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Exotic B_c -like molecules in QCD sum rules

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ABSTRACT

We use the QCD sum rules to study possible B_c -like molecular states. We consider isoscalar $J^P = 0^+$ and $J^P = 1^+$ $D^{(*)}B^{(*)}$ molecular currents. We consider the contributions of condensates up to dimension eight and we work at leading order in α_s . We obtain for these states masses around 7 GeV.

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The study of states with configuration more complex than the conventional $q\bar{q}$ meson and qqq baryon is quite old and, despite decades of progress, no exotic hadron has been conclusively identified. Famous examples of possible nonconventional meson states are the light scalars and the $X(3872)$ [1]. While, from a theoretical point of view the most acceptable structure for the light scalars is a tetraquark (diquark–antidiquark) configuration [2], in the case of the $X(3872)$ there is an agreement in the community that it might be a $D\bar{D}^*$ molecular state. Establishing the structure of these states and identifying other possible exotic states represents a remarkable progress in hadron physics.

Besides the $X(3872)$, in the past decade, more and more charmonium-like or bottomonium-like states were observed in the e^+e^- collision [3–5], B meson decays [1,6–8] and even $\gamma\gamma$ fusion processes [9–11], which have stimulated the extensive discussion of exotic hadron configurations (for a review see Refs. [12–15]). An important question that arises is that if some of these observed states are molecular states, then many others should also exist. In a very recent publication [16], a one boson exchange (OBE) model was used to investigate hadronic molecules with both open charm and open bottom. These new structures were labelled as B_c -like molecules, and were categorized into four groups: \mathcal{DB} , \mathcal{D}^*B^* , \mathcal{D}^*B and \mathcal{DB}^* , where these symbols represent the group of states: $\mathcal{D}^{(*)} = [D^{(*)0}, D^{(*)+}, D^{(*)+}]$ for charmed mesons and $\mathcal{B}^{(*)} = [B^{(*)+}, B^{(*)0}, B^{(*)0}]$ for bottom mesons. A complete analysis, based on the approach developed in Refs. [17–24], was done in Ref. [16] to study the interaction of these B_c -like molecules. These states were categorized using a hand-waving notation, with

Table 1

Currents describing possible B_c -like molecules.

State	$I(J^P)$	Current
DB	$0(0^+)$	$j = (\bar{q}\gamma_5 c)(\bar{b}\gamma_5 q)$
D^*B^*	$0(0^+)$	$j = (\bar{q}\gamma_\mu c)(\bar{b}\gamma^\mu q)$
D^*B	$0(1^+)$	$j_\mu = i(\bar{q}\gamma_\mu c)(\bar{b}\gamma_5 q)$
DB^*	$0(1^+)$	$j_\mu = i(\bar{q}\gamma_5 c)(\bar{b}\gamma_\mu q)$

five-stars, four-stars, etc. A five-star state implies that a loosely molecular state probably exists. They find five five-star states, all of them isosinglets in the light sector, with no strange quarks.

Here we use the QCD sum rules (QCDSR) [14,25–27], to check if some of the five-star states found in Ref. [16] are supported by a QCDSR calculation. The states we will consider are the isosinglets $J^P = 0^+$ $DB = (D^0B^+ + D^+B^0)$, $J^P = 1^+$ $D^*B = (D^{*0}B^+ + D^{*+}B^0)$, $J^P = 1^+$ $DB^* = (D^0B^{*+} + D^+B^{*0})$ and the $J^P = 0^+$ $D^*B^* = (D^{*0}B^{*+} + D^{*+}B^0)$. The QCDSR approach is based on the two-point correlation function

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T [j(x) j^\dagger(0)] | 0 \rangle, \quad (1)$$

where the current $j(x)$ contains all the information about the hadron of interest, like quantum numbers, quarks contents and so on. Possible currents for the states described above are given in Table 1, where we have used a short notation for the isoscalars since we are considering the light quarks, $q = u, d$, degenerate. We use the same techniques developed in Refs. [28–39].

The QCD sum rule is obtained by evaluating the correlation function in Eq. (1) in two ways: in the OPE side, we calculate the correlation function at the quark level in terms of quark and gluon fields. We work at leading order in α_s in the operators, we consider the contributions from condensates up to dimension eight. In

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the phenomenological side, the correlation function is calculated by inserting intermediate states for the hadronic state, H , and parameterizing the coupling of these states to the current $j_\mu(x)$, in terms of a generic coupling parameter λ , so that

$$\langle 0 | j | H \rangle = \lambda, \quad (2)$$

for the scalar states and

$$\langle 0 | j_\mu | H \rangle = \lambda \varepsilon_\mu, \quad (3)$$

for the axial currents, where ε_μ is the polarization vector. In the case of the axial current, we can write the correlation function in Eq. (1) in terms of two independent Lorentz structures:

$$\Pi_{\mu\nu}(q) = -\Pi(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \Pi'(q^2) \frac{q_\mu q_\nu}{q^2}. \quad (4)$$

The two invariant functions, Π and Π' , appearing in Eq. (4), have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure $g_{\mu\nu}$, since it projects out the 1^+ state.

