Assembly Tolerance Analysis Method Based on the Real Machine Model with Three Datum Planes Location

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Abstract

An assembly tolerance analysis method based on the real machine model is proposed, aiming to investigate the effect of the geometric errors of datum feature on the result of tolerance chain. The real parts are simulated by the variation of the geometric features, which are represented by the control point variation model and generated by the Monte Carlo simulation. The investigated objective is the assembly success rate of pin-hole assembly at the location with the three datum planes, the assembly clearance or interference is calculated according to the assembly requirements and assembly process. The part coordinate system in assembly is constructed firstly by the datum embodiment principles, then the position of the junction planes are determined according to the assembly requirements, and finally, the position of the parts relative to machine are determined by the coordinate transformation. The assembly success rate is represented by the probability distribution of clearance or interference between pin and hole, and the influences of the datum errors on the assembly result are discussed by the example assembly.

1. Introduction

1.1. The motivation for this study

The purpose of this work is as follows:
1. To propose the establishment method of coordinate system of part in assembly. In this method, the datum features are represented by the associated geometry, the position and orientation deviations of datum feature are included and the form errors are neglected, the deviation information of the associated geometry is simulated by the Monte Carlo method, the tolerance zone of datum features is the sample space of associated geometry, so the establishment of datum reference frame is based on the real machine model.
2. To establish the position calculation method of part in assembly based on the real machine model. In this method, the datum embodiment principles, position errors of datum features, and datum precedence order are taken into consideration, which was usually ignored in the previous methods.
3. To investigate the effect of geometric errors of datum feature on the result of tolerance chain at different datum precedence order, and to investigate the correlations between the tolerance specifications of datum plane under consideration of the datum precedence. It has not involved in the past tolerance analysis.

1.2. Literature review

Variations and tolerance accumulation are unavoidable in product design. In order to check if the resultant tolerance accumulations meet the functional requirements, tolerance analysis is used for this purpose. A substantial amount of research has been devoted to the development of tolerance analysis. Therefore, we will briefly review assembly tolerance analysis in design in this section. In all efforts to represent GD&T in computer systems, the "offset models"[1] by Requicha were introduced firstly at the beginning of the 1980s, and since then various models and methods are proposed by the researcher. The tolerance chain technique is the most popular method in industry by draftsmen and designers to conduct stack up analysis. The designer typically works with engineering drawings and interprets the tolerance standard symbols. The dimension chain is constructed in the
direction of analysis, the geometric tolerances are taken as zero dimension + variation, the plus tolerance and minus tolerance are assigned on each arc, and the accumulation is calculated by the closed tolerance chain. It presents the two most common models for tolerance accumulation; worst case and statistical respectively. The analyst positions parts in assemblies to yield each of the worst cases or statistical values in each analysis direction, i.e., separate dimension chain have to be constructed for each worst case. Since impact factors not in the direction of analysis are ignored, this may yield significant errors in most cases.

Most CAT packages take advantage of the same parametric/variational approach used in CAD systems and apply the Monte Carlo simulation to tolerance analysis [2–4]. In parametric approach, the analyzed dimension is expressed as an algebraic function or a set of algebraic equation that relates the analyzed dimension to those on which it depends. The function is either linearized or directly used for the Monte Carlo simulation in the nonlinear analysis. Results commonly available are the lists of contributors, sensitivities, and percentage contributions, and the tolerance accumulation. The parametric approach is easy to integrate with CAD system. However, parametric analysis directly using the constraint model in CAD has some inherent limitations. The model variables do not necessarily correspond directly to the tolerances that may be specified on the drawings, it is difficult to automate the parametric approach.

Chase and Gao [5, 6] developed a vector loop model; dimensions are represented by vectors, and the length of the vector is the magnitude of the dimension. Kinematics variations are small adjustments between contact surfaces, which occur at the assembly time in response to the dimensional and geometric variations, the contact surface is a joint with relevant mating relations. Geometric tolerances are considered by adding micro DOFs to particular ones of the joints. The kinematics approach is geometrically simple and computationally efficient. But kinematics is not enough to model all types of geometric variations, it is not totally consistent with tolerance standard, and the construction of the vector loop is difficult, so this method heavily depends on the user’s expertise and experience to obtain correct results.

