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Differential Approximation Results for the Steiner Tree Problem

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Abstract—We study the approximability of three versions of the Steiner tree problem. For the first one where the input graph is only supposed connected, we show that it is not approximable within better than $|V \setminus N|^{-\epsilon}$ for any $\epsilon \in (0,1)$, where V and N are the vertex-set of the input graph and the set of terminal vertices, respectively. For the second of the Steiner tree versions considered, the one where the input graph is supposed complete and the edge distances are arbitrary, we prove that it can be differentially approximated within 1/2. For the third one defined on complete graphs with edge distances 1 or 2, we show that it is differentially approximable within 0.82. Also, extending the result of Bern and Plassmann [1], we show that the Steiner tree problem with edge lengths 1 and 2 is **MaxSNP**-complete even in the case where $|V| \leqslant r|N|$, for any $r > 0$. This allows us to finally show that the Steiner tree problem with edge lengths 1 and 2 cannot by approximated by polynomial time differential approximation schemata. © 2003 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

Given a connected graph $G(V,E)$, a length function d on its edges, and a set $N \subseteq V$ (we call it the terminal set whereas $V \setminus N$ is called optional set), an optimal Steiner tree is a shortest tree spanning all vertices in N (the length of a tree is given by $d(T) = \sum_{e \in E(T)} d(e)$). The Steiner tree problem, denoted by STEINER in what follows, is **NP**-complete [2]. It has many real-world applications since it is admitted in routing in VLSI layout, in the design of communication networks, etc.

We consider in this paper four versions of STEINER: the general one where the input graph is only supposed connected and the edge distances are supposed arbitrary (this is the version called STEINER in the sequel), the one where the input graph is supposed complete and the edge distances are once more supposed arbitrary (called COMPLETE STEINER in the sequel),

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the one where the input graph is supposed complete and the edge distances are either 1 or 2 (COMPLETE STEINER(1,2)), and finally, one further restriction of COMPLETE STEINER(1,2) where the terminal vertices verify $|N| \leqslant r|V|$ for some $r > 0$; we will call this version BOUNDED TERMINALS COMPLETE STEINER(1,2).

Given an instance I of an optimization problem and a feasible solution S of I produced by some algorithm A, we denote by $m_{\mathtt{A}}(I,S)$ the value of the solution S, by opt (I) the value of an optimal solution of I, and by $\omega(I)$ the value of a worst solution of I. The *standard performance, or approximation, ratio* of A when running on I is defined as $\rho_{\mathtt{A}}(I,S) = \max\{m_{\mathtt{A}}(I,S)/\mathrm{opt}\,(I), \mathrm{opt}\,(I)/m_{\mathtt{A}}(I,S)\}$, while the *differential performance, or approximation, ratio* of S is defined as $\delta_{\mathtt{A}}(I,S) = |m_{\mathtt{A}}(I,S) - \omega(I)|/|\mathrm{opt}\,(I) - \omega(I)|$.

Dealing with STEINER, since the early 1990s, several authors published algorithms with decreasing standard performance ratio [3–6]. The best known standard approximation ratio is 1.55 [6]. For COMPLETE STEINER(1,2) Bern and Plassmann [1] have proved that it is **MaxSNP**-complete; this implies that, unless $\mathbf{P} = \mathbf{NP}$, it cannot be approximated by a polynomial time standard approximation schema, i.e., that the best standard approximation ratio for COMPLETE STEINER(1,2) (and, consequently, also for COMPLETE STEINER) cannot get arbitrarily close to 1. In fact, a lower bound of 1.0074 for the standard approximation ratio of COMPLETE STEINER has been provided very recently in [7]. The best known standard approximation ratio for COMPLETE STEINER(1,2) is 1.28 [6].

Here we study the differential approximability of STEINER and of its versions defined above. In what follows, we consider as worst solution a maximum total-distance spanning tree of the input-graph. For STEINER itself, we show that it is not approximable within better than $|V \setminus N|^{-\epsilon}$ for any $\epsilon \in (0,1)$. For COMPLETE STEINER, we prove that it is differentially approximable within $1/2$. For COMPLETE STEINER(1,2) we show that it is differentially approximable within 0.82. We next extend the inapproximability result of [1], and show that even BOUNDED TERMINALS COMPLETE STEINER(1,2) is **MaxSNP**-complete. This allows us to show that COMPLETE STEINER(1,2) cannot by approximated by polynomial time differential approximation schemata.

