The influence of constructional detail to lateral-torsional buckling of beams

T. J. Živner

Abstract

The resistance of steel members in bending can be influenced by the lateral-torsional stability phenomenon. This paper investigates the influence of end plates in beam to column joint on the value of elastic buckling moment \( M_{cr} \). The investigated beam is considered as simply supported in bending about horizontal and vertical axis. The rotation about its longitudinal axis is prevented. The warping is partially restrained by the use of a constructional detail which is represented by warping restraints on both ends of the beam. The investigated beam is loaded with equal and opposite end moments, which deduce the constant bending moment diagram, the worst one in the theory of LTB. The National Annex to STN EN 1993-1-1 [1] recommends for the design of steel members to LTB to use for the buckling length factor for torsion 1.0 for a simple support and for warping fixity of both ends 0.5. Both assumptions are very frequently used in design practice, but the real behaviour of steel joints is different. The parametric study considering only the influence of end plates on LTB was presented in [2, 3]. The presented parametric study takes into account the end plates together with the torsional stiffness of columns. The results are in the form of ratios \( M_{cr,approx} / M_{cr,exact} \). Both values of \( M_{cr} \) were calculated with the influence of constructional detail, but each value with a different manner. The \( M_{cr,approx} \) is approximative value calculated via the proposed two steps procedure and \( M_{cr,exact} \) is calculated with the computer program [1P]. The buckling length factors in torsion \( k_w \) which are inputs for the two steps analysis of \( M_{cr,approx} \) proposed in this paper are also presented.

Keywords: Elastic lateral-torsional buckling of beams; columns’ torsional stiffness; warping end restraint, buckling length factor in torsion

1. Introduction

Lateral-torsional buckling of the simply supported beam causes warping at its ends as a result of torsion. The effects of warping and of end warping restraint on the behaviour of isolated members have been studied extensively by authors such as Austin et al., Bleich, Galambos, Timoshenko and Gere, Trahair, Vlasov,
Zbirohowski-Koscia and many others as is stated in [4]. According to Timoshenko and Gere [5] the stiffness of the I-section beam to torsion is significantly increased, if the warping is restrained. Probably the most extensive study on the influence of end plates on the elastic buckling moment was presented by Lindner [6], where the author proposed approximate formulas, which are a modification of formulas given in DIN 18 800: 1990. Except of these approximate formulas Lindner has proposed graphs for six corresponding IPE cross-sections and three different thicknesses of end plates (10; 25; 40mm) which are also published in the rules for member stability in EN 1993-1-1: 2006, Background documentation and design guidelines [7]. The influence of end plates on $M_{cr}$ was also studied in [2] and on the elastic and plastic buckling moment resistance in [8]. One of the latest research on the lateral-torsional stability of beams with effect of end connection restraints based on finite element calculations was presented in [9]. The authors proposed theoretical expressions of buckling length factors $k_z$ and $k_o$, which take into account the minor axis flexural restraint and the torsional restraint at the supports. These factors were derived for elastic buckling moment formula used in EC3 ENV. However the proposed analytical solution of the buckling length factor in torsion $k_o$ does not take into account the effects of warping torsion, the expression consider only the torsional stiffness of the support.

1.1. Eurocode 3

Many years of research developments on lateral-torsional buckling of steel members have been accompanied by the realization of updated design codes and standards such as the American AISC LRFD, British BS 5950-1 and European EC3 final draft. These codes are based on the limit state concept. One of those limit state concept verifications is the lateral-torsional buckling resistance of beams subjected to bending. EC3 offers five different approaches how to estimate the lateral-torsional buckling resistance of a beam. Two of them employ the equivalent member method, where the elastic buckling moment calculation on an ideal member is required. Geometric and physical imperfections of a real steel member are taken into account through the use of buckling curves. Final version of a general design code EC3 [10] in contrast to EC3 ENV and to design code for aluminium structures EC9 does not give any advice how to calculate the value of $M_{cr}$, every user should decide whether to use a computer program or formulas from specific sources. The Slovak National Annex to EC3 [1] offers the same general formula for $M_{cr}$ for beams with uniform cross-sections symmetrical about the minor or major axis. The same approach is given also in Annex I of the European design code for aluminium structures. Both are the result of a long-term extensive study of Ivan J. Baláž from STU in Bratislava on the problem of LTB. In the case of a beam of uniform cross-section which is symmetrical about the minor or major axis, for bending about the major axis the elastic critical moment for LTB is given by the general formula

$$M_{cr} = \mu_{cr}\frac{\pi\sqrt{EI_zGI_t}}{L}$$  \hspace{1cm} (1)

where the relative non-dimensional critical moment $\mu_{cr}$ for doubly symmetrical cross-sections loaded with equal and opposite end moments investigated in this paper can be simplified to the following expression

$$\mu_{cr} = C_1\frac{k_z}{k_w}\sqrt{1+\kappa_{wt}^2}$$  \hspace{1cm} (2)

where the non-dimensional torsion parameter is

$$\kappa_{wt} = \frac{\pi}{k_wL}\frac{EI_w}{GI_t}$$  \hspace{1cm} (3)

The $C_1$ is a factor which depends on the form of bending moment diagram, end restraint conditions and also
on $\kappa_w$. The end restraint conditions are also expressed in buckling length factors $k_y, k_z, k_w$. All the buckling length factors can generally have values between 0.5 and 1.0. This paper investigates only the buckling length factor for torsion $k_w$. Special provision for warping fixity on both ends of the beam should be made for $k_w = 0.5$. The direct method of how to take into account the warping stiffness of the supporting structure on both ends of the investigated beam is not given in [1].

