Determination of regressive relation binding the theoretical and observed final values of curvatures for geological and mining conditions the one of JSW coal mines

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Abstract

The aim of this paper is obtaining of dependence between practical and theoretical values of curvatures which were calculated at a given level of forecast’s safety. The practical curvatures were determined on the basis of results of geodetic measurements conducted on observation points situated in form of line. Values of theoretical curvatures were determined by the usage of EDN – OPN computer program, applying the Budryk – Knothe theory assuming the typical values of its parameters ($a = 0.8$, $tgb = 2.0$, $B = 0.32r$). Then calculated an unreliability of forecast of curvatures’ final values. The regression relation between the observed and theoretical final values of curvatures was determinate assuming the probability of 50% that the measured value will not exceed the predicted value. The values of standard deviation are between 22.84 [m^{-1} 10^{-6}], when all final values of measured and theoretical curvatures are simultaneously taken into account in a linear regression analysis, and 25.32 [m^{-1} 10^{-6}], when a linear regression is carried out for the curvatures measured after the exploitation of the third longwall. The lower value of standard deviation (16.38 [m^{-1} 10^{-6}]) was obtained when a linear regression was made for the curvatures observed after the exploitation of the first longwall. On the basis of the undertaken analysis it can be concluded that for the geological and mining conditions prevailing in the area of measuring line, a regression relation between the measured and theoretical values of curvatures can be expressed by some equation. However, the predicted curvatures are characterized by the calculation error.

1. Introduction

Curvatures’ values of subsidence troughs are significantly impacted by random factors and as a result, the description of factor’s distribution is characterised by a number of errors (Popiolek, 2009). At the same time, due to current medium and large depths of mining works, the category of mining area is determined by values of horizontal deformation and not by curvatures. Consequently, the specialists working on issues...
connected with mining damages have not been particularly interested in this deformation indicator. The indicator is also omitted while preparing working plans which are the basis used by regional mining authorities to allow underground workings.

Such approach is in principle correct, however, as emphasized by the specialists in building in mining areas, in the case of some buildings, the values of curvatures impacting the buildings largely determine the extent of damages and the possibility of their safe use (Ostrowski, 2006).

Modern software allows to calculate values for mining works of any shape of directional and main curvatures, as well as to calculate final and extreme values in time on the basis of the formula by S. Knothe and the extension of the theory presented by J. Białek. The formula allows to include exploitation edges and so-called distant impacts.

The aim of this paper is to determine the difference between measured curvatures and theoretical curvatures calculated by the formula presented by S. Knothe (Knothe, 1953), with the parameters’ values proposed by S. Knothe. The calculation was carried out for final surface curvatures at measuring lines placed perpendicularly to directions of four longwall located in 338/2 coal bed in one of the mines of Jastrzębska Spółka Węglowa.

2. Theoretical curvatures distribution

Calculation for curvatures distribution were conducted for theoretical mining exploitations in three longwalls at different phases of exploitation development (Białek & Mielimąka, 2003) in order to demonstrate the theoretical curvatures along a profile of a trough, crosswise to workings direction. The calculations were evaluated using EDBJ series programs (Białek, 2003) and included implementation of the formula provided by the Budryk-Knothe theory – which also can be used to forecasting deformations of gateroads (Prusek & Jędrzejec, 2008) and proposed extension by J. Białek (Białek, 1991). The results are presented in Fig. 1.

On the basis of theoretical curvatures distribution presented in Fig. 1, it can be concluded that (Mielimąka, 2009):

- after the exploitation of the first longwall, positive curvatures, even twice larger than extremes negative curvatures outside the mining exploitation area, may occur above the gobs. It takes place when the width of the mining exploitation band (longwall length) reaches the critical value, which in the case of Knothe theory is 0.8r,
- after the exploitation of the second longwall, the extreme curvatures, both positive as well as negative, on each sides of the trough have the same values as negative extreme curvatures after the mining exploitation of the first longwall,
- the mining exploitation of the third longwall did not cause changes of values of extreme curvatures on any side of the trough, however, it resulted in a formation of a flat trough bottom above the area of the second longwall, where the curvatures are equal to 0.