In standard approximation, STEINER reduces to COMPLETE STEINER. In this sense, the same standard approximation ratio is guaranteed for both of them. In fact, from a network (G,d) where $G(V,E)$ is connected, we can polynomially construct the network $D_G(V) = (K_n, d')$ where, for every pair $(w,v) \in V \times V$, the distance $d'(v,w)$ of the edge vw is the cost of the shortest path from v to w in (G,d). The network $D_G(V)$ is usually called *the distance network* of (G,d) and is well defined since G is connected. Moreover, it verifies the following properties:

(i) d' satisfies the triangular inequality,
(ii) $d'(e) \leqslant d(e)$ for any edge $e \in E$, and
(iii) the cost of an optimal Steiner tree in $D_G(V)$ equals the cost of an optimal Steiner tree in (G,d).

A well-known basic heuristic for STEINER works as follows: construct the network $D_G(N)$; find a minimum spanning tree T' of $D_G(N)$; replace any edge vw of T' by a shortest path between v and w in G and denote by G' the subgraph of G so obtained; finally, compute a minimum spanning tree T of G', repeatedly remove any optional vertex of degree one, and output the resulting tree. The reduction just specified preserves the approximation ratio for both STEINER reduces to COMPLETE STEINER. Moreover, the spanning tree T' of $D_G(N)$ is a 2-standard approximation for COMPLETE STEINER. Therefore, for standard approximation, we can always suppose that the graph is complete and the length function verifies the triangular inequality.

2. THE COMPLEXITY OF DIFFERENTIALLY APPROXIMATING STEINER

Unfortunately, the equiapproximability shown just above between STEINER and COMPLETE STEINER does not hold when dealing with the differential approximation. In fact, the cost of a worst solution of $D_G(V)$ is not equal to the cost of a worst solution of (G, d). The ratio between these two values can be arbitrarily large.

THEOREM 1. STEINER *is not approximable within differential ratio greater than* $|V \setminus N|^{-\varepsilon}$, *for any* $\varepsilon \in]0,1[$ *unless* $\mathbf{NP} = \mathbf{ZPP}$.

PROOF. We first reduce STEINER to SET COVER[1] and show that this reduction transforms any differential approximation ratio for the former into an equal-value differential approximation ratio for the latter. Let $I(\mathcal{S}, X)$ be an instance of set cover where $\mathcal{S} = \{S_1, \dots, S_p\}$ is the set-system and $X = \{x_1, \dots, x_n)$ is the ground set. We build the network $I' = (G(V, E), d)$ as follows:

- $V = N \cup V_1$, where $N = \{v_1, \dots, v_n\}$ and $V_1 = \{w_1, \dots, w_p\}$;
- $xy \in E$, iff $x \in V_1$ and $y \in V_1$, or if $x = w_i$, $y = v_j$, and $x_j \in S_i$;
- $d(e) = 1$, $\forall e \in E$.

Obviously, G is connected and, moreover, T is a Steiner tree of G iff $S = \{S_i : i \in J\}$, where $J = \{i : \exists j, (v_i, w_j) \in T\}$ is a set cover of X; thus, $\text{opt}(I') = \text{opt}(I) + (|X|-1)$, $\omega(I') = \omega(I) + (|X|-1)$, and $d(T) = |S| + (|X|-1)$. So, if $d(T) \le \delta \,\text{opt}(I') + (1-\delta)\omega(I')$, then $|S| \le \delta \,\text{opt}(I) + (1-\delta)\omega(I)$. It is easy to see that by the reduction just described, any differential approximation ratio of value δ for STEINER transforms into a differential approximation ratio of value δ for SET COVER.