2. Parametric study

The main objective of this paper is to give an instruction how to obtain a reduced buckling length factor for torsion $k_w$ in (3) and an increased elastic buckling moment $M_{cr}$ in (1) of the investigated beam which is supported by adjacent construction with its not negligible bending and torsional rigidities. However, only the torsional rigidity of the column together with end plates is taken into account in this case. The presented parametric study on the influence of columns and end plates torsional stiffness on $k_w$ and $M_{cr}$ is based on finite element calculations performed with [1P]. In the first step of the parametric study $k_w$ is calculated on a simply supported member subjected to normal force furnished with warping restraints $C_w$ on both ends. The value of $k_w$ is preferably calculated from the elastic torsional buckling force because the calculation of $k_w$ from the formula of $M_{cr}$ depends also on factor $C_1$ which is coupled with $\kappa_w$, as it was indicated in section 1.2. In the second step the factor $k_w$ calculated with the help of [1P] is substituted into the approach employed in section 1.2. All the calculations on a LTB are for simply supported beams with warping restraints $C_w$ subjected to equal and opposite end moments which deduce a constant bending moment as this loading gives the smallest ratios of elastic buckling moments (see [2]). With approaching towards torsional and warping fixity the value of 1.0 for factor $C_1$ is not more valid. Therefore, the value of $C_1$ was linearly interpolated according to $k_w$ and to $\kappa_w$. To estimate the accuracy of the proposed method the results are in the form of elastic buckling moment ratios $M_{cr,approx}/M_{cr,exact}$, where $M_{cr,approx}$ is calculated via the two steps calculation and $M_{cr,exact}$ is calculated with the program DRILL. For the scheme of the above described procedure see also Fig. 1.

Fig. 1. The scheme of the proposed procedure

Because of limited amount of pages the reader should find reference to investigated lengths, cross-sectional dimensions and other parameters of beams in Table 1 and Table 2 in [8]. Non-dimensional values of $\pi L/\sqrt{EI_w/GI}$ were used in parametric study. Warping stiffnesses $C_w$ of different dimensions and types of supporting columns are given in Table 1 and Table 2 and were used for the sake of parametric study together with warping stiffnesses of four different thicknesses of end plates studied in [2, 3]. These were applied to both ends of the investigated simple supported member.
Table 1. Warping stiffness $C_w = GI_{t,\text{column}}h_{\text{beam}}$ of some hot rolled cross-sections of columns in [kNm$^3$], for $h_{\text{beam}}$ see Fig. NB.3.1 in [1]

<table>
<thead>
<tr>
<th>Hot rolled cross-section of beams</th>
<th>Hot rolled cross-section of columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPE 80</td>
<td>IPE 120 IPE 160 IPE 200 IPE 240 IPE 300 IPE 360 IPE 400 IPE 500 IPE 600</td>
</tr>
<tr>
<td>IPE 200</td>
<td>0,108 0,269 0,559 1,083 1,998 3,121 5,789 7,923 13,85 25,66</td>
</tr>
<tr>
<td>IPE 400</td>
<td>0,218 0,543 1,128 2,185 4,032 6,298 11,68 15,99 27,95 51,79</td>
</tr>
</tbody>
</table>

The hot rolled cross-sections of columns were investigated in the whole range of sections IPE and HEB. The IPE cross-sections of beams were paired with IPE (see Table 1) and also HEB cross-section of columns (see Table 2). The HEB cross-sections of beams where connected just to the HEB cross-section of columns as indicates Table 2.

Table 2. Warping stiffness $C_w = GI_{t,\text{column}}h_{\text{beam}}$ of some hot rolled cross-sections of columns in [kNm$^3$]

<table>
<thead>
<tr>
<th>Hot rolled cross-section of beams</th>
<th>Hot rolled cross-section of columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEB 100</td>
<td>HEB 200 HEB 260 HEB 300 HEB 340 HEB 360 HEB 400 HEB 600 HEB 800 HEB 1000</td>
</tr>
<tr>
<td>IPE 200</td>
<td>1,434 9,195 19,2 28,70 39,90 45,36 55,18 103,5 146,7 194,6</td>
</tr>
<tr>
<td>HEB 200</td>
<td>1,386 8,883 18,55 27,73 38,54 43,82 53,31 99,98 141,8 188,0</td>
</tr>
<tr>
<td>IPE 400</td>
<td>2,895 18,56 38,75 57,93 80,52 91,56 111,4 208,9 296,2 392,7</td>
</tr>
<tr>
<td>HEB 400</td>
<td>2,816 18,06 37,70 56,36 78,33 89,07 108,4 203,2 288,1 382,1</td>
</tr>
</tbody>
</table>

In the range of parametric study some of the combinations of the beams and columns may not occur in practice, but the values of $C_w$ in Table 1 and 2 were calculated for the sake of complete parametric study.

