{CKM}. Unfortunately it is a prediction that we lost in the case of realistic versions of $SU(5)$. However, if this modification of the theory does not change the relation $Y_U = Y_U^T$, we could test a $SU(5)$ theory from the nucleon decays into an antineutrino (see Eq. (29)).

4.2. A GUT model with symmetric Yukawa couplings

There are many examples of grand unified theories with symmetric Yukawa couplings. This is the case of $SO(10)$ [4] theories with two Higgses 10_H and 126_H, including the minimal supersymmetric $SO(10)$ model [24,25].

In Ref. [26] has been investigated the dependence of the $d = 6$ gauge contributions on fermion mixings. They consider two different cases, the naive minimal $SO(10)$, where all fermion masses arise from Yukawa couplings to 10_H, and the case where we have the Higgses 10_H and 126_H. Assuming only two generations, and neglecting the possible mixings which appear when the neutrino mass matrix is diagonalized, they showed approximately that the predictions for the decay channels $p \to \pi^+ \bar{\nu}$ and $p \to K^0 l^+$ do not change in the different models for fermion masses. At the same time, it has been showed that the predictions for the decays $p \to K^0 e^+$, and $p \to \mu^+ \pi$ are quite different in these two scenarios for fermion masses.

In this section we will analyze the properties of all decays in those theories, using the fact that the Yukawa matrices are symmetric. We will take into account the mixings of the third generation and all possible CP violation effects.

In theories with symmetric Yukawa couplings we get the following relations for the mixing matrices, $U_C = U K_u$, $D_C = D K_d$ and $E_C = E K_e$, where K_u, K_d and K_e are diagonal matrices containing three CP violating phases. In those cases $V_1 = K_u^*$, $V_2 = K_e^* V_{DE}^\dagger$, $V_3 = K_d^* V_{DE}$ and $V_4 = K_d^*$. Using these relations the coefficients in Eqs. (9)–(12) are given by:

$$c\left(e_\alpha^C, d_\beta\right)_{\text{sym}} = \left(K_u^*\right)^{11} \left(K_e^*\right)^{\alpha\alpha} \left[\delta^{\beta i} + V_{CKM}^{1\beta} K_2^{\beta\beta} \left(K_2^*\right)^{ii} \left(V_{CKM}^\dagger\right)^{i1}\right] \left(V_{DE}^*\right)^{i\alpha}, \tag{33}$$

$$c\left(e_\alpha, d_\beta^C\right)_{\text{sym}} = \left(K_u^*\right)^{11} \left(K_d^*\right)^{\beta\beta} \left[k_1^2 \delta^{\beta i} + k_2^2 \left(K_2^*\right)^{\beta\beta} \left(V_{CKM}^\dagger\right)^{\beta 1} V_{CKM}^{1i} K_2^{ii}\right] \left(V_{DE}^{i\alpha}\right), \tag{34}$$

$$c\left(\nu_l, d_\alpha, d_\beta^C\right)_{\text{sym}} = \left(K_u^*\right)^{11} K_1^{11} \left[k_1^2 \delta^{\alpha i} \delta^{\beta j} + k_2^2 \delta^{\alpha\beta} \delta^{ij} \left(K_d^*\right)^{\alpha\alpha} K_d^{ii}\right] \left(V_{CKM} K_2\right)^{1i} \left(K_d^* V_{DE} V_{EN}\right)^{jl}, \tag{35}$$

$$c\left(\nu_l^C, d_\alpha, d_\beta^C\right)_{\text{sym}} = \left(K_d^*\right)^{\beta\beta} \left(K_1^*\right)^{11} \left[\left(K_2^*\right)^{\beta\beta} \left(V_{CKM}^\dagger\right)^{\beta 1} \delta^{\alpha i} + \delta^{\alpha\beta} \left(K_2^*\right)^{ii} \left(V_{CKM}^\dagger\right)^{i1}\right] \left(U_{EN}^\dagger K_e^* V_{DE}^\dagger\right)^{li}, \tag{36}$$

with $\alpha = \beta \neq 2$.