The DOF analysis method of tolerance analysis is become a mainly approach in recent research. DOF models treat geometric entities as if they were rigid bodies with DOFs. Geometric relations are treated as constraints on DOFs. According to the representation of tolerance and geometry feature, this approaches are classified as variation models [7], TTRS[8-10], SDT[11-14], T_Map [15,16]. The mathematic tools are used to represent tolerance types and variations, these mathematic tools are included vector representation, screw theory, torsor, affine geometry etc., the coordinate transformation matrix are used to construct the tolerance accumulation and propagation models. Most tolerance approaches based on DOF analysis are indifferent to tolerance standard, such as ASME Y14.5 Rule #1, floating zones, effects of bonus and shift, form tolerance, or datum precedence. Although T-Map model is consistent with ASME Y14.5 Standard, but the mathematic tool and calculation procedure applied are complex and not straightforward.

Each of the models represents an advantage over the other in at least one aspect or another, but none of the models are perfect in representing all geometrical tolerances specified in the standard. The present methods have not considered the issue about the error transformation by assembly contact, and have not investigated the effects of assembly precedence order to the tolerance accumulation systematically; it is the common defects of all models and methods. In assembly tolerance analysis, the errors of assembly datum have been discussed rarely, although the kinematic method and deviation domain have modeled the assembly clearance, but didn't include the error of contact surface.

This literature survey gives an overview of research that had been performed in the area of tolerance analysis. It shows that none of the methods have been concerned on the tolerance transformation by assembly datum. The rest of this paper will elaborate on the tolerance transformation model and its implementation with an assembly example.

2. Approach overview

2.1. The definition of geometric feature position and its simulation

The position information hierarchy of geometric feature is outlined in Fig. 1, there are four procedures to define the position of a target geometry feature from bottom to up: 1) The mating model construction, it establishes the positional relationship among mating parts or components in a stated order of precedence; 2) DRF establishment, a DRF is established according to three datum features at an order of precedence and datum embodiment principle; 3) The ideal position determination, the ideal position of target feature and its tolerance zone are positioned by the nominal dimensions relative to the DRF; 4) The generation of the sample position, the sample geometry is represented by the control point variation model (CPVM) [18] and generated by the Monte Carlo simulation method.

![Diagram of geometric feature position](image)

Fig. 1 the definition of position of geometric features
2.2. The influence of assembly requirement on the propagation of part's geometric errors

In the current research, the assembly location datums of a part are three pairs of plane datum. Because of the existing of the orientation and position derivation of geometric feature, not all three pairs of datum planes keep coplanar contact with each other. The contacts are determined by the assembly order of precedence and the constraint ability of assembly datum.

According to the theory of degree of freedom (DOF), the assembly requirements for three pairs of planar datum are given as follows: 1) the first datum plane "face-face coincidence", two datum planes are coplanar; 2) the second datum "face-face alignment", two intersection lines of two second datum planes with their first datum plane must be parallel; 3) the third datum "face-face contact", two third datum feature contact each other at least one point. The mating "face-face coincidence" constrains 3 DOFs of the part, two rotational DOFs and one translation DOF. The mating "face-face alignment" constrains two DOFs, one is rotational DOF and the other is translation DOF. Because there are different contact states for different order datum feature, the influence of their errors to the location of assembled part are different.

3. The establishment of DRF based on the CPVM models

3.1. The criteria of the datum planes determination

Because of the manufacturing errors of the actual datum feature, the datum planes of a DRF cannot align to the actual datum feature completely, the DRF should be constructed according to the datum embodiment principles and datum precedence order. In order to construct the DRF from the actual datum feature, the datum feature simulator [17] is used. In the studied example of this paper, the datum feature simulators are three planes. The actual datum planes are represented with the associated plane and defined by four control parameters of CPVM model [18]. According to the datum embodiment principle and the fact that the actual datum planes are represented by the associated plane, the coordinate planes of a DRF are determined: 1) The primary coordinate plane of a DRF is the associated plane of primary datum feature; 2) the Z axis of the DRF is parallel to the normal of the associated plane; 3) the secondary coordinate plane of a DRF is perpendicular to the primary datum plane and passes through the intersection line of the second datum feature simulator with the primary coordinate plane, the intersection line is the X axis of the DRF; 4) the tertiary coordinate plane of a DRF the intersection point of the third datum feature simulator with the X axis of the DRF, the intersection point is the origin of the DRF.

Because primary coordinate plane of a DRF are coplanar with the associated plane of primary datum feature, the establishment of DRF includes the determination of second coordinate plane and tertiary coordinate plane. According to the assumption of three planar datums, the establishment of a DRF is done by determining secondary datum feature simulator and tertiary datum feature simulator.