SET COVER contains VERTEX COVER[2] as subproblem and this latter problem is approximate equivalent to the INDEPENDENT SET[3] [8] for the differential approximation (i.e., both problems have the same differential approximation ratio). Furthermore, the standard and the differential approximation ratios coincide for INDEPENDENT SET which cannot be approximated within standard approximation ratio better than $|V|^{\varepsilon}$ for any $\varepsilon \in]0,1[$ unless $\mathbf{NP} = \mathbf{ZPP}$ [9]. Putting all this together, one gets the result claimed. ∎

3. THE DIFFERENTIAL APPROXIMATION OF COMPLETE STEINER

Let $(K_{|V|}, d)$ be an instance of COMPLETE STEINER, denote by N the terminals in V and consider the following algorithm, denoted by `C_Steiner`, whose complexity is $O(|V|^2 \log |V|)$:

1. compute a minimum spanning tree $T_V(V, E(T_V))$ on $K_{|V|}$;
2. while there exists an edge $e_x \in E(T_V)$ adjacent to a leaf $x \notin N$ set $V(T_V) = V(T_V) \setminus \{x\}$, $E(T_V) = E(T_V) \setminus \{e_x\}$;
3. compute a minimum spanning tree T_N on the subgraph of $K_{|V|}$ induced by N;
4. output $T = \arg\min\{d(T_V); d(T_N)\}$.

THEOREM 2. *Algorithm* `C_Steiner` *achieves differential approximation ratio 1/2 for* COMPLETE STEINER. *This ratio is tight.*

PROOF. Let T^* be an optimal Steiner tree on $(K_{|V|}, d)$ and assume a minimum spanning tree T_V of $(K_{|V|}, d)$ as computed by algorithm `C_Steiner`. Then, $|E(T_N)| \leqslant |E(T^*)| \leqslant |E(T_V)|$. Starting from T_V, we will show that there exists a forest $T_F = (T_1, \dots, T_p)$ included to T_V and verifying the following properties:

(i) $p = |V(T^*)|$ and $d(T_V) - d(T_F) \leqslant d(T^*) = \text{opt}(K_{|V|}, d)$;

(ii) $\forall i \le p$, T_i contains at most one terminal vertex.

PROOF OF PROPERTY (i). We add the edges of T^* in T_V and iteratively construct the set E_1 as follows: let $e \in E(T^*)$; if $e \in T_V$, then $E_1 = E_1 \cup \{e\}$, else in $E(T_V) \cup \{e\}$, we have a cycle μ_e (we

[1]Given a collection $\mathcal{S}$ of subsets of a finite set X, a set cover is a subcollection $\mathcal{S}' \subseteq \mathcal{S}$ such that $\bigcup_{S_i \in \mathcal{S}'} S_i = C$, and the SET COVER problem is to find a set cover of minimum size.

[2]Given a graph $G(V, E)$, a vertex cover is a subset $V' \subseteq V$ such that, $\forall uv \in E$, either $u \in V'$, or $v \in V'$, and the VERTEX COVER problem is to determine a minimum-size vertex cover.

[3]Given a graph $G(V, E)$, an independent set is a subset $V' \subseteq V$ such that whenever $\{v_i, v_j\} \subseteq V'$, $v_i v_j \notin E$, and the INDEPENDENT SET problem is to find an independent set of maximum size.

see it as the set of its edges) containing e; suppose that we are in the r^{th} iteration and denote by E_1^r the state of E_1 got at the end of the r^{th} iteration; we then have two cases:

- if there exists an edge $e_i \in \mu_e \setminus \{e\}$ not belonging to the current set E_1^{r-1}, then $E_1^r = E_1^{r-1} \cup \{e_i\}$;
- else, at iteration r, we have $\mu_e \setminus \{e\} \subseteq E_1$; consider some $r' < r$ where an edge, denoted by $e_{r'}$, has been included in $E_1^{r'}$; since any cycle created by the introduction of an edge of T^* in T_V contains at least three edges, there exists at least one edge, say $e'_{r'}$, that could be included in $E_1^{r'}$ instead of $e_{r'}$; so, if we consider the modification of $E_1^{r'}$ to $E_1^{r'} \setminus \{e_{r'}\} \cup \{e'_{r'}\}$ and assuming that for any $i \in \{r'+1, \ldots, r\}$ the sets E_1^i remain unchanged, then $(E(T_V) \setminus E_1^{r-1}) \cap (\mu_e \setminus \{e\}) \neq \emptyset$ and, in order to produce E_1^r, one can apply the previous item.

