USAMI Control with a Higher Order Harmonics Elimination Strategy based on the Resultant Theory

R. Taleb*, A. Derrouazin
Laboratoire de Génie Electrique et Energies Renouvelables (GEER), University Hassiba Benbouali of Chlef, Algeria.

Abstract

This paper proposed a method to determinate the switching angles for a Uniform Step Asymmetrical Multilevel Inverter (USAMI) by eliminating specified higher order harmonics while maintaining the required fundamental voltage. This method is based on the resultant theory and can be applied to USAMI with any number of levels. As an example, a seven-level USAMI is considered in this paper and the optimum switching angles are calculated to generate a desired fundamental component and to eliminate the fifth and seventh harmonic components.

1. Introduction

Multilevel inverters have been widely used in last years for high-power applications [1]. Variable-speed drives have reached a wide range of standard applications such as pumps, fans and others. Many of these applications use medium-voltage motors (2300, 3300, 4160 or 6600V), due to their lower current ratings in higher power levels [2]. Static Var compensators and active filters are other applications that use multilevel converters [3].

* Corresponding author. Tel.: +213 (0)5 54 90 15 21; fax: +213 0(27) 72 17 94
E-mail address: r.taleb@univ-chlef.dz

Several topologies of multilevel inverters have been studied and presented. Among them, neutral point clamped inverters [4], flying capacitors inverters also called imbricated cells [5], and series connected cells inverters also
called cascaded inverters [6]. The industry often has used the neutral-point-clamped inverter. However, the topology that uses series connected cells inverters presents some advantages, as smaller voltage rate \(dU/dt\) due to existence of higher number levels, producing less common-mode voltage across motor windings [7]. Furthermore, this topology is simple and its modular configuration makes it easily extensible for any number of desired output voltage levels. Fig.1a) shows the basic diagram of this topology with \(k\) partial cells represented by Fig.1b). The \(j^{th}\) single-phase inverter is supplied by a dc-voltage source \(U_{dj}\) \((j = 1...k)\). The relationship between the number of series-connected single-phase inverters in each phase and the number of output voltage levels generated by this topology, respectively \(k\) and \(N\), is given by: \(N = 2k + 1\), in the case where there are equal voltages in all partial inverters.

Fig.1. a) A series-connected multilevel inverter topology with \(k\) partial cells; b) Partial cell configuration

In all the well-known multilevel converter topologies, the number of power devices required depends on the output voltage level needed. However, increasing the number of power semiconductor switches also increases the converter circuit and control complexity and the costs. To provide a large number of output levels without increasing the number of converters, a uniform step asymmetrical multilevel inverters (USAMI) can be used [8, 9].

The key issue in designing an effective multilevel inverter is to ensure that the Total Harmonic Distortion (THD) of the output voltage waveform is within acceptable limits. Harmonic Elimination Strategy (HES) has been intensively studied in order to achieve low THD [10]. The output voltage waveform analysis using Fourier theory produces a set of non-linear transcendental equations. The solution of these equations, if it exists, gives the switching angles required for a fundamental component and a selected harmonic profile. Iterative procedures such as Newton-Raphson method has been used to solve these sets of equations [11]. This method is derivative-dependent may end in local optima, and its convergence can only be guaranteed by a judicious choice of the initial values [12]. Another approach based on converting the transcendental equation into polynomial equations can be applied to determine the switching angles to eliminate specific harmonics. The approach, called resultant theory [13, 14], gives a complete solution to the problem in that all possible solutions are found.

The paper is organized as follows. USAMIs are described and modeled in Section 2. In Section 3, the resultant theory is applied to a seven-level USAMI with the problem formulated as achieving the fundamental while not generating the fifth and seventh harmonics. Simulation results are presented in Section 4, and a summary of the results is presented in Section 5.

2. Uniform step asymmetrical multilevel inverter

Fig.1b shows a detail of the partial cells and the main notations used. Each couple of switches \(S_{jx}\) and \(S'_{jx}\) \((x = 1, 2; j = 1...k)\) is controlled by a couple of switching functions \(M_{jx}\) and \(M'_{jx} \in \{0, 1\}\) such that:

\[M_{jx} + M'_{jx} = 1\] (1)
The conversion of the switch commutations into a voltage is described by a conversion function, \( F_j \) such that:

\[
F_j = M_{j_1} - M'_{j_1} \Rightarrow F_j \in \{-1, 0, 1\}
\]  

The output voltage of each cell is given by:

\[
U_{pj} = F_j \ast U_d \Rightarrow U_{pj} \in \{-U_d, 0, U_d\}
\]  