![Fig. 2 the position relation of f2 relative to F1](image)

3.2. The construction of second datum feature simulator plane

The determination mechanism of secondary datum feature simulator is described in Fig. 2, the second datum associated plane is denoted by f2 and the primary coordinate plane of the DRF is denoted by F1, the second datum plane is selected at an inclined position on propose of the procedure's generality. In this figure, F1 is coplanar with its associated datum plane f1, and f2 is defined by four parameter s1, s2, s3 and s4 controlled by the CPVM model. The line λ is the intersection line of f2 with f1, the vectors n1 and n2 are the normal of plane F1 and the nominal plane of f2 respectively. According to definition of the datum feature simulator, E2 is determined under three conditions: 1) the angle between E2 with F1 is a nominal angle; 2) E2 is parallel to λ, it ensures that the intersection line of E2 with f2 is parallel to F1; 3) E2 passes through the vertex or edge of f2. As a result, all three planes E2, f2 and f1 are parallel to λ. According to above analysis, E2 can be calculated by the projective geometry.

3.3. The construction of tertiary datum feature simulator plane

For the purpose of generality, the tertiary associated datum plane f3 is set at an inclined position relative to primary and secondary datum plane. Fig. 3 shows the position of the primary coordinate plane F1 and the tertiary associated datum plane f3. In Fig. 3, line L is the intersection line of secondary
coordinate plane with the primary datum plane, i.e. the X axis of the DRF being constructed, and the vector \( n_3 \) is the normal of ideal tertiary datum feature. The datum feature simulator \( E_3 \) has the nominal angle with line \( L \) and has one contact point with \( f_3 \), and \( F \) is a plane parallel to \( f_3 \) at an arbitrary position. To move \( F \) until it contact \( f_3 \) along direction \( L \), the origin \( O \) is the intersection point of \( L \) with the position of \( F \).

4. The establishment of the coordinate relation of mating plane based on the CPVM model

Due to manufacturing errors, the position relation of coordinate system of two parts must be established according to the actual contact of mating surface pairs. For the assembly by three plane datum that investigated in this paper, the actual surfaces are represented by the associated plane, which is generated by the CPVM, so the contact of three pair of planes are determined by three pairs of associated plane. According to the precedence order, the assembly requirements for three pairs of associated plane are "face-face coincidence", "face-face alignment", and "face-face contact".

The assembly requirement "face-face coincidence" refers to the superposition of two primary associated planes, and for this reason, there are only the contact positions of other two pairs of associated planes must be determined, that is the contact position of secondary datum associated planes and that of two tertiary datum associated planes.

4.1. The contacting position calculation of secondary associated plane

Figure 4 shows the two secondary associated planes of the base part and the plate part before making contact. Two planes \( F_{2a} \) and \( F_{2b} \) denote the two secondary associated planes respectively, and the two quadrilateral enclosed by four double dot dash lines denote their normal planes. Two primary datum associated plane \( F_{1b} \) and \( F_{1p} \) are coplanar, and lines \( \lambda_1, \lambda_2 \) are the intersection of \( F_{1b} \) with \( F_{2a} \) and \( F_{1p} \) with \( F_{2b} \) respectively. All planes \( F_{1b}, F_{1p}, F_{2a} \) and \( F_{2b} \) are generated by the CPVM model. When the secondary datum surface is presented with associated plane, the contact requirements are turned into three conditions: 1) plane \( F_{1b} \) is coplanar with \( F_{1p} \); 2) \( \lambda_1 \) is parallel with \( \lambda_2 \); 3) plane \( F_{2a} \) contacts to plane \( F_{2b} \). Once the contact position of the plane \( F_{2a} \) and \( F_{2b} \) is determined, the distance between two secondary datum plane of DRF belonging to two parts respectively is determined. The distance is determined by the variation parameters \( (u_1, u_2, u_3, u_4) \) and \( (s_1, s_2, s_3, s_4) \) belonging to two associated planes \( F_{2a} \) and \( F_{2b} \) respectively.

4.2. The contacting position calculation of tertiary associated plane

The assembly requirement "face-face contact" of the tertiary datum refers to that two tertiary associated planes make contact at one point in respect to the assembly requirements for the first two assembly datum. Fig. 5 shows the two tertiary associated planes \( F_{3a} \) and \( F_{3b} \) of two parts before they make contact, the coordinate system \( o-x_1y_1z_1 \) are the global coordinate system of the first part, the axis \( x_1 \) and coordinate plane \( x_1o_1y_1 \) have been determined according to first two assembly condition, and only the origin \( o_t \) should be determined by the assembling requirement "face-face contact" of two tertiary associated planes.

