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ScienceDirect

Procedia - Social and Behavioral Sciences 182 (2015) 703 – 709

Procedia
Social and Behavioral Sciences

4th WORLD CONFERENCE ON EDUCATIONAL TECHNOLOGY RESEARCHES,

Workspace analysis and geometric modeling of 6 DOF Fanuc 200iC Robot

Kamel Bouzgou^{a*}, Zoubir Ahmed-Foitih^b^a PhD Student at L.E.P.E.S.A Laboratory USTO-MB Oran ALGERIA^b Professor at L.E.P.E.S.A Laboratory USTO-MB Oran ALGERIA

Abstract

In this paper, we propose a study of criteria for choosing the best solution among the solutions of the inverse geometric model of the 6 DOF robot arm, FANUC 200iC Lr Mate. Knowledge of these parameters can help us the control and the generation of motions without that the task will be redundant with a minimization of the execution time, the effort and energy consumed by actuators. For this, the solving of the inverse kinematics by an analytical method is necessary, and the Jacobian matrix give us the nonlinear equation for find the singular configurations. We validate our work by conducting a simulation software platform that allows us to verify the results of manipulation in a virtual reality environment based on VRML and Matlab software, integration with the CAD model.

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Peer-review under responsibility of Academic World Research and Education Center.

Keywords: Forward geometric model, Inverse kinematic model, Singularity, Jacobian matrix, 6 DOF manipulator arms, Workspace, VRML, Matlab

1. Introduction

In telerobotic, the current problem of robotic systems has resulted in the reduction of physical workload of the operator, coupled with an increase in mental load.

To carry out a teleoperation action, it's necessary to provide the operator in the command / control situation, information on the progress of the task in the worksite, i.e. an assistance to the operator for perception, decision and control. That largely explains the development of the current artificial assistance techniques.

* Kamel Bouzgou. Tel.: +4-345-434-343.

E-mail address: bouzgou_kamel@hotmail.fr

The purpose of this paper, is to present the results of a graphical simulation of robot control industrial manipulator Fanuc 6 DOF in an environment with a synthetic representation of the scene (virtual world), which consists of all relevant objects models from the task site.

Initially, the robot was programmed and modeled by the manufacturer, with opaque and limited software for possible extensions.

For our use, we will try to develop a software model to an open system using the Matlab mathematical software in a virtual reality (VRML). We will determine the boundaries of the work-space of the robot arm, that are defined by the mechanical articulation limits and by singularities (Khalil and Dombre, 1999; Lallemand and Zeghloul, 1994), several studies on this subject have been made (Paul, 1981; Bouzgou and Ahmed-foitih, 2014), For this we proceed by a theoretical study of the robot in order to identify geometric parameters ,to determine the geometric and kinematic models required to our study.

2. Description of the geometry of Fanuc 200IC robot

The kinematics of the wrist is a RRR type, has three revolute joints with intersecting axes, equivalent to a ball socket (*Fanucrobotics*).

Where: $d2 = 75$ $d3 = 400$ $d4 = 75$ $R4 = 410$ $R6 = 80$

From a methodological viewpoint, firstly we place z_j axes on the joint axes, then the x_j axes, the geometric parameters of the robot are determined.

Axes 4,5 et 6 are concurrent axes, they presented the orientation of the end-effector, and they don't affect its position, for this, we can be defined a *E* matrix that represents the translation of the coordinate system frame of the end-effector relative to the R_6 frame, this translation along the z axis is equal to $R_6 + r$, such that *r* is the length along the same axis of the terminal member attached to the tool

All tables should be numbered with Arabic numerals. Every table should have a caption. Headings should be placed above tables, left justified. Only horizontal lines should be used within a table, to distinguish the column headings from the body of the table, and immediately above and below the table. Tables must be embedded into the text and not supplied separately. Below is an example which the authors may find useful.