The phenomenological side of Eq. (1), in the $g_{\mu\nu}$ structure in the case of the axial currents, can be written as

$$\Pi^{phen}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \int_0^\infty ds \frac{\rho^{cont}(s)}{s - q^2}, \quad (5)$$

where M_H is the hadron mass and the second term in the RHS of Eq. (5) denotes the contribution of the continuum of the states with the same quantum numbers as the current. In general, in the QCDSR method it is assumed that the continuum contribution to the spectral density, $\rho^{cont}(s)$ in Eq. (5), vanishes below a certain continuum threshold s_0 . Above this threshold, it is given by the result obtained in the OPE side. Therefore, one uses the ansatz [40]

$$\rho^{cont}(s) = \rho^{OPE}(s) \Theta(s - s_0). \quad (6)$$

The correlation function in the OPE side can be written as a dispersion relation:

$$\Pi^{OPE}(q^2) = \int_{(m_c+m_b)^2}^\infty ds \frac{\rho^{OPE}(s)}{s - q^2}, \quad (7)$$

where $\rho^{OPE}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{OPE}(s) = \text{Im}[\Pi^{OPE}(s)]$.

After transferring the continuum contribution to the OPE side, and performing a Borel transform, the sum rule can be written as

$$\lambda^2 e^{-M_H^2 \tau} = \int_{(m_c+m_b)^2}^{s_0} ds e^{-s\tau} \rho^{OPE}(s), \quad (8)$$

where we have introduced the Borel parameter $\tau = 1/M^2$, with M being the Borel mass. To extract M_H we take the derivative of Eq. (8) with respect to Borel parameter τ and divide the result by Eq. (8), so that

$$M_H^2 = \frac{\int_{(m_c+m_b)^2}^{s_0} ds s e^{-s\tau} \rho^{OPE}(s)}{\int_{(m_c+m_b)^2}^{s_0} ds e^{-s\tau} \rho^{OPE}(s)}. \quad (9)$$

The expressions for $\rho^{OPE}(s)$ for the currents in Table 1, using factorization hypothesis, up to dimension-eight condensates, are given in Appendix A.

Table 2
QCD input parameters.

Parameters	Values
m_b	(4.24 – 4.60) GeV
m_c	(1.23 – 1.47) GeV
$\langle \bar{q}q \rangle$	$-(0.23 \pm 0.03)^3 \text{ GeV}^3$
$\langle g_s^2 G^2 \rangle$	$(0.88 \pm 0.25) \text{ GeV}^4$
$m_0^2 \equiv \langle \bar{q}Gq \rangle / \langle \bar{q}q \rangle$	$(0.8 \pm 0.1) \text{ GeV}^2$
$\langle g_s^3 G^3 \rangle$	$(0.58 \pm 0.18) \text{ GeV}^6$
$\rho \equiv \langle \bar{q}q\bar{q}q \rangle / \langle \bar{q}q \rangle^2$	$(0.5 - 2.0)$

To extract reliable results from the sum rule, it is necessary to establish the Borel window. A valid sum rule exists when one can find a Borel window where there are a OPE convergence, a τ -stability and the dominance of the pole contribution. The maximum value of τ parameter is determined by imposing that the contribution of the higher dimension condensate is smaller than 15% of the total contribution. The minimum value of τ is determined by imposing that the pole contribution is equal to the continuum contribution. To guarantee a reliable result extracted from sum rules it is important that there is a τ stability inside the Borel window.

The continuum threshold is a physical parameter that should be determined from the spectrum of the mesons. Using a harmonic-oscillator potential model, it was shown in Ref. [41] that a constant continuum threshold is a very poor approximation. The actual accuracy of the parameters extracted from the sum rules improves considerably when using a Borel dependent continuum threshold. It also allows to estimate realistic systematic errors [41]. However, to be able to fix the form of the Borel dependent continuum threshold (and the values of the parameters in the function) one needs to use the experimental value of the mass of the particle [42]. Since in our study we do not know the experimental value of the masses of the states, it is not possible to fix the Borel dependent continuum threshold. For this reason, although aware of the limitations of the values we are going to extract from the sum rule, to have a first estimate for the values of the masses of the states, we are going to use a constant continuum threshold. In many cases, a good approximation for the value of the continuum threshold is the value of the mass of the first excited state squared. In some known cases, like the ρ and J/ψ , the first excited state has a mass approximately 0.5 GeV above the ground state mass. Since here we do not know the spectrum for the hadrons studied, we will fix the continuum threshold range starting with the smaller value which provides a valid Borel window. The optimal choice for s_0 will be taken when there is a τ -stability inside the Borel window.

For a consistent comparison with the results obtained for the other molecular states using the QCDSR approach, we have considered here the same values used for the quark masses and condensates as in Refs. [29–35,43], listed in Table 2. For the heavy quark masses, we could use the range spanned by the running \overline{MS} mass $\overline{m}_Q(M_Q)$ and the on-shell mass from QCD (spectral) sum rules compiled in [27] and more recently obtained in Ref. [44]. However, we do not obtain a valid Borel window with the usual on-shell mass for b quark, $m_b = 4.70$ GeV. For this reason, we have considered as the maximum value for b quark mass $m_b = 4.60$ GeV, as indicated in Table 2. For the $\langle G^3 \rangle$ condensate, we have used the new numerical value estimated in Ref. [44]. To take into account the violation of the factorization hypothesis we introduced in Table 2 the parameter ρ .