According to the assembling requirement "face-face contact", the position of tertiary associated plane \( F_{3a} \) is that \( F_{3b} \) passed through the real contacting point. To find a vertex of plane \( F_{3b} \) that contacts with the tertiary DRF plane \( y_1o_1z_1 \) of part plate using the method described in third section, and to project the vertex on axis \( x_1 \), the projection point is the origin \( o_t \) of \( o-x_1y_1z_1 \).

5. The assembly example

5.1. The assembly example and its specification

The current approach will be illustrated by a pin-hole assembly of two parts, the base and the plate, which is shown in Fig. 6. The true position of both pin and hole specifies a cylindrical tolerance zone with the diameter 0.1 mm respectively. The mating requirement of the contact surface for both base and plate are as follows: 1) the contact condition for datum A is "face-face coincidence"; 2) the contact condition for datum B is "face-face alignment"; 3) the contact condition for datum C is "face-face contact".

The assembly surface and the datum plane are represented by CPVM model, and the assembly clearance and interference are defined based on the CPVM model. The assembly success rate is represented by the probability
distribution of clearance or interference between pin and hole by the Monte Carlo simulation method.

5.2. The definition of clearance and interference of the pin-hole assembly

For the simplification of the calculation and consider the fact that the length and the diameter of the pin and hole much larger than the diameter of true position zone, it is assumed that the end planes of the pin and hole be coplanar with their ideal end plane, and that the end outline of the pin and hole be circle with radius \( R_1 \) or \( R_2 \) approximately. The clearance or interference of the pin-hole assembly is calculated at their end plane, and only one end point is taken into account because of the same variations of both end points. According to the definition of CPVM model, the pin-hole assembly at the end plane is shown in Fig.7, and the clearance or interference is calculated between two surfaces on the side at the direction from the center of hole toward the center of the pin, the value is defined as follows:

\[
e = R_h - (J + R_p)
\]

Where,

\( e \): the clearance or interference value.
\( J \): the distance of two centers of the hole and the pin.
\( R_h, R_p \): two radiuses of both hole and pin respectively.

5.3. The calculation results of the pin-hole assembly

For the assembly with three orthogonal location planes, the perpendicularity errors of the datum feature are the main factors to influence the assembly clearance and interference. In the example assembly, the effects of the perpendicularity errors are investigated mainly, the perpendicularity includes those of the secondary datum and tertiary datum plane relative to the primary datum plane. The effects investigated include two aspects, one is the deviation of the coordinate system of the plate from the base and, and another one is the assembly clearance and interference between two surfaces of pin and hole. In the example assembly, the results are computed by the Monte Carlo simulation at the simulation number 200 thousands.

Figure 8 is the relation of the mean clearance with the perpendicularity tolerance of both planes B and C relative to their primary datum plane. The mean clearance will decrease along with the increase of the perpendicularity tolerance. The design intent requires a clearance between two mating surfaces, but it will be found that only zero perpendicularity tolerance can reach the design intent, and the possibility of interference will increase along with the increase of the perpendicularity tolerance.

6. Conclusions

The positions of the parts or components on a machine are determined by the assembly datum and assembly sequence,
and the geometric errors of a machine are the results of the accumulation and propagation of the geometric error of the parts or components. The effects of geometric errors of the assembly scheme on the mating clearance between two mating surfaces are investigated in this paper; the research objective is the clearance calculation of the pin-hole assembly. The mating parts are located by three plane feature with different the perpendicularity error. The contributions of this paper are as follows.

1) The assembly datum and assembly sequence influence the stack up of the geometric error and the geometric errors of component are transmitted through the mating contact of assembly datum. The contact is determined by the assembly sequence of mating datum, and the assembly datum in different assembly sequence has different influence on the stack up of geometric errors. The geometric error of the component is transformed by the assembly scheme firstly, and then contributes to the assembly position of machine.

2) The real machine model is defined by the functional geometric feature of the component. The defined model of the real position of geometric feature is proposed. The representing method of real position of geometric feature is consisted of following procedure: the establishments of assembly coordinate relations, the construction of DRF of a tolerance, the determination of basic position and the model of control point variations of geometric elements.

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