Table 1 . Modified geometric parameters D-H of the FANUC robot. (Denavit, 1955)

Joint J	σ_j	α_j	d_j	θ_j	R_j
1	0	0	0	θ_1	0
2	0	90	d2	$\frac{\pi}{2} + \theta_2$	0
3	0	0	d3	θ_3	0
4	0	90	d4	θ_4	R4
5	0	-90	0	θ_5	0
6	0	90	0	θ_6	0

3. Geometric model of the FANUC robot:

The homogeneous transformation matrices are given by general matrix such as:

$${}^{j-1}T_j = \begin{bmatrix} C\theta_j & -S\theta_j & 0 & d_j \\ C\alpha_j S\theta_j & C\alpha_j C\theta_j & -S\alpha_j & -r_j S\alpha_j \\ S\alpha_j S\theta_j & S\alpha_j C\theta_j & C\alpha_j & r_j C\alpha_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Where: $C\theta_i = \cos(\theta_i)$ And $S\theta_i = \sin(\theta_i)$

1. Direct geometric model

The direct geometric model (DGM) is the set of relations which express the position of the end-effector, i.e. operational coordinates of the robot, according to its joint coordinates. In the case of a simple open-chain, it can be represented by the transformation matrix 0T_k . Such as ${}^0T_k = \prod_{i=1}^k {}^{i-1}T_i(q_i)$ (2)

Realizing the composition of transformations universal frame R_0 until frame R_6 of equation (2) we obtain:

$${}^0T_6 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \cdot {}^5T_6$$

Let us note: ${}^fT_E = {}^0T_6 \cdot E$ (3)

$${}^fT_E = \begin{bmatrix} s_x & n_x & a_x & P_x \\ s_y & n_y & a_y & P_y \\ s_z & n_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } {}^fT_E = \begin{bmatrix} {}^fR_E & {}^fP_E \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^f s_E & {}^f n_E & {}^f a_E & {}^f P_E \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$E = {}^0T_6$: Transformation matrix of tool frame in the end-effector frame.

That is: $T = {}^0T_6 \cdot E$ With: $\begin{cases} P_x = {}^fT_E & (1, 4); \\ P_y = {}^fT_E & (2, 4); \\ P_z = {}^fT_E & (3, 4); \end{cases}$ (5)

After calculation and identification of the terms of two matrices of the equation (3) (4), we will have:

$$\begin{cases} {}^fT_E(1:3,4) = T(1:3,4) \\ P_x = C1d2 + R6[C5C1C23 + S5(S1S4 - C4C1S23)] - d4C1S23 + R4C1C23 - C1S2d3 \\ P_y = d2S1 - d4S1S23 + R6[C5S1C23 - S5(C1S4 + C4S1S23)] + R4S1C23 - d3S1S2 \\ P_z = C2d3 + d4C23 + R4S23 + R6[C5S23 + C4S5C23] \end{cases} \tag{6}$$

Elements of the orientation matrix are represented by the ${}^f s_E$ ${}^f n_E$ ${}^f a_E$ column vectors.

$$\begin{aligned} {}^f s_E &= \begin{bmatrix} s6(c4s1 + s4c1s23 + c6(c5(s1s4 - c4c1s23) - s5c1c23)) \\ -s6(c1c4 - s4s1s23) - c6(c5(c1s4 + c4s1s23) + s5s1c23) \\ -c6(s5s23 - c4c5c23) - s4s6c23 \end{bmatrix} \\ {}^f n_E &= \begin{bmatrix} c6(c4s1 + s4c1s23) - s6(c5(s1s4 - c4c1s23) - s5c1c23) \\ s6(c5(c1s4 + c4s1s23) + s5s1c23) - c6(c1c4 - s4s1s23) \\ s6(s5s23 - c4c5c23) - c6s4c23 \end{bmatrix} \\ {}^f a_E &= \begin{bmatrix} s5(s1s4 - c4c1s23) + c5c1c23 \\ -s5(c1s4 + c4s1s23) + c5s1c23 \\ c5s23 + c4s5c23 \end{bmatrix} \end{aligned} \tag{7}$$

Where: $Ci = \cos(\theta_i)$ And $Si = \sin(\theta_i)$

2. Inverse kinematic model

The inverse problem is to calculate the joint coordinates corresponding to a given situation of the end-effector. When it exists, the form which gives all the possible solutions constitutes what one calls the inverse kinematic model (IKM). we can distinguish three methods of calculating of:

-Paul's method. (Paul , 1981).

- Pieper’s method. (Pieper , 1968).
- General method of Raghavan & Roth.