Let us consider first the molecular current for the $DB(0^+)$ state. In Fig. 1 a), we show the relative contribution of the terms in the OPE side of the sum rule, for $\sqrt{s_0} = 7.20$ GeV. From this figure we

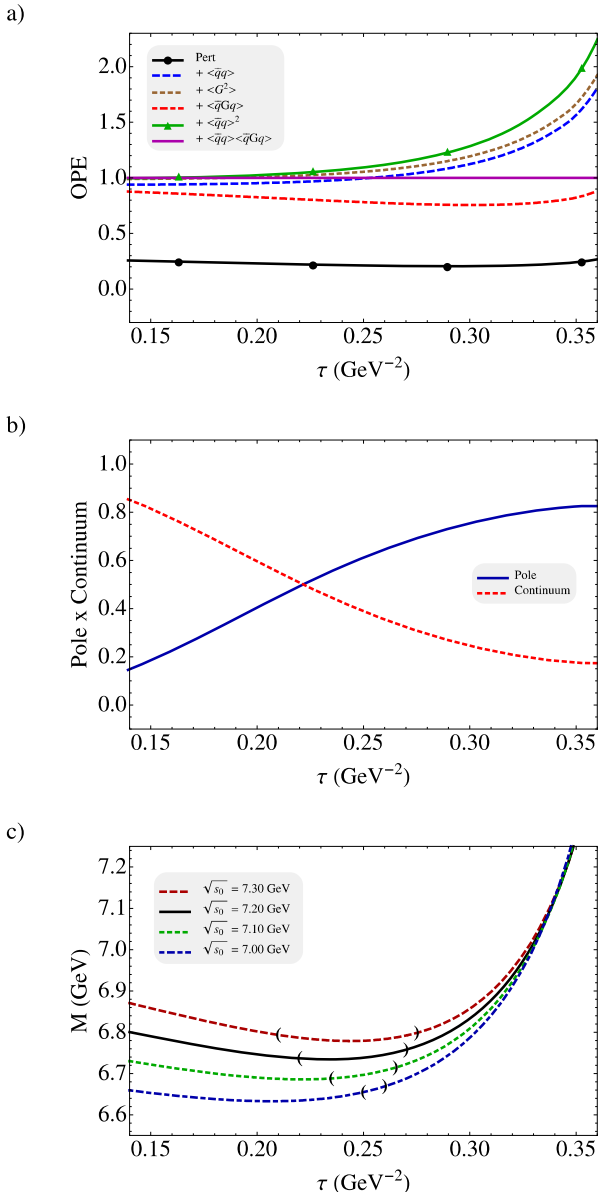


Fig. 1. $DB(0^+)$ molecular state up to dimension-8 contribution for $m_c = 1.23$ GeV and $m_b = 4.24$ GeV. a) OPE convergence in the region $0.14 \leq \tau \leq 0.36$ GeV^{-2} for $\sqrt{s_0} = 7.20$ GeV. We plot the relative contributions starting with the perturbative contribution and each other line represents the relative contribution after adding of one extra condensate in the expansion: $+\langle\bar{q}q\rangle$, $+\langle G^2\rangle$, $+\langle\bar{q}Gq\rangle$, $+\langle\bar{q}q\rangle^2$ and $+\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle$. b) The pole and continuum contributions for $\sqrt{s_0} = 7.20$ GeV. c) The mass as a function of the sum rule parameter τ , for different values of $\sqrt{s_0}$. For each line, the region bounded by parenthesis indicates a valid Borel window.

see that the contribution of the dimension-8 condensate is smaller than 15% of the total contribution for values of $\tau \leq 0.27$ GeV^{-2} , which indicates a good OPE convergence. From Fig. 1 b), we also see that the pole contribution is bigger than the continuum contribution only for $\tau \geq 0.22$ GeV^{-2} . Therefore, we fix the Borel window as: $(0.22 \leq \tau \leq 0.27)$ GeV^{-2} . The results for the mass are shown in Fig. 1 c), as a function of τ , for different values of s_0 . As we can see from Fig. 1 c), the Borel window (indicated through the parenthesis) gets smaller as the value of $\sqrt{s_0}$ decreases. So, we can only work with values for $\sqrt{s_0}$ bigger than 7.00 GeV, otherwise we do not obtain a valid Borel window for this sum rule. We also observe that the optimal choice for the continuum threshold is $\sqrt{s_0} = 7.20$ GeV, because it provides the best τ -stability inside

of the Borel window, including the existence of a minimum point for the value of the mass.