Equation (3) shows that each partial cell can generate three different levels. The output voltage of the multilevel converter is given by:

\[
U_s = U_{p1} + U_{p2} + \ldots + U_{pk}
\]  

A series-connected multilevel inverter is known as asymmetric, if at least one to the dc-voltage sources feeding the partial inverters is different of the others. Three conditions have been established for the design of a uniform or regular step AMI (i.e., the steps \( \Delta U \) between all voltage levels are equal, in this case the step is equal to the smallest dc-voltage \( U_{d1} \)) [15]:

1) the dc-voltage sources must be arranged in an increasing way \( U_{d(h-1)} \leq U_{dh} \forall h = 2 \ldots k\);
2) the ratio between two consecutive inverters must be an integer \( U_{d(h)} / U_{d(h-1)} = \delta_h, \delta_h \in N\);
3) the \( j\)th partial cell must be fed by the voltage \( U_{dj} \) such that

\[
U_{dj} \leq 1 + 2\sum_{l=1}^{j-1} U_{dl}
\]

If these three conditions are satisfied, the multilevel inverter will generate an output voltage \( U_s \) with \( N \) regular different levels such that:

\[
N = 1 + 2\sum_{j=1}^{k} \frac{U_{dj}}{U_{d1}}
\]

For example the generation of a eleven-level output voltage can be achieved with the following dc-voltage sources: \( U_{d1} = 1 \text{ p.u.} \) and \( U_{d2} = 2 \text{ p.u.} \). According to (5) and (6), more than one possible dc-voltage setting can be chosen to generate the same number of levels. Fig.2 shows the possible output voltage of the two partial cells. There are two possible commutation sequences that result in \( U_s = 1 \text{ p.u.} \): \( (U_{p1}, U_{p2}) \in \{(-1, 2); (1, 0)\} \). The dashed lines in Fig.2 show the commutation sequence \( (U_{p1}, U_{p2}) = (1, 0) \). The possible redundant switching states in a multilevel converter are a degree of freedom which is usually used to optimise its performances with an appropriate modulation strategy [16].

![Fig. 2. Possible output voltages of each partial inverter to generate, \( N = 7 \) levels with \( k = 2 \) cells, \( U_{d1} = 1\text{ p.u.} \) and \( U_{d2} = 2\text{ p.u.} \).](image-url)
3. Modulation control

Generally, traditional PWM control methods and space vector PWM methods are applied to symmetrical multilevel inverter (SMI) modulation control [17, 18, 19], and can also be used to control asymmetrical multilevel inverter (AMI). These methods will cause extra losses due to high switching frequencies. For this reason, low-switching frequency control methods, such as harmonic elimination strategy, represent interesting and alternative solutions [20, 21].

Since the generalized output waveform of a USAMI $U_s$ is non-sinusoidal, it may be expressed in terms of a Fourier series expansion. The output waveform is represented by Fig. 3 and can be expressed by:

$$
U_s = \sum_{n=1,3,5,...}^\infty U_n \sin(n\omega t)
$$

$$
U_n = \frac{4U_d}{n\pi} \sum_{i=1}^{p} \cos(n\theta_i)
$$

where

- $n = 1, 5, 7, \ldots, 3p - 2$, for three-phase-systems where $p$ is odd;
- $n = 1, 5, 7, \ldots, 3p - 1$, for three-phase-systems where $p$ is even;
- $p = (N - 1)/2$ is the number of switching angles per quarter waveform;
- $U_n$ is the amplitude of the harmonic term of rank $n$;
- $\theta_i$ is the $i$th switching angle.

Equation (7) has $p$ variables, i.e.; $\theta_1, \theta_2, \theta_3, \ldots \theta_p$, where $0 \leq \theta_1 < \theta_2 < \ldots < \theta_p \leq \pi/2$. A solution set can be obtained by assigning a specific value to the fundamental component $U_1$ and equating $p - 1$ harmonics to zero. For example, in the case of a three phase seven-level USAMI composed of $k = 2$ partial inverters per phase supplied by the dc-voltages $U_{d1} = 1p.u.$ and $U_{d2} = 2p.u.$, the goal is to achieve the fundamental component and eliminate the fifth and seventh harmonics. Using (7), this can be formulated as the solution to the following equations:
\[
\begin{align*}
\sum_{j=1}^{p=3} \cos(\theta_j) &= \frac{\pi U_1}{4 U_{d1}} \\
\sum_{j=1}^{p=3} \cos(n \theta_j) &= 0 & \text{for } n \in \{5, 7\}
\end{align*}
\]  

(8)

This is a system of three transcendental equations with \( \theta_1, \theta_2, \) and \( \theta_3 \) as the unknowns. One approach for solving the set of nonlinear transcendental equation (8) is to use an iterative method such as the Newton-Raphson method [22, 23]. Iterative methods give only one specific solution which depends on initial conditions. Furthermore, there is no guarantee that optimum solutions are obtained. Therefore, it is worth considering other and simple procedures such as the resultant theory method which produces all possible solutions [13, 14].