3. Conclusions

On the basis of the parametric study the following conclusions can be drawn:

- The maximum value of $C_w = 51,79$ kNm$^3$ for IPE cross-sections of columns together with IPE cross-sections of beams (see Table 1) does not reach the maximum value of $C_w = 124,42$ kNm$^3$ of four different thicknesses of end plates in [2]. For example the column IPE 330 has approximately the same $C_w$ as 20 mm thick end plate in the case of IPE 200 used as a beam.
- The higher torsional stiffness of columns HEB combined with IPE beams results in a higher maximal value of warping stiffness $C_w = 392,7$ kNm$^3$ (see Table 2), which is higher than maximum $C_w$ of end plates.
- The constructional detail of beam to column joint with end plates, resulting from higher $C_w$ on both ends of the beam gives naturally smaller values of $k_w$ and higher values of $M_{cr}$.
- For different lengths and different cross-sectional dimensions the $k_w$ is different, but important is, that $M_{cr}$ can be increased with a moderate computational effort.
- Generally, as the non-dimensional torsion parameter $\pi L/\sqrt{E I_w G I_t}$ decreases (the length of the investigated beams increases) also the influence of warping torsion decreases, that is the reason why the $k_w$ values are smaller with smaller values of $\pi L/\sqrt{E I_w G I_t}$ and vice-versa.
- For all investigated IPE and HEB cross-section beams the increased $C_w$ due to columns torsional stiffness still does not, in no case, reach the limit of warping fixity ($k_w = 0,5$), see Fig. 2.
- It was observed that at some point of a very high $C_w$, the further increase of $C_w$ does not have a big influence on the $k_w$. The value of $k_w$ with further increasing of $C_w$ asymptotically approaches the value 0,5.
Theoretically, to reach the warping fixity we would need an infinite value of \( C_w \). Probably closed form cross-sections would approach this target value.

The lowest values of \( k_w \) were observed on IPE 200 cross-section of beams. In both IPE 200 and IPE 400 practical cases the values of \( k_w \) were smaller than 0.6, see Fig. 2. With the use of whole range of HEB profiles, the maximal \( C_w \) leads to a minimum value of \( k_w \) equal to 0.501, which confirms the previous implication.

Naturally, the highest \( k_w \) has the HEB 400 cross-section, which varies from 0.9 for short beams to 0.6 for longer beams with high values of \( C_w \), see Fig. 3. The lower values of \( k_w \) can be obtained with column HEB 1000 together with end plate thickness up to 40 mm.

Generally, the higher cross-section with wider flanges, the influence to \( k_w \) is less significant.
The most commonly used end plates in practice with thicknesses of 10, 20 mm investigated in [2] gave the ratio of $M_{cr}$ with influence of $C_w$ to $M_{cr}$ without $C_w$ of fork support maximal 15%. This time with increased $C_w$ due to torsional stiffness of columns the ratio is much higher.

The proposed approach to calculation of $M_{cr}$ by means of presented paper (see section 2) is not the most correct one, but until now the only possible as in the formula (2) there are two unknown values and each one depends on the other.

The value of factor $C_1$ for a higher precision should be linearly interpolated in square according to the calculated value of $k_w$ and the value of parameter $N_{wt}$. The value of $C_1$ should be found in specific sources like [3] or produced by a more exact calculation with a computer program as the value of $C_1$ for investigated loading and warping fixity is not given in [1].

The influence of higher $C_w$ on the value of $M_{cr}$ has the same principles which were described above in the case of $k_w$, but in an opposite direction.

The elastic buckling moment ratios $M_{cr,approx}/M_{cr,exact}$ vary from 1,002 to 1,072, see Fig. 2, 3. This means that the elastic buckling moment calculated according to the proposed procedure gives higher values than elastic buckling moment calculated by the use of software DRILL. The values of $M_{cr,approx}$ are hence on an unsafe side, but maximum 7,2%. This results from a not accurate linear interpolation of the values $C_1$ as the relation between $C_1$ and $k_w$ is not linear. The highest errors were observed with intermediate and higher values of $k_w$, as the value of $k_w$ approaches to 0,5 the error is smaller.

Acknowledgements

The author acknowledges support by the Slovak Scientific Grant Agency under the contracts No. 1/1101/12 and by the program: Program na podporu mladých výskumných pracovníkov – LTBCD 2012.

References


Programs