Therefore, varying the value of the continuum threshold in the range $\sqrt{s_0} = (7.00 - 7.30)$ GeV, and the others parameters as indicated in Table 2, we get

$$M_{DB}^{(8)} = (6.77 \pm 0.11) \text{ GeV}. \quad (10)$$

The quoted uncertainty is the OPE uncertainty. The most important source of uncertainty is the values of the heavy quark masses. As discussed in Ref. [42], there is another kind of uncertainty, called systematic uncertainty, related to the intrinsic limited accuracy of the method. The systematic uncertainty of the physical quantity extracted from the QCDSR represents, perhaps, the most subtle point in the application of the method. Without an estimate of the systematic uncertainty, the numerical value of the physical quantity one reads off from the Borel window might differ significantly from its true value. In Ref. [42] it was shown that the use of the Borel dependent continuum threshold allows to estimate the systematic uncertainty. In particular, for the case of the D and D_s mesons studied in [42], the systematic uncertainty turns out to be of the same order of the OPE uncertainty. Since here we do not have how to estimate the Borel dependent continuum threshold, in an attempt to obtain some information about the systematic uncertainty, we will repeat the analysis considering only terms up to dimension 6 in the OPE. These new results are shown in Fig. 2.

As one can see in Fig. 2 a), when we remove the dimension-8 condensate contribution we lose the OPE convergence, since the most important contributions to the OPE come from $\langle\bar{q}q\rangle$ and $\rho\langle\bar{q}q\rangle^2$ contributions. Thus to be able to extract some results from this analysis we determine the maximum value of τ parameter imposing that the contribution of the dimension-6 condensate is smaller than 25% of the total contribution, otherwise we do not have a valid Borel window for this sum rule. The minimum value of τ is not changed since the pole dominance behavior remains the same. Finally, we obtain the results shown in Fig. 2 c), from where we get

$$M_{DB}^{(6)} = (6.63 \pm 0.09) \text{ GeV}. \quad (11)$$

Note that the value in Eq. (11) differs at maximum only $\sim 5.0\%$ to that in Eq. (10). Besides, the inclusion of dimension-8 condensate provides a better OPE convergence, τ -stability and an improved Borel window. Therefore, even being aware that this is only part of the dimension-8 contribution, here we consider it as a form to estimate the systematic uncertainty. One should note that a complete evaluation of the dimension-8 contributions require more involved analysis including a non-trivial choice of the factorization assumption basis [45]. Then, the final value for the DB molecular state is given by

$$M_{DB} = (6.75 \pm 0.14) \text{ GeV}. \quad (12)$$

The mass in Eq. (12) is ~ 400 MeV below the DB threshold indicating that such molecular state would be tightly bound. This result, for the binding energy, is very different than the obtained in Ref. [16] for the $DB(0^+)$ molecular state. The authors of Ref. [16] found that the $DB(0^+)$ molecular state is loosely bound with a binding energy smaller than 14 MeV. However, it is very important to notice that since the molecular currents given in Table 1 are local, they do not represent extended objects, with two mesons separated in space, but rather a very compact object with two singlet quark-antiquark pairs. Therefore, the result obtained here may suggest that, although a loosely bound $DB(0^+)$ molecular state can exist, it may not be the ground state for a four-quark exotic state with the same quantum numbers and quark content.