3. A short characteristic of the observation material

Measuring line No. 1 was placed in order to register deformations caused by mining works of 001, 002, 005 and 007 longwall in 338/2 coal bed (Fig. 2). Systematic altitude observations (using a precision levelling method) and linear (using a precise electronic rangefinder) was conducted during the exploitation along line No. 1.

Mining works at 002, 005 and 007 longwalls were conducted using a longitudinal caving method, in accordance with the

![Fig. 1 – Theoretical graph of curvatures along the line perpendicular to the direction of three longwalls.](image-url)
numbering, at 580 m–700 m. The length of the longwalls was about 250 m, and their runs increase from 750 m, for longwall 001, to 1080 m in the case of longwall 007. An inclination of the seam in that region amounts to about 6°–8° and its average thickness, almost equal to the height of the longwalls, is 2.0 m.

Line No. 1 runs from north to south according to the seam’s dip through the middle of the workings. It consists of 53 stabilized points which are located at a distance of about 24 m to about 50 m.

The analysis of curvatures registered on this measuring line included the selection of geodetic results of observations from series 3, 7, 14 and 17 which had been conducted after the exploitation of 001, 002, 005 and 007 longwalls, 338/2 coal bed.

4. Evaluation of curvatures measured after subsequent phases of exploitation and theoretical curvatures

The values of curvatures for finally formed subsidence trough at individual points of observation lines were calculated on the basis of measured depression and measured lengths of its sections. Knowing the depressions \( w_i \) at points \( P_i \) and the distance between the points \( l_{i,i+1} \), the inclination of individual points is determined at first \( T_{i,i+1} \) by the formula:

\[
T_{i,i+1} = \frac{w_{i+1} - w_i}{l_{i,i+1}}.
\]  

(1)

When a particular section is twisted or bend at an angle \( \alpha \) in relation to a perpendicular edge of the exploitation, the inclination calculated by the formula (1) must be increased by dividing its value by \( \cos \alpha \).

The curvature \( K_{i,i+2} \) at point \( P_{i+1} \) is calculated by the following formula:

\[
K_{i,i+2} = 2\times\frac{T_{i,i+1} - T_{i,i+2}}{l_{i,i+1} + l_{i,i+2}}.
\]  

(2)

The final course of surface curvatures along the measuring line No. 1 after individual phases of exploitation is shown in Fig. 3.
The values of curvatures which theoretically occurred at individual points of the measuring line No. 1 were determined using EDBJ program created by J. Białek. The calculation was conducted using the formula of subsidence by S. Knothe. The formula adopted values typical for projections of parameters values deformation of the formula: subsidence factor $a = 0.8$ for exploitation with caving, $a = 0.7$ for exploitation with caving and gob sealing by post power plant dust, and the parameter $\tg = 2.0$ (Scigała, 2013).

Final values for each stage of exploitation of curvatures along the measuring line were determined using the adopted theoretical model. The values were defined for each calculation point.

The course of surface and theoretical curvatures along the measuring line No. 1 after the mining exploitation of individual longwalls is shown in Fig. 4.

5. Determination of relations between theoretical and measured curvatures

Final projected curvatures $K_{pred}$ are determined by the relation (Białek, 2013):

$$K_{pred} = \gamma K_{theor} + \lambda \sigma_K$$

where:

- $\gamma$ – a parameter determined on the basis of a set of measured curvatures' values and related theoretical curvatures while minimizing the value of the formula $(K_{meas} - \gamma K_{theor})^2 = \text{minimum}$,
- $\lambda$ – a parameter which value depends on the adopted level of projection certainty (Kowalski, 2007); assuming that deviations of measured values from projected values meet the requirements of a standard distribution, depending on parameter $\lambda$, the result will include:
  - $\lambda = 0$ – standard deviation of the deformation parameter is not included in the projection, however, the probability that the measure value will be smaller than projected values is 50%,
  - $\lambda = 1.0{\text{--}100\%}$ of the standard deviation is added to projected values, which gives 84% probability that the measure values will be smaller than projected,
  - $\lambda = 1.64{\text{--}164\%}$ of the standard deviation is added to projected values, which gives 95% probability that the measure values will be smaller than projected,
  - $\lambda = 2.24{\text{--}224\%}$ of the standard deviation is added to projected values, which gives 99% probability that the measure values will be smaller than projected.