Hence, we have exhibited a set $E_1 \subseteq E(T_V)$ verifying $|E_1| = |E(T^*)|$. Revisit one of the cycles μ_e considered in the first of the items just above and set $e_M = \arg\max\{d(e_i) : e_i \in \mu_e\}$. Then distance $d(e)$ of the edge $e \in E(T^*)$ added in T_V is at least equal to $d(e_M)$ (the case $e \in E(T_V) \cap E(T^*)$ implies equality), otherwise $E(T_V) \setminus \{e_M\} \cup \{e\}$ would be the edge-set of a new spanning tree of value smaller than the one of T_V supposed to be the minimum one. Since the edges of E_1 are all edges of $E(T_V)$, we conclude $d(E_1) \leqslant d(T^*) = \operatorname{opt}(K_{|V|}, d)$. Set now $E(T_F) = E(T_V) \setminus E_1$. Since removal of an edge of E_1 disconnects T_V into two subtrees, removal of the $|E_1| = |E(T^*)|$ edges of E_1 will create a forest T_F of $|E(T^*)| + 1 = |V(T^*)|$ trees. Therefore, the first statement of Property (i) is proved. Moreover, $d(E_1) = d(T_V) - d(T_F)$ and since $d(E_1) \leqslant d(T^*)$, the second statement and Property (i) are proved. ∎

PROOF OF PROPERTY (ii). Assume that, for some $i \leqslant p$, T_i contains two terminals v_1 and v_2. Let μ^* be the path (seen as a set of edges) in T^* from v_1 to v_2 (by construction of T_F no subset of μ^* belongs to $E(T_i)$). For any edge $e \in \mu^*$ let μ_e be the unique path in T_V linking the endpoints of e. Moreover, by the construction of T_F, $\mu_e \cap E(T_i) = \emptyset$. So, $E(T_i) \cup (\bigcup_{e \in \mu^*} \{\mu_e\})$ contains a cycle, a contradiction with the fact that T_V tree. This completes the proof of Property (ii). ∎

An immediate consequence of Property (ii) is that $T_F \cup T_N$ is a forest; therefore,

$$d(T_F) + d(T_N) \leqslant \omega(K_{|V|}, d). \tag{1}$$

Combining the second statement of Property (ii) and (1) we get $m_{\texttt{C\_Steiner}}((K_{|V|}, d), T) = d(T) \leqslant (d(T_V) + d(T_N))/2 \leqslant (\operatorname{opt}(K_{|V|}, d) + \omega(K_{|V|}, d))/2$, in other words, $\delta_{\texttt{C\_Steiner}} \geqslant 1/2$.

We now show that the ratio obtained above is tight. Consider the following instance $(K_{|V|}, d)$ with $V = V_1 \cup N$, $|N| = |V_1| = n$ and such that the edge-distances of the subgraph induced by N (respectively, V_1) are $2n+2$ (respectively, n). Moreover, any edge of the bipartite graph between N and V_1 has distance equal to $n+1$. Then, $d(T_V) = 2n^2$ and $d(T_N) = 2n^2 - 2$. Moreover, $\operatorname{opt}(K_{|V|}, d) = n^2 + n$ and $\omega(K_{|V|}, d) = 3n^2 + n - 2$. So, for $n \to \infty$ the tightness follows. ∎

4. THE DIFFERENTIAL APPROXIMATION OF COMPLETE STEINER(1,2)

Before studying the approximation of COMPLETE STEINER(1,2), we prove the following auxiliary lemma.

LEMMA 1. *Consider an instance $K_{|V|}$ of* COMPLETE STEINER(1,2) *and denote by E_1 the set of edges of $K_{|V|}$ of distance 1 and by E_2, the one of distance 2. We can always assume that the partial graphs $G_2(V, E_2)$ and $G_1(V, E_1)$ are both connected.*

PROOF. Let T^* be an optimal Steiner tree on $K_{|V|}$. Assume first that G_2 is not connected. Then, obviously, $d(T^*) \leq |N|$ and this case is solvable in polynomial time.