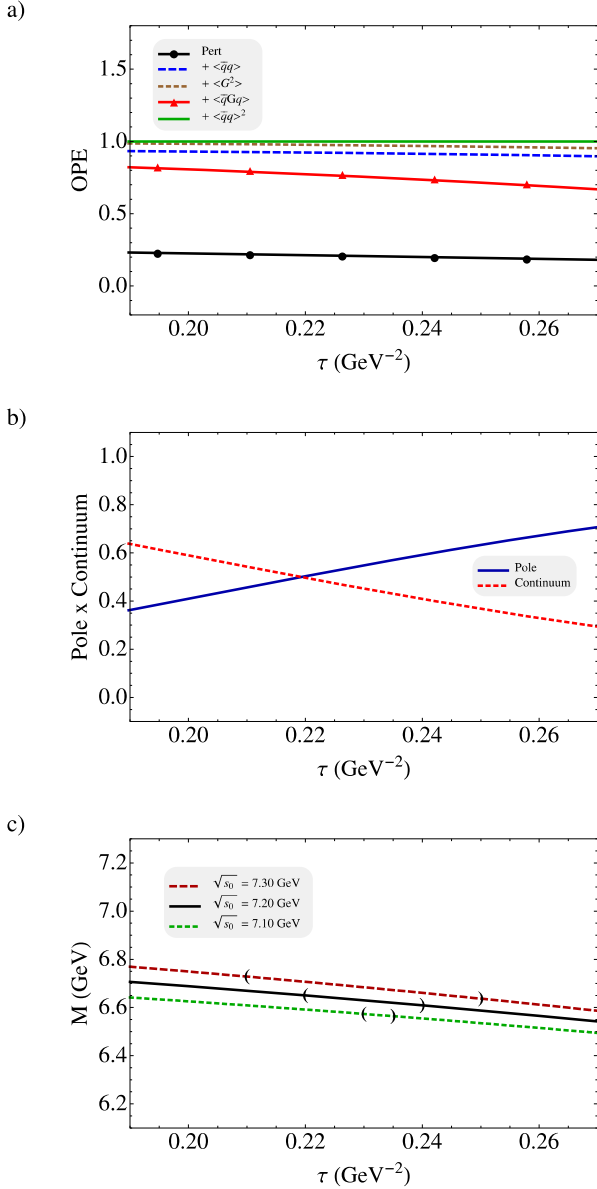


Fig. 2. $DB(0^+)$ molecular state up to dimension-6 contribution for $m_c = 1.23$ GeV and $m_b = 4.24$ GeV. a) OPE convergence in the region $0.19 \leq \tau \leq 0.27$ GeV $^{-2}$ for $\sqrt{s_0} = 7.20$ GeV. We plot the relative contributions starting with the perturbative contribution and each other line represents the relative contribution after adding of one extra condensate in the expansion: $\langle \bar{q}q \rangle$, $\langle G^2 \rangle$, $\langle \bar{q}Gq \rangle$ and $\langle \bar{q}q \rangle^2$. b) The pole and continuum contributions for $\sqrt{s_0} = 7.20$ GeV. c) The mass as a function of the sum rule parameter τ , for different values of $\sqrt{s_0}$. For each line, the region bounded by parenthesis indicates a valid Borel window.

Having the hadron mass, we can also evaluate the coupling parameter, λ , defined in Eq. (2). We get

$$\lambda_{DB} = (0.029 \pm 0.008) \text{ GeV}^5. \quad (13)$$

The parameter λ gives a measure of the strength of the coupling between the current and the state. The result in Eq. (13) has the same order of magnitude as the coupling obtained for the $X(3872)$ [29], for example. This indicates that such state could be very well represented by the respective current in Table 1.

We can extend the same analysis to study the others molecular states presented in Table 1. For all of them we get a similar OPE convergence in a region where the pole contribution is bigger than the continuum contribution. We obtain the results shown in Fig. 3.

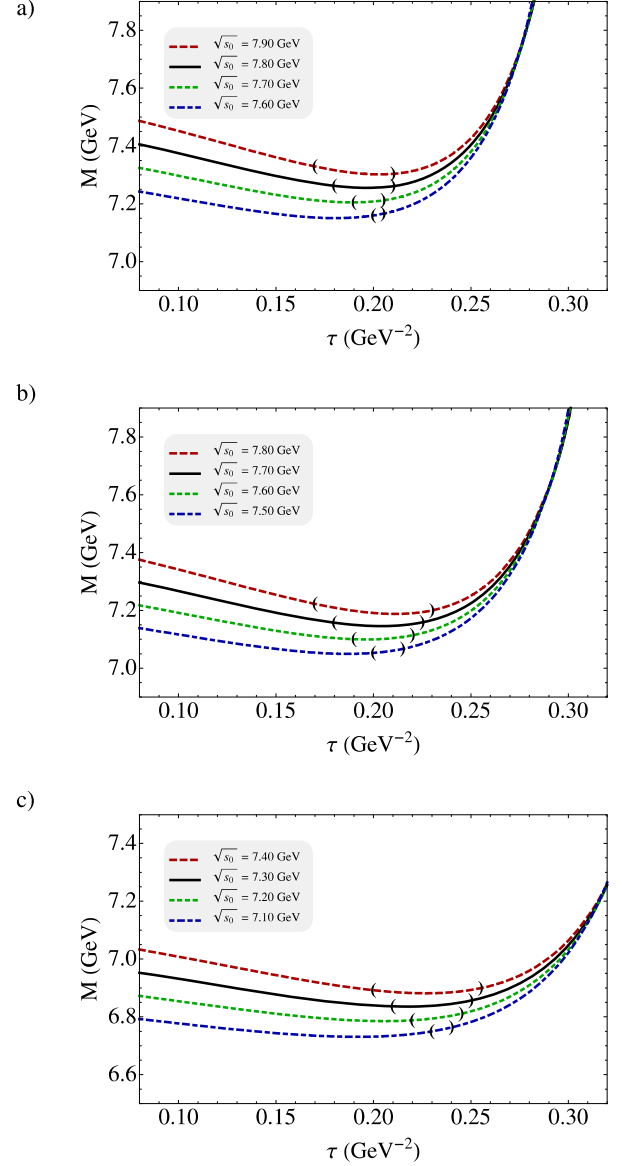


Fig. 3. The mass as a function of the sum rule parameter τ , for $m_c = 1.23$ GeV and $m_b = 4.24$ GeV, considering different values for $\sqrt{s_0}$: a) for D^*B^* , 0^+ molecular current; b) for D^*B , 1^+ molecular current; c) for DB^* , 1^+ molecular current. For each line, the region bounded by parenthesis indicates a valid Borel window.