Several iterative methods to find the IKM (Benhabib and Goldenberg and Al, 1985; Toyosaku and Nagasaka, and Al, 1992) have been made, in our case, Pieper's method is suitable for manipulator arms with concurrent wrist axes are used.

2.1. Inverse kinematic model of FANUC robot

$$U_0 = \begin{bmatrix} & P_x \\ {}^0A_E & P_x \\ & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

With 0A_E orientation matrix of frame R_E/R_0

2.1.1. Calculation of θ_1, θ_2 and θ_3

$$U_0 = {}^0T_6 \cdot E \rightarrow U_0 \cdot E^{-1} = {}^0T_6 \rightarrow \dot{U}_0 = U_0 E^{-1}$$

With \dot{U}_0 a new orientation matrix ${}^1T_0 \cdot \dot{U}_0 \cdot [0 \ 0 \ 0 \ 1]^T$ Implies:

$${}^1T_0 \cdot \dot{U}_0 \cdot [0 \ 0 \ 0 \ 1]^T = {}^1T_6 \cdot [0 \ 0 \ 0 \ 1]^T \quad \text{and} \quad {}^1T_0 \cdot \dot{U}_0 \cdot [0 \ 0 \ 0 \ 1]^T \quad \text{Implies:}$$

$${}^1T_6 \cdot [0 \ 0 \ 0 \ 1]^T = {}^1T_4 \cdot [0 \ 0 \ 0 \ 1]^T$$

Because we have a three intersecting axes , While using Matlab mathematical software we found:

$$\begin{cases} C1P_x - R6C1 + P_yS1 = d2 - d3S2 - d4S23 + R4C23 \\ R6S1 + C1P_y - P_xS1 = 0 \\ P_z = C2d3 + d4C23 + R4S23 \end{cases} \tag{9}$$

With: $C23 = \cos(\theta_2 + \theta_3)$ and $S23 = \sin(\theta_2 + \theta_3)$ By using the 2nd equality of (9):

$$-S1(P_x - R6) + C1P_y = 0 \quad \text{Thus:}$$

$$\begin{cases} \theta_1 = ATAN2 (P_y, P_x - R6) \\ \theta_1 = \theta_1 + \pi \end{cases} \tag{10}$$

From a 1st equality of (9) we make: $C1P_x - R6C1 + P_yS1 - d2 = A$ And the all become:

$$\begin{cases} A = -d3S2 - d4S23 + R4C23 \\ P_z = C2d3 + d4C23 + R4S23 \end{cases} \tag{11}$$

From a 1st equality of (11) we draw $S2 \quad S2 = \frac{R4C23-d4S23-A}{d3}$ (12)

From a 2nd equality of (11) we draw: $C2 \quad C2 = \frac{P_z-d4C23+R4S23}{d3}$ (13)

Therefore:

$$\begin{aligned} d3^2S2^2 &= R4^2C23^2 + d4^2S23^2 - 25R4d4C23S23 + A^2 - 2AR4C23 + 2Ad4S23 \\ d3^2S2^2 &= P_z^2 + d4^2C23^2 - 2P_zd4C23 + R4^2S23^2 + 2P_zR4S23 + 2R4d4C23S23 \\ d3^2 &= C23^2[R4^2 + d4^2] + S23^2[R4^2 + d4^2] + C23[-2AR4 - 2P_zd4] + S23[2Ad4 - 2P_zR4] + P_z^2 \end{aligned} \tag{14}$$

We pose: $X = -2AR4 - 2P_zd4 \rightarrow Y = 2Ad4 - 2P_zR4 \rightarrow H = R4^2 + d4^2 + P_z^2 - d3^2 + A^2$

We replace in (13): $XC23 + YS23 + H = 0 \rightarrow -YS23 = XC23 + H$

$$Y^2 - Y^2C23^2 = X^2C23^2 + 2XHC23 + H^2 \rightarrow (X^2 + Y^2)C23^2 + 2XHC23 + (H^2 - Y^2) = 0$$

Equation (According to C23) of the second degree admits two real solutions if $\Delta \geq 0$ with:

$$\Delta = (2XH)^2 - 4(X^2 + Y^2)(H^2 - Y^2) \quad \text{Thus:} \quad C23 = \frac{-2XH \pm \sqrt{\Delta}}{2(X^2 + Y^2)} \rightarrow S23 = \sqrt{1 - C23^2}$$

