2.12. Markov processes

Fluctuation Theory for Systems of Signed and Unsigned Particles with Interaction Mechanisms based on Intersection Local Times

Robert J. Adler, *Israel Institute of Technology, Haifa, Israel*

We consider two distinct models of particle systems. In the first we have an infinite collection of identical Markov processes starting at random throughout Euclidean space. In the second a random sign is associated with each process. An interaction mechanism is introduced in each case via intersection local times, and the fluctuation theory of the systems studied as the processes become dense in space. In the first case the fluctuation theory always turns out to be Gaussian, irregardless of the order of the intersections taken to introduce the interaction mechanism. In the second case, an interaction mechanism based on k-th order intersections leads to a fluctuation theory akin to a Φ^k Euclidean quantum field theory. We consider the consequences of these results and relate them to different models previously studied.

A Markovian Sequence; Stationarity and Extremal Properties

Maria Teresa Alpuim, *University of Lisbon, Portugal*

Let $\{Y_n\}$ be a sequence of i.i.d. random variables with common d.f. $F(x)$ and X_0 a random variable independent of the Y_i's with d.f. $F_0(x)$. Define the Markovian sequence

$$X_i = k \max\{X_{i-1}, Y_i\} \text{ if } i \geq 1, \quad X_i = X_0, \text{ if } i = 0, \qquad k \in R, \quad 0 < k < 1.$$

We study the main properties of this sequence and give conditions to obtain stationarity. It is shown that for any d.f. $H(x)$ with left end point greater than or equal to zero for which $\log H(\mathrm{e}^x)$ is concave it is possible to construct such a sequence, being stationary and with marginal distributions equal to $H(x)$.

For this type of sequences the distribution of the maximum term and time between consecutive exceedances of a fixed level are trivial and we also give the distribution of the minimum. We study the limit law of extreme and kth order statistic in the stationary case and show that, in some cases, these sequences provide an interesting example of a stationary sequence verifying Leadbetter's $D(u_n)$ condition and, thus, whose maximum term has a nondegenerated limit law, but possessing extremal index lesser than one for which it is possible to specify completely the limit laws for the kth order statistic.

Similar results hold when we consider sequences defined by

$$Z_i = \max\{Z_{i-1}, Y_i\} - k, \quad k > 0,$$

or even

$$U_i = k \min\{U_{i-1}, Y_i\}, \ k > 1, \quad \text{and} \quad V_i = \min\{V_{i-1}, Y_i\} + k, \ k > 0.$$