In Fig. 3 a), we show the ground state mass, for the D^*B^* , 0^+ molecular current, as a function of τ . For $\sqrt{s_0} = 7.80$ GeV, we can fix the Borel window as: $(0.18 \leq \tau \leq 0.21)$ GeV $^{-2}$. From this figure we again see that there is a very good τ -stability in the determined Borel window.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = (7.60 - 7.90)$ GeV, the others parameters as indicated in Table 2 and also estimating the uncertainty by neglecting the dimension-8 contribution we get

$$M_{D^*B^*} = (7.27 \pm 0.12) \text{ GeV}, \quad (14)$$

$$\lambda_{D^*B^*} = (0.115 \pm 0.021) \text{ GeV}^5. \quad (15)$$

The obtained mass indicates a binding energy of the order of ~ 50 MeV below the D^*B^* threshold. Considering the uncertainties, it is even possible that this state is not bound. In this case, our central result is in a good agreement with the result obtained for the D^*B^* , 0^+ , molecular state obtained in Ref. [16]. However,

since we do not have a trustable estimate for the systematic error, as discussed above, any conclusion about the possible existence of this state would be premature.

We now consider the $D^*B(1^+)$ molecular current. In Fig. 3 b), we show the ground state mass, as a function of τ . For $\sqrt{s_0} = 7.70$ GeV, we can fix the Borel window as: $(0.18 \leq \tau \leq 0.22)$ GeV $^{-2}$.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = (7.50 - 7.80)$ GeV, the others parameters as indicated in Table 2 and also estimating the uncertainty by neglecting the dimension-8 contribution we get

$$M_{D^*B} = (7.16 \pm 0.12) \text{ GeV}, \quad (16)$$

$$\lambda_{D^*B} = (0.058 \pm 0.013) \text{ GeV}^5. \quad (17)$$

The obtained mass indicates a central binding energy for the $D^*B, 1^+$ state of the order of ~ 130 MeV. Considering the uncertainty, this result might be compatible with the one obtained by the authors in Ref. [16], or can even be unbound, as the D^*B^* state. Therefore, also in this case, any conclusion about the possible existence of this state would be premature.

Finally, we study the molecular current for $DB^*, 1^+$ state. As one can see from Fig. 3 c), we have a very good τ -stability inside of the Borel window: $(0.21 \leq \tau \leq 0.25)$ GeV $^{-2}$, for $\sqrt{s_0} = 7.30$ GeV. Doing the same procedure to estimate the uncertainties in the range $\sqrt{s_0} = (7.10 - 7.40)$ GeV we get

$$M_{DB^*} = (6.85 \pm 0.15) \text{ GeV}, \quad (18)$$

$$\lambda_{DB^*} = (0.036 \pm 0.011) \text{ GeV}^5 \quad (19)$$

which indicates a binding energy of the order ~ 330 MeV, much bigger than that obtained in Ref. [16].

We can compare our results with the ones presented in Ref. [46]. First of all we would like to point out that we have found some disagreements in the spectral densities expressions for the DB, D^*B and DB^* molecular currents. In particular, we have found some missing terms in the $\langle G^2 \rangle, \langle \bar{q}Gq \rangle$ and $\langle G^3 \rangle$ contributions, due to some diagrams that have been neglected in their calculations. We have found that the $\langle \bar{q}Gq \rangle$ contribution plays an important role to the final result, and this can explain why the mass values found in Ref. [46] differ from ours. Another important point, in which our calculations differ, is the fact that the Borel window $(0.10 \leq \tau \leq 0.14)$ GeV $^{-2}$, considered by the authors in Ref. [46], does not have pole dominance, as can be seen in Figs. 1 c), 3 a), 3 b) and 3 c). The only result for the mass, which is in agreement with Ref. [46], is the one for the D^*B^* molecular current. For this current we found disagreements only for the $\langle G^3 \rangle$ contribution. Since the $\langle G^3 \rangle$ contribution is very small, as compared to the others, the differences found could not modify the final result.

In conclusion, we have studied the mass of the exotic B_c -like molecular states using QCD sum rules. We find that for the molecular currents $D^*B^*(J^P = 0^+)$ and $D^*B(J^P = 1^+)$, the QCDSR central results lead approximately to the same predictions made by the authors in Ref. [16], for the respective molecular states in a OBE model. However, since our uncertainties are underestimated due to our crude model for the continuum threshold, any conclusion about the possible existence of these states would be premature.

In the case of the $DB(J^P = 0^+)$ and $DB^*(J^P = 1^+)$ molecular currents, from the QCD sum rule point of view, the masses of the corresponding states are smaller than the masses obtained for the respective molecular states studied in Ref. [16]. We interpret this result as an indication of the possible existence of four-quark states, with the same quark content and quantum numbers as

the $DB(J^P = 0^+)$ and $DB^*(J^P = 1^+)$ molecular states, but with smaller masses.

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Appendix A. Spectral densities

The spectral densities expressions for the molecular currents given in Table 1, were calculated up to dimension-6 condensates, at leading order in α_s . To keep the heavy quark mass finite, we use the momentum-space expression for the heavy quark propagator. We calculate the light quark part of the correlation function in the coordinate-space, and we use the Schwinger parameters to evaluate the heavy quark part of the correlator. To evaluate the d^4x integration in Eq. (1), we use again the Schwinger parameters, after a Wick rotation. Finally we get integrals in the Schwinger parameters. The result of these integrals are given in terms of logarithmic functions, from where we extract the spectral densities and the limits of the integration. The same technique can be used to evaluate the condensate contributions. To evaluate the systematic uncertainty we also include a part of the dimension-8 contribution, related with the mixed-condensate times the quark condensate. In Ref. [38] it was shown that the contribution of this condensate is much bigger than other dimension-8 condensates, related with the gluon condensate.