We replace its in (12) and (13) we find:

$$\begin{cases} \theta_2 = ATAN2(S2, C2) \\ \theta_3 = ATAN2(S23, C23) - \theta_2 \end{cases} \tag{15}$$

2.1.2. Calculation of θ_4 , θ_5 and θ_6

We have found θ_1 , θ_2 and θ_3 ; therefore the 0T_3 matrix is known:

$$U_0 = {}^0T_6 \Rightarrow {}^3T_0 U_0 = {}^3T_6 \rightarrow {}^4T_3 {}^3T_0 U_0 = {}^4T_6 = \begin{bmatrix} {}^4A_6 & P \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow {}^3T_6 U_0 = \begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = {}^4T_3 {}^3T_0 U_0$$

$$M = \begin{bmatrix} C4V_{13} + S4V_{33} & -C4V_{12} - S4V_{32} & C4V_{11} + S4V_{31} \\ C4V_{33} - S4V_{13} & -C4V_{32} + S4V_{12} & C4V_{31} - S4V_{11} \\ -V_{23} & V_{22} & -V_{21} \end{bmatrix} \quad \text{and} \quad {}^4A_6 = \begin{bmatrix} C5C6 & -C5S6 & S5 \\ S6 & C6 & 0 \\ -C6S5 & S5S6 & C5 \end{bmatrix}$$

${}^4A_6(2,3) = 0$; $M(2,3) = C4V_{31} - S4V_{11}$ and so by identifying:

$$C4V_{31} = S4V_{11} \rightarrow \frac{S4}{C4} = \frac{V_{33}}{V_{11}}$$

$$\begin{cases} \theta_4 = ATAN2(V_{31}, V_{11}) \\ \theta_4 = \theta_4 + \pi \end{cases} \tag{16}$$

$$M(3,3) = -V_{21} \rightarrow {}^4A_6(3,3) = C5 \rightarrow M(1,3) = C4V_{11} + S4V_{31} \rightarrow {}^4A_6(1,3) = S5 \tag{17}$$

$$\theta_5 = ATAN2(M(1,3), M(3,3))$$

$$M(2,1) = C4V_{33} - S4V_{13} \rightarrow {}^4A_6(2,1) = S6 \rightarrow M(2,2) = -C4V_{32} + S4V_{12} \rightarrow {}^4A_6(2,2) = C6$$

Thus: $\frac{S6}{C6} = \frac{M(2,1)}{M(2,2)}$

$$\theta_6 = ATAN2(M(2,1), M(2,2)) \tag{18}$$

We will have up to 8 solutions outside the singular positions; some of these configurations may not be accessible because of joint limits.

3. Control interface used in the application of the teleoperation:

We developed for this study a graphical interface with Matlab, where we integrated the robot designed with CAD (<http://grabcad.com>; Bouzgou and Ahmed-foitih, 2014) in the application under VRML for visualizing well and to handle the arm, our interface will help us for simulation and could us to show all positions of the end effector in its workspace. And by the generation of motion in real time between two stations, we can predict and filtering some tasks, where our arm could not do it with its parameters. (Bouzgou & Ahmed-Foitih, 2014).

3.2. Workspace of Fanuc 200IC robot arm:

Table 2: Series robot specifications of the FANUC robot.

Items	LR MATE 200iC/5L		
Axes	6		
Reach [mm]	892		
Motion (Degrees)	range	Axe 1	340/360
		Axe 2	230
		Axe 3	373
		Axe 4	380
		Axe 5	240
		Axe 6	720

Robot arm joints are limited by mechanical stops, and a real value of angle is applied for a position in workspace of end effector, the following table shown us value range of all joints of manipulator arm. Several works of the workspace and singularity analysis were studied (Vaezi and Al, 2011; Djuric and Al, 2013), Knowledge of size and boundary of the workspace can smooth teleoperation and the control of manipulator arm from a distance.