In order to prove that we can restrict ourselves to the case where G_1 is connected, we reduce COMPLETE STEINER(1,2) with G_1 not connected to COMPLETE STEINER(1,2) with G_1

connected. Let $G_1^1, G_1^2, \ldots,$ be the connected components of G_1 and assume that p of them, say $G_1^1, G_1^2, \ldots, G_1^p$, contain terminal vertices while the rest (if any) does not contain any terminal. Polynomially split $K_{|V|}$ into $K_{|V|}^1, \ldots, K_{|V|}^{p+1}$ where $K_{|V|}^j$ is the subgraph of $K_{|V|}$ induced by $V(G_1^j)$, $j = 1, \ldots, p$, while $K_{|V|}^{p+1}$ is the subgraph of $K_{|V|}$ induced by $\bigcup_{j \geqslant p+1} V(G_1^j)$ (note that $K_{|V|}^{p+1}$ does not contain any terminal). If one computes a Steiner tree T_j of $K_{|V|}^j$, for $j \leqslant p$, then one can simply obtain a Steiner tree T for $K_{|V|}$ by simply connecting by one edge T_{j-1} and T_j for $j \in \{2, \ldots, p\}$; so, for both T and $\operatorname{opt}(K_{|V|})$

$$d(T) = 2(p-1) + \sum_{j=1}^{p} d(T_j), \tag{2}$$

$$\operatorname{opt}\left(K_{|V|}\right) = 2(p-1) + \sum_{j=1}^{p} \operatorname{opt}\left(K_{|V|}^j\right). \tag{3}$$

Conversely, assume that the restriction of an optimal Steiner tree T^* on $K_{|V|}^j$ is a forest $F_1, \ldots,$ F_{k_j}. Then, one can add $k_j - 1$ edges, one between F_1 and F_r, $r \leqslant k_j$ and delete $k_j - 1$ other edges of $T^* \setminus F_1 \setminus \cdots \setminus F_{k_j}$, in such a way that the new tree is also an optimal Steiner tree. Thus, without loss of generality, we can assume that the restriction of an optimal Steiner tree on $K_{|V|}^j$ is always a tree. The same argument holds for any $j \leqslant p$. Dealing with the worst solutions, we have the following:

$$\omega\left(K_{|V|}\right) \geqslant 2(p-1) + \sum_{j=1}^{p} \omega\left(K_{|V|}^j\right). \tag{4}$$

Now, assume that a δ-differential approximation algorithm A solves COMPLETE STEINER(1,2) when G_1 is connected. One can obtain a solution T by computing $T_j = A(K_{|V|}^j)$ and by properly linking the different T_js as described previously. Then, (2)–(4) imply

$$\begin{aligned} d(T) &= 2(p-1) + \sum_{j=1}^{p} d(T_j) \leqslant 2(p-1) + \sum_{j=1}^{p} \left((1-\delta)\omega\left(K_{|V|}^j\right) + \delta \operatorname{opt}\left(K_{|V|}^j\right)\right) \\ &\leqslant (1-\delta)\left(\sum_{j=1}^{p} \omega\left(K_{|V|}^j\right) + 2(p-1)\right) + \delta\left(\sum_{j=1}^{p} \operatorname{opt}\left(K_{|V|}^j\right) + 2(p-1)\right) \\ &\leqslant (1-\delta)\omega\left(K_{|V|}\right) + \delta \operatorname{opt}\left(K_{|V|}\right), \end{aligned}$$

and the result claimed follows. ∎

THEOREM 3. *There exists a reduction from* COMPLETE STEINER(1,2) *to itself transforming any standard approximation ratio* ρ *to differential approximation ratio* $\delta \geqslant \rho/(2\rho - 1)$.