For the $DB, 0^+$ molecular current we get

$$\begin{aligned} \rho_{DB}^{pert}(s) &= \frac{3}{2^{11}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) F(\alpha, \beta)^4, \\ \rho_{DB}^{(\bar{q}q)}(s) &= -\frac{3\langle \bar{q}q \rangle}{2^7\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (\beta m_c + \alpha m_b) F(\alpha, \beta)^2, \\ \rho_{DB}^{(G^2)}(s) &= \frac{\langle g_s^2 G^2 \rangle}{2^{12}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} F(\alpha, \beta) [3\alpha\beta(\alpha+\beta)F(\alpha, \beta) \\ &\quad + 2(1-\alpha-\beta)(\beta^3 m_c^2 + \alpha^3 m_b^2)], \\ \rho_{DB}^{(\bar{q}Gq)}(s) &= -\frac{3\langle \bar{q}Gq \rangle}{2^8\pi^4} \left[\int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)} (m_c + \alpha(m_b - m_c)) H(\alpha) \right. \\ &\quad \left. - 2 \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (\beta^2 m_c + \alpha^2 m_b) F(\alpha, \beta) \right], \\ \rho_{DB}^{(\bar{q}q)^2}(s) &= \frac{m_b m_c \rho \langle \bar{q}q \rangle^2}{16\pi^2} \lambda_{bc} v, \\ \rho_{DB}^{(G^3)}(s) &= \frac{\langle g_s^3 G^3 \rangle}{2^{13}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) [2(\beta^4 m_c^2 + \alpha^4 m_b^2) \\ &\quad + (\alpha^3 + \beta^3) F(\alpha, \beta)], \end{aligned}$$

$$\rho_{DB}^{(8)}(s) = \frac{m_c m_b \rho(\bar{q}q) \langle \bar{q}Gq \rangle}{2^5 \pi^2} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \left[1 - \alpha + \alpha^2 - (m_c^2 - \alpha(m_c^2 - m_b^2))\tau \right] \delta\left(s - \frac{m_c^2 - \alpha(m_c^2 - m_b^2)}{\alpha(1-\alpha)}\right).$$

For the $D^*B, 1^+$ molecular current we get

$$\begin{aligned} \rho_{D^*B}^{pert}(s) &= \frac{3}{2^{12} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) \\ &\quad \times (1 + \alpha + \beta) F(\alpha, \beta)^4, \\ \rho_{D^*B}^{(\bar{q}q)}(s) &= -\frac{3 \langle \bar{q}q \rangle}{2^7 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} [\beta m_c + \alpha(\alpha + \beta) m_b] \\ &\quad \times F(\alpha, \beta)^2, \\ \rho_{D^*B}^{(G^2)}(s) &= \frac{\langle g_s^2 G^2 \rangle}{2^{12} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} F(\alpha, \beta) [\alpha\beta(3\alpha + \beta) \\ &\quad - \beta(2 - \alpha - \beta)] F(\alpha, \beta) + (\beta^3 m_c^2 + \alpha^3 m_b^2) \\ &\quad \times (1 - \alpha - \beta)(1 + \alpha + \beta)], \\ \rho_{D^*B}^{(\bar{q}Gq)}(s) &= -\frac{3 \langle \bar{q}Gq \rangle}{2^8 \pi^4} \left[\int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)} (m_c - \alpha(m_c - m_b)) H(\alpha) \right. \\ &\quad \left. - m_b \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\alpha + 3\beta) F(\alpha, \beta) \right], \end{aligned}$$

$$\begin{aligned} \rho_{D^*B}^{(\bar{q}q)^2}(s) &= \frac{m_c m_b \rho(\bar{q}q)^2}{16\pi^2} \lambda_{bc} v, \\ \rho_{D^*B}^{(G^3)}(s) &= \frac{\langle g_s^3 G^3 \rangle}{2^{14} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)(1 + \alpha + \beta) \\ &\quad \times [2(\beta^4 m_c^2 + \alpha^4 m_b^2) + (\alpha^3 + \beta^3) F(\alpha, \beta)], \\ \rho_{D^*B}^{(8)}(s) &= \frac{m_c m_b \rho(\bar{q}q) \langle \bar{q}Gq \rangle}{2^5 \pi^2} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \\ &\quad \times \delta\left(s - \frac{m_c^2 - \alpha(m_c^2 - m_b^2)}{\alpha(1-\alpha)}\right) \\ &\quad \times [\alpha^2 - (m_c^2 - \alpha(m_c^2 - m_b^2))\tau]. \end{aligned}$$