Notice all overall phases in the different coefficients. In order to compute the decay rate into an antineutrino we need the following expression:

$$\sum_{l=1}^{3} c(\nu_l, d_\alpha, d_\beta)^*_{\text{sym}} c(\nu_l, d_\gamma, d_\delta)_{\text{sym}} = \Big[k_1^2 \delta^{\alpha i}\delta^{\beta j} + k_2^2 \delta^{\alpha\beta}\delta^{ij} K_d^{\alpha\alpha}\big(K_d^*\big)^{ii}\Big]\Big[k_1^2 \delta^{\gamma i'}\delta^{\delta j} + k_2^2 \delta^{\gamma\delta}\delta^{i'j}\big(K_d^*\big)^{\gamma\gamma} K_d^{i'i'}\Big]\big(V_{CKM}^* K_2^*\big)^{1i}(V_{CKM}K_2)^{1i'}. \tag{37}$$

Using the above expression, and Eq. (13) we find that it is possible to determine the factor $k_1 = g_{\text{GUT}}/M_{(X,Y)}$:

$$k_1 = \frac{Q_1^{1/4}}{[|A_1|^2|V_{CKM}^{11}|^2 + |A_2|^2|V_{CKM}^{12}|^2]^{1/4}}, \tag{38}$$

where:

$$Q_1 = \frac{8\pi m_p^3 f_\pi^2 \Gamma(p \to K^+\bar{\nu})}{(m_p^2 - m_K^2)^2 A_L^2 |\alpha|^2}, \tag{39}$$

$$A_1 = \frac{2m_p}{3m_B} D, \tag{40}$$

$$A_2 = 1 + \frac{m_p}{3m_B}(D + 3F). \tag{41}$$

Notice that we have an expression for k_1, which is independent of the unknown mixing matrices and the CP violating phases. In other words, we find that the amplitude of the decay $p \to K^+\bar{\nu}$ is independent of all unknown mixings and CP violating phases, this only depends on the factor k_1. Therefore it is a possibility to test any grand unified theory with symmetric Yukawa matrices through this channel.

Once we know k_1, and using the expression (14) we can find the factor k_2, solving the following equation:

$$k_2^4 + 2k_2^2 k_1^2 \big|V_{CKM}^{11}\big|^2 + k_1^4 \big|V_{CKM}^{11}\big|^2 - \frac{8\pi f_\pi^2 \Gamma(p \to \pi^+\bar{\nu})}{m_p A_L^2 |\alpha|^2 (1 + D + F)^2} = 0, \tag{42}$$

$$k_2 = k_1 \big|V_{CKM}^{11}\big| \big\{-1 + \sqrt{Q_2}\big\}^{1/2}, \tag{43}$$

with:

$$Q_2 = 1 + \frac{8\pi f_\pi^2 \Gamma(p \to \pi^+\bar{\nu})}{k_1^4 |V_{CKM}^{11}|^4 m_p A_L^2 |\alpha|^2 (1 + D + F)^2} - \big|V_{CKM}^{11}\big|^{-2}. \tag{44}$$

Using the condition $Q_2 > 1$, we get the following relation:

$$\frac{\tau(p \to K^+\bar{\nu})}{\tau(p \to \pi^+\bar{\nu})} > \frac{m_p^4 |V_{CKM}^{11}|^2 (1 + D + F)^2}{(m_p^2 - m_K^2)^2 [|A_1|^2 |V_{CKM}^{11}|^2 + |A_2|^2 |V_{CKM}^{12}|^2]}. \tag{45}$$

It is a clear prediction of a GUT model with symmetric Yukawa couplings.

Using the expressions (13), (14), (18), (19), and (20) we can get the following relations:

$$\frac{\tau(n \to K^0\bar{\nu})}{\tau(p \to K^+\bar{\nu})} = \frac{m_n^3 (m_p^2 - m_K^2)^2 [|A_1|^2 |V_{CKM}^{11}|^2 + |A_2|^2 |V_{CKM}^{12}|^2]}{m_p^3 (m_n^2 - m_K^2)^2 [|A_3|^2 |V_{CKM}^{11}|^2 + |A_2|^2 |V_{CKM}^{12}|^2]}, \tag{46}$$

$$\frac{\tau(n \to \pi^0\bar{\nu})}{\tau(p \to \pi^+\bar{\nu})} = \frac{2m_p}{m_n}, \tag{47}$$

$$\frac{\tau(n \to \eta^0\bar{\nu})}{\tau(p \to \pi^+\bar{\nu})} = \frac{6m_p m_n^3 (1 + D + F)^2}{(m_n^2 - m_\eta^2)^2 (1 - D - 3F)^2}, \tag{48}$$

with:

$$A_3 = 1 + \frac{m_n}{3m_B}(D - 3F). \tag{49}$$

Notice that using the expressions for k_1 and k_2 (Eqs. (38) and (43)), and the relation between the different decay rates of the neutron and the proton into an antineutrino (Eqs. (45)–(48)), we can conclude that it is possible to make a clear test of a grand unified theory with symmetric Yukawa couplings.