PROOF. Let $K_{|V|}$ be an instance of COMPLETE STEINER(1,2) verifying Lemma 1 and let T' be a Steiner tree guaranteeing ρ-standard approximation ratio. Set $T = \arg\min\{d(T_V), d(T')\}$. From the fact that G_2 is connected (Lemma 1), we get $\omega(K_{|V|}) = 2d(T_V)$; therefore,

$$\begin{aligned} d(T) &\leqslant \frac{1}{2\rho-1} d(T') + \left(1 - \frac{1}{2\rho-1}\right) d(T_V) \leqslant \frac{1}{2\rho-1} \rho \operatorname{opt}\left(K_{|V|}\right) + 2\frac{\rho-1}{2\rho-1} d(T_V) \\ &\leqslant \frac{\rho}{2\rho-1} \operatorname{opt}\left(K_{|V|}\right) + \left(1 - \frac{\rho}{2\rho-1}\right)\omega\left(K_{|V|}\right), \end{aligned}$$

and the proof of the theorem is complete. ∎

Using the 1.28-standard approximation algorithm of [6] and applying Theorem 3, we obtain the following corollary.

COROLLARY 1. COMPLETE STEINER(1,2) *is 0.82-differential approximable.*

We now prove that COMPLETE STEINER(1,2) is not solvable by polynomial time differential approximation schemata unless $\mathbf{P} = \mathbf{NP}$. In order to obtain this result, we first establish an intermediate result interesting by itself, namely, that COMPLETE STEINER(1,2) is **MaxSNP**-complete even if $|V| \leqslant r|N|$, for any $r > 0$ (the **MaxSNP**-completeness of the general COMPLETE STEINER(1,2) is proved in [1]).

PROPOSITION 1. *For any $r > 0$,* COMPLETE STEINER(1,2) *in graphs verifying $|V| \leqslant r|N|$ is* **MaxSNP**-*complete.*

PROOF. Let $r > 0$ be a constant. We show that the transformation of Theorem 1 (denoted by $\propto (I)$) can be viewed as an L-reduction from SET COVER. Let $t \geqslant 3$ and $q \geqslant 2$ be two constants and consider the problem SET COVER(t, q) where any set has size at most t and any element of the ground set belongs to at most q sets. This particular version of SET COVER is **MaxSNP**-complete [10] and without loss of generality, we assume that the sets have exactly t elements. Consider the transformation $\propto (I)$, complete the graph G of Theorem 1 in order to obtain a $K_{|V|}$ and set the distances of the edges added to 2. Let T be a Steiner tree of K_n. If T does not contain any optional vertex, then one can add an optional vertex and three edges of distance 1 and she/he can delete two edges of distance 2, so obtaining a new Steiner tree with lower total distance. Therefore, we can assume that T contains some optional vertices. If T contains an edge e of distance 2, then we can delete it and add at most two edges of distance 1 (that may be adjacent to a new optional vertex); we so obtain a new Steiner tree with the same total distance. Thus, we can always assume that the Steiner trees we deal with do not contain any edge of cost 2. With these assumptions, we have finally: $\operatorname{opt}(\propto (I)) \leqslant 2q \operatorname{opt}(I)$ and $\operatorname{opt}(I) - |S| \leqslant \operatorname{opt}(\propto (I)) - d(T)$. Consequently, the reduction described is indeed an L-reduction. Moreover, we have $|V| \leq (1 + (q/t))|N|$. Taking $q/t \leqslant r$, the result claimed follows. ∎

THEOREM 4. *Consider $r = 2$, i.e., $|V| \leq 2|N|$. Then, there exists a reduction from* COMPLETE STEINER(1,2) *to itself transforming any differential approximation ratio δ into standard approximation ratio $\rho \leqslant 4 - 3\delta$.*

PROOF. Let $K_{|V|}$ be an instance of COMPLETE STEINER(1,2) verifying Lemma 1. We have $\omega(K_{|V|}) = 2(|V| - 1) \leqslant 4|N| \leqslant 4 \operatorname{opt}(K_{|V|})$. On the hypothesis that a Steiner tree T in $K_{|V|}$ guarantees differential approximation ratio δ, we get $d(T) \leqslant \delta \operatorname{opt}(K_{|V|}) + (1 - \delta)\omega(K_{|V|}) \leqslant \delta \operatorname{opt}(K_{|V|}) + 4(1 - \delta) \operatorname{opt}(K_{|V|}) \leqslant (4 - 3\delta) \operatorname{opt}(K_{|V|})$. ∎

Using Proposition 1 and Theorem 4, we obtain the following.

COROLLARY 2. COMPLETE STEINER(1,2) *does not admit a polynomial time approximation schema unless* $\mathbf{P} = \mathbf{NP}$.

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