For the $DB^*, 1^+$ molecular current we get

$$\begin{aligned} \rho_{DB^*}^{pert}(s) &= \frac{3}{2^{12} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) \\ &\quad \times (1 + \alpha + \beta) F(\alpha, \beta)^4, \\ \rho_{DB^*}^{(\bar{q}q)}(s) &= -\frac{3 \langle \bar{q}q \rangle}{2^7 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} [\beta(\alpha + \beta) m_c + \alpha m_b] \\ &\quad \times F(\alpha, \beta)^2, \\ \rho_{DB^*}^{(G^2)}(s) &= \frac{\langle g_s^2 G^2 \rangle}{2^{12} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} F(\alpha, \beta) [\alpha\beta(3\beta(\alpha + \beta) \end{aligned}$$

$$- \alpha(2 - \alpha - \beta)] F(\alpha, \beta) + (\beta^3 m_c^2 + \alpha^3 m_b^2) \times (1 - \alpha - \beta)(1 + \alpha + \beta)],$$

$$\rho_{DB^*}^{(\bar{q}Gq)}(s) = -\frac{3 \langle \bar{q}Gq \rangle}{2^8 \pi^4} \left[\int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)} (m_c - \alpha(m_c - m_b)) H(\alpha) - m_c \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} d\beta (3\alpha + 2\beta) F(\alpha, \beta) \right],$$

$$\begin{aligned} \rho_{DB^*}^{(\bar{q}q)^2}(s) &= \frac{m_c m_b \rho(\bar{q}q)^2}{16\pi^2} \lambda_{bc} v, \\ \rho_{DB^*}^{(G^3)}(s) &= \frac{\langle g_s^3 G^3 \rangle}{2^{14} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)(1 + \alpha + \beta) \\ &\quad \times [2(\beta^4 m_c^2 + \alpha^4 m_b^2) + (\alpha^3 + \beta^3) F(\alpha, \beta)], \end{aligned}$$

$$\begin{aligned} \rho_{DB^*}^{(8)}(s) &= \frac{m_c m_b \rho(\bar{q}q) \langle \bar{q}Gq \rangle}{2^5 \pi^2} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \\ &\quad \times \delta\left(s - \frac{m_c^2 - \alpha(m_c^2 - m_b^2)}{\alpha(1-\alpha)}\right) \\ &\quad \times [(1 - \alpha)^2 - (m_c^2 - \alpha(m_c^2 - m_b^2))\tau]. \end{aligned}$$

For the $D^*B^*, 0^+$ molecular current we get

$$\begin{aligned} \rho_{D^*B^*}^{pert}(s) &= \frac{3}{2^9 \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) F(\alpha, \beta)^4, \\ \rho_{D^*B^*}^{(\bar{q}q)}(s) &= -\frac{3 \langle \bar{q}q \rangle}{2^6 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (\beta m_c + \alpha m_b) F(\alpha, \beta)^2, \\ \rho_{D^*B^*}^{(G^2)}(s) &= \frac{\langle g_s^2 G^2 \rangle}{2^9 \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) \\ &\quad \times (\beta^3 m_c^2 + \alpha^3 m_b^2) F(\alpha, \beta), \\ \rho_{D^*B^*}^{(\bar{q}Gq)}(s) &= -\frac{3 \langle \bar{q}Gq \rangle}{2^7 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)} (m_c - \alpha(m_c - m_b)) H(\alpha), \\ \rho_{D^*B^*}^{(\bar{q}q)^2}(s) &= \frac{m_c m_b \rho(\bar{q}q)^2}{4\pi^2} \lambda_{bc} v, \\ \rho_{D^*B^*}^{(G^3)}(s) &= \frac{\langle g_s^3 G^3 \rangle}{2^{11} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) [2(\beta^4 m_c^2 + \alpha^4 m_b^2) \\ &\quad + (\alpha^3 + \beta^3) F(\alpha, \beta)], \\ \rho_{D^*B^*}^{(8)}(s) &= -\frac{m_c m_b \rho(\bar{q}q) \langle \bar{q}Gq \rangle}{8\pi^2} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \\ &\quad \times \delta\left(s - \frac{m_c^2 - \alpha(m_c^2 - m_b^2)}{\alpha(1-\alpha)}\right) \\ &\quad \times [\alpha(1 - \alpha) + (m_c^2 - \alpha(m_c^2 - m_b^2))\tau]. \end{aligned}$$

In all these expressions we have used the following definitions:

$$H(\alpha) = m_b^2 \alpha + m_c^2 (1 - \alpha) - \alpha(1 - \alpha)s, \quad (\text{A.1})$$

$$F(\alpha, \beta) = m_b^2 \alpha + m_c^2 \beta - \alpha \beta s, \quad (\text{A.2})$$

$$\lambda_{bc} = 1 + (m_c^2 - m_b^2)/s, \quad (\text{A.3})$$

$$v = \sqrt{1 - \frac{4m_c^2/s}{\lambda_{bc}^2}}, \quad (\text{A.4})$$

and the integration limits are given by

$$\beta_{min} = \frac{\alpha m_b^2}{\alpha s - m_c^2}, \quad (\text{A.5})$$

$$\alpha_{min} = \frac{\lambda_{bc}}{2}(1 - v), \quad (\text{A.6})$$

$$\alpha_{max} = \frac{\lambda_{bc}}{2}(1 + v). \quad (\text{A.7})$$

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