As we say before, there are realistic $SO(10)$ theories with symmetric Yukawa couplings. In a $SO(10)$ theory all fermions of a family live in the 16_F spinor representation [4]. In this case the coefficients for the gauge $d = 6$ operators are given by Eqs. (9)–(12).

Let us analyze the most realistic and studied $SO(10)$ theories, where all Yukawa couplings are symmetric. It is the case of theories with the 10_H and/or 126_H Higgses [24,25,27–34]. We have already studied the case of GUT models with symmetric Yukawa couplings, where we pointed out the possibility to make a consistent check of these theories. In order to predict the decay rates into charged antileptons in this case, we have to know the matrices K_2 and V_{DE} (see Eqs. (33) and (34)). In those $SO(10)$ theories there is a specific expression for the matrix V_{DE}:

$$4V_{UD}^T K_u^* Y_U^{\text{diag}} V_{UD} - (3\tan\alpha_{10} + \tan\alpha_{126}) K_d^* Y_D^{\text{diag}} = V_{DE}^* K_e^* Y_E^{\text{diag}} V_{DE}^\dagger (\tan\alpha_{10} - \tan\alpha_{126}). \tag{50}$$

In the above expressions $\tan\alpha_{10} = v_{10}^U/v_{10}^D$, and $\tan\alpha_{126} = v_{126}^U/v_{126}^D$. In Eq. (50) we see explicitly the relation between the different factors entering in the proton decay predictions.

To compute the amplitude for proton decay into charged antileptons we need the following expression:

$$\sum_{\alpha=1}^{2} c\left(e_\alpha^C, d_\beta\right)^*_{\text{sym}} c\left(e_\alpha^C, d_\gamma\right)_{\text{sym}}$$
$$= \left[\delta^{\beta i} + V_{CKM}^{1\beta} K_2^{\beta\beta} \left(K_2^*\right)^{ii} \left(V_{CKM}^\dagger\right)^{i1}\right]\left[\delta^{\gamma j} + V_{CKM}^{1\gamma} K_2^{\gamma\gamma} \left(K_2^*\right)^{jj} \left(V_{CKM}^\dagger\right)^{j1}\right] \sum_{i=1}^{2} V_{DE}^{i\alpha} \left(V_{DE}^{j\alpha}\right)^*. \tag{51}$$

Therefore the amplitude of the channels with charged antileptons always depend on the matrices K_2 and V_{DE}. Therefore it is not possible to make a clear test of the theory through those channels, they are useful to distinguish between different models for fermion masses with symmetric Yukawa matrices. Notice that in Ref. [26] has been showed that the predictions for the decay channel $p \to l^+ K^0$ are the same in different models for fermion masses, however as we can appreciate from Eq. (51) it is not true in the general case when we consider all generations and the extra CP violating phases.

5. Conclusions

We have studied in detail the predictions coming from the gauge $d = 6$ operators, the less model dependent contributions for proton decay. Analyzing the different decay channels, we find that there are only seven independent equations for the coefficients involved in the two bodies decay channels for proton decay. In general we could say that the number of physical parameters involved in those predictions must be less than seven.

We have pointed out that it is possible to make a clear test of any grand unified theory with symmetric Yukawa couplings through the decay of the nucleon, since in these cases the decay rates of the nucleon into an antineutrino are independent of the mixings matrices and the new sources of CP violation beyond V_{CKM} and V_l, they depend only on the factors k_1 and k_2. The relations between the decays of the proton and the neutron into an antineutrino have been found. Notice that it is the case of realistic grand unified theories based on the $SO(10)$ gauge group. The predictions for the decay channels with charged leptons are not the same in different models for fermion masses with symmetric Yukawa couplings, therefore they could be useful to distinguish between different models. Our results are valid in supersymmetric and non-supersymmetric scenarios.

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