The value of the standard deviation characterizing the data set composed of $n$ pair of elements $(K_{meas}, K_{theor})$ is determined by the formula:

$$\sigma_K = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (K_{meas} - \gamma K_{theor})^2}$$

The standard deviation of deformation indicators determined in such a way will depend on random dispersion of the curvatures, errors in determination of measured curvatures and errors in the applied projection model.

An analysis of a linear regression was carried out for surface curvatures along line No. 1 after the finish of mining.
works at subsequent longwalls and relating theoretical curvatures. The calculation were based on S. Knothe theory. Table 1 presents the results of the analysis and basic data concerning:

- the set of curvatures which were the subject of the analysis and marked by a number of pairs of curvatures included in the linear regression analysis, and definitely greatest final values of measured and projected curvatures,
- the results of linear regression analysis $K_{\text{meas}} = \gamma K_{\text{theor}}$ by presenting the values of $\gamma$ coefficient and the values of $R^2$, determination coefficient,
- certainty levels of description of adopted model for measured curvature characterised by $\sigma_K$ standard deviation and $M_K$ variation coefficient.

The comparison of curvatures measured crosswise to the direction of the measuring line No. 1, after the mining exploitation of subsequent longwalls with relating theoretical curvatures calculated on the basis of S. Knothe theory, allowed to formulate the following statement:

1. Extreme values of curvatures measured during every stage of the exploitation are always significantly greater than the theoretical curvatures (max. in the range of positive curvatures $K_{\text{max,meas}} = 2.93 * K_{\text{max,theor}}$).

2. According to the theory of extreme impacts, the curvatures left in an incomplete trough should become, even twice smaller after the complete trough is formed. Extreme curvatures measured at the observation line No. 1, which had occurred in the incomplete trough formed over the area of the first exploited longwall, were not significantly or not at all reduced in the result of further exploitation. Therefore, in this case permanent deformations were formed which cannot be compensated.

3. The values of $\gamma$ coefficients of linear regression formulae $K_{\text{meas}} = \gamma K_{\text{theor}}$ determined after the mining exploitation of the second and third longwall are similar and their values are within the range of 1.0359 to 1.0649. The value of this coefficient for the regression formula determined for each carried out measurements is also close to one and amounts to 0.9466. Only in the case of regression analysis $K_{\text{meas}} = \gamma K_{\text{theor}}$ conducted for theoretical and measured curvatures after the mining exploitation of the first longwall, the values of this coefficient is smaller – 0.7664.

4. Obtained values of the deformation coefficient are decreasing along with the mining exploitation halt and smaller distance to the complete trough at 0.6098 to 0.3049 and 0.3399. It indicates that the final curvatures measured at the measuring line No. 1 are progressively less determined by the theoretical curvatures when the selected area becomes wider and the complete trough is formulated.

5. In all analysed cases, the values of standard deviation calculated by the Formula (4) are similar. They amount from 22.84 [m$^{-1}$ 10$^{-6}$] if all final measured curvatures and relating theoretical curvatures are include in the analysis of the linear regression. When the regression was conducted for curvatures measured at the line No. 1 after the

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### Table 1 — Basic data characterizing the results of the linear regression analysis.