Workspace, singularities or the nearest solution to the current configuration that minimizes the number of joints actuated; Using the equations of the direct geometric model (DGM), we plot the points of the workspace based on θ_2 and θ_3 .

$${}^f a_E(1,1) = s5(s1s4 - c4c1s23) + c5c1c23$$

The solving of this equation for initial position angle of manipulator robot, to know all value of θ_5 angle such us:

$$s5(s1s4 - c4c1s23) + c5c1c23=0. \tag{19}$$

We found:

$$\theta_5 = \pi/2 \quad \text{For} \quad {}^f R_E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{or} \quad \theta_5 = -\pi/2 \quad \text{For} \quad {}^f R_E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

4. Singularities of FANUC robot

We can find singularities from any Jacobian matrix, but we often choose the projection of the 6J_6 matrix in the R_i reference frame which gives us the simplest iJ_6 matrix. (Khalil and Dombre, 1999)

So the J^{th} column of 6J_6 Jacobian matrix is:

$${}^6J_j = \begin{bmatrix} - \begin{bmatrix} s_x \\ n_x \\ a_x \end{bmatrix} \cdot P_y + \begin{bmatrix} s_y \\ n_y \\ a_y \end{bmatrix} \cdot P_x \\ \begin{bmatrix} s_z \\ n_z \\ a_z \end{bmatrix} \end{bmatrix} \tag{20}$$

In our case this matrix will be the projection of 6J_6 in the R_3 reference frame, thus we will obtain the Jacobian matrix 3J_6 which defines the R_6 reference frame in the R_3 frame.

The J matrix has particular form as:

$$J = \begin{bmatrix} A & 0_3 \\ B & C \end{bmatrix} \tag{21}$$

The inverse kinematic of manipulator arm is defined where J^{-1} exist. (Bouzgou, and Ahmed-foitih, 2014 september).

$$\text{Therefore: } \det(J) = \det(A) \cdot \det(C) \tag{22}$$

$$\det(C) = -S5C4^2 - S5S4^2 \rightarrow \det(C) = 0 \Rightarrow S5 = 0$$

$$\text{Thus: } \theta_5 = k \cdot \pi \quad \text{where} \quad k = \{-1,0,1\}$$

$$\theta_5 = \pi, \text{ because of obstinate mechanics, } \theta_5 \text{ angle can be taking a first solution only, so } \theta_5 = 0$$

$$\text{Thus: } \det(C) = 0 \Rightarrow \theta_5 = 0 \tag{23}$$

With this configuration, the two articulations θ_4 and θ_6 have their confused axes, which makes lose a degree of freedom to the robot, the rotation of the end-effector can be done is by the rotation of θ_4 or θ_6 , the robot thus has practically 5 DOF.

$$\begin{aligned} \det(A) &= d3[C3R4 - d4S3][d2 + d3S2 + C2C3R4 - C2d4S3 + C3d4S2 + R4S2S3] \\ &= d3[C3R4 - d4S3][d2 + d3S2 + C2(R4C3 - d4S3) + S2(R4S3 + d4C3)] \\ C3R4 - d4S3 = 0 &\Rightarrow \frac{S3}{C3} = \frac{R4}{d4} \end{aligned}$$

$$\text{Thus: } \begin{cases} \theta_3 = \text{ATAN2}(R4, d4) = 1.3899 \\ \theta_3 = \theta_3 + \pi = 4.5315 \end{cases} \quad (24)$$

$$d2 + d3S2 + C2(R4C3 - d4S3) + S2(R4S3 + d4C3) = 0 \quad (25)$$

That it becomes to solving a non-linear equation with two unknown θ_2, θ_3 , analytically is difficult, we use the mathematical software Matlab to find the solutions geometrically.

The mathematical solution is:

Where θ_2 and $\theta_3 \in [-\pi, +\pi]$, and as we have constraints at the articular space because: $(-1.7453 \leq \theta_2 \leq 2.2689$ and $-4.0143 \leq \theta_3 \leq 1.5708)$.

5. Conclusion

The inverse kinematic model gives us the eight solutions of the positions of the end-effector apart from the singularities, we could visualize them in a virtual environment by using a software other than the manufacturer's software, the space work of the arm is limited by the articular thrusts and the branches of the singularities, which are represented in the form of curves and right-hand sides while solving an equation with two unknown. Several mathematical tools are used in this study, whose validation of our work was made using Matlab. We can thereafter consider other research orientations such as the generation of motion and the planning of trajectory (Bouzgou and Ahmed-foitih and Al, 2013; Bouzgou and Ahmed-foitih, 2014).

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