<table>
<thead>
<tr>
<th>Exploitation stage</th>
<th>Number of analysed pair of curvatures</th>
<th>Theoretical extreme curvature $K_{\text{theor}}$ [m$^{-1}$ 10$^{-6}$]</th>
<th>Measured extreme curvature $K_{\text{meas}}$ [m$^{-1}$ 10$^{-6}$]</th>
<th>Coefficient of regression $\gamma$</th>
<th>Determination coefficient $R^2$</th>
<th>Standard model deviation $\sigma_K$ [m$^{-1}$ 10$^{-6}$]</th>
<th>Curvature variation coefficient $M_K = \frac{s_K}{K_{\text{max,meas}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>After longwall 001</td>
<td>31</td>
<td>83.3</td>
<td>81.1</td>
<td>0.7664</td>
<td>0.9466</td>
<td>56.8</td>
<td>91.1</td>
</tr>
<tr>
<td>After longwall 002</td>
<td>37</td>
<td>81.1</td>
<td>83.3</td>
<td>0.6098</td>
<td>0.4631</td>
<td>91.1</td>
<td>31.1</td>
</tr>
<tr>
<td>After longwall 005</td>
<td>47</td>
<td>89.9</td>
<td>88.9</td>
<td>1.0359</td>
<td>0.3049</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>After longwall 007</td>
<td>47</td>
<td>87.2</td>
<td>88.9</td>
<td>1.0649</td>
<td>0.3399</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>All measurements</td>
<td>160</td>
<td>91.1</td>
<td>58.8</td>
<td>0.3950</td>
<td>0.7732</td>
<td>22.8458</td>
<td>0.7732</td>
</tr>
</tbody>
</table>

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mining exploitation of the third longwall they amount to up to 25.32 [m\(^{-1}\) 10\(^{-6}\)]. Lower value of the standard deviation, 16.38 [m\(^{-1}\) 10\(^{-6}\)], was obtained only when the regression had been conducted for the curvatures registered after the mining exploitation of the first longwall.

6. The coefficients of variation of final curvatures measured along the line No. 1 are very large and have similar values. The coefficients were calculated after the mining exploitation of the second, third and fourth longwall and amount to 0.7436 (after the second longwall), 0.7732 (after the third) and 0.8149 (after the fourth).

A significant dispersion of measured final curvatures and relating theoretical curvatures in relation to the line of linear tendency \(K_{\text{meas}} = \gamma K_{\text{theor}}\) for each of the analysed measuring lines No. 1 is visible in the diagram presented in Fig. 5.

On the basis of conducted analysis, it can be concluded that the Formula (3) for geological and mining conditions of the mine, in the mining area where the line No. 1 is located, with the coefficient \(l = 0\), will adopt the following pattern:

\[
K_{\text{pred}} = 1.04 K_{\text{theor}}
\]

(5)

Projected curvatures calculated by this formula are characterized by a significant error that amounts to \(\pm 0.74 K_{\text{max}}\),

where: \(K_{\text{max}}\) – an extreme final curvature determined for a complete subsidence trough.

Obtained regression Formula (5) is the best estimator for measured values in terms of the method of least squares. The application gives 50% of probability that the observed values will not exceed the projected values. Nevertheless, it should be emphasized that notable differences between projected values and the ones determined theoretically may occur in some particular areas.

6. Conclusions

The conducted analysis of linear regression of final curvatures measured at the line No. 1 at one of the mines after the completion of mining exploitation of subsequent four walls in the 338/2 coal bed and relating final theoretical curvatures allowed to conclude that the final curvatures values calculated by the S. Knothe theory, while adapting typical final parameters of the formula \((a = 0.8, \tan b = 2.0)\), multiplied by the coefficient \(\gamma = 1.04\), are the best estimator for measured values in terms of the method of least squares. Values of final theoretical curvatures determined in this way at particular measuring points may be significantly different from final values of measured curvatures in such points, because the average error level of this type of projections is high. It is the result of the random dispersion of the curvatures, errors in determination of measured curvatures as well as errors in the applied projection model.

A method that could effectively lower the error level should be searched through an application of changes to the projection model and formulation of a model which would include exploitation edges that are located far from impacts and the influence of order and direction of the mining works on the deformation distribution in the mining areas. The new method must reduce the error level mainly in the areas of extreme curvatures occurrence, which is particularly important in the case of curvatures impact on buildings (Ostrowski, 2006) and in the area of trough bottom.

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References