### 3.1. Solution using the resultant theory

The resultant theory method converts the transcendental equations of (8) into polynomial equations. This is achieved by using variable substitutions, i.e., \( x_1 = \cos(\theta_1), x_2 = \cos(\theta_2), x_3 = \cos(\theta_3) \), and trigonometric identities such as \( \cos(5 \theta) = 5 \cos(\theta) - 20 \cos^3(\theta) + 16 \cos^5(\theta) \) and \( \cos(7 \theta) = -7 \cos(\theta) + 56 \cos^3(\theta) - 112 \cos^5(\theta) + 64 \cos^7(\theta) \) and leads to the following equivalent conditions:

\[
p_5(x) = x_1 + x_2 + x_3 - m = 0 \]
\[
p_3(x) = \sum_{i=1}^{3} (5x_i - 20x_i^3 + 16x_i^5) = 0 \]  
\[
p_7(x) = \sum_{i=1}^{3} (-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7) = 0.
\]  

where \( x = (x_1, x_2, x_3) \) and \( m = \pi U_1/4 U_{d1} \). The system defined by (9) is a set of three polynomial equations with three unknowns, \( x_1, x_2, x_3 \). Furthermore, the solutions must satisfy \( 0 \leq x_3 < x_2 < x_1 \leq 1 \). By making the substitution \( x_3 = m - (x_1 + x_2) \) into \( p_5, p_7 \), it can be deduced that

\[
p_5(x_1, x_2) = 5x_1 - 20x_1^3 + 16x_1^5 + 5x_2 - 20x_2^3 + 16x_2^5 + 5(m - x_1 - x_2) - 20(m - x_1 - x_2)^3 + 16(m - x_1 - x_2)^5
\]
\[
p_7(x_1, x_2) = -7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7 - 7x_2 + 56x_2^3 - 112x_2^5 + 64x_2^7 - 7(m - x_1 - x_2)
\]
\[
\quad + 56(m - x_1 - x_2)^3 - 112(m - x_1 - x_2)^5 + 64(m - x_1 - x_2)^7.
\]  

(10)

The system composed of \( p_5(x_1, x_2) = 0, p_7(x_1, x_2) = 0 \) is a system of two polynomial equations with two unknowns, i.e., \( x_1 \) and \( x_2 \), which has to be solved. In order to explain how to compute the zero sets of the polynomial system, a brief discussion is provided. One systematic procedure to do this is referred to as the elimination theory and uses the notion of resultants. This approach considers \( a(x_1, x_2) \) and \( b(x_1, x_2) \) as polynomial equations containing an unknown variable \( x_2 \) whose coefficients are also polynomial terms where \( x_1 \) is the unknown variable. Thus, \( a(x_1, x_2) \) and \( b(x_1, x_2) \), respectively of degrees 3 and 2 according to \( x_2 \), may be written in the form

\[
a(x_1, x_2) = a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1)
\]
\[
b(x_1, x_2) = b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1).
\]  

(11)

The \( n \times n \) Sylvester matrix, where \( n = \deg_{x_2} \{a(x_1, x_2)\} + \deg_{x_2} \{b(x_1, x_2)\} = 3 + 2 = 5 \), can be deduced as
The resultant polynomial is then defined by

\[ r(x_1) = \text{Res}(a(x_1, x_2), b(x_1, x_2, x_3)) = \det S_{a,b}(x_1) \]

and is the result of solving \( a(x_1, x_2) = 0 \) simultaneously for \( x_1 \), i.e., by eliminating \( x_2 \).

### 3.2. Switching angle solutions

As briefly explained, the theory of resultants provides a systematic method to find the polynomial \( r(x_1) = \text{Res}(p_5, p_7, x_2) \) that results when \( x_2 \) is eliminated by simultaneously solving \( p_5(x_1, x_2) = 0 \) and \( p_7(x_1, x_2) = 0 \). First, the roots of \( r(x_1) = 0 \) are numerically computed for the roots \( \{x_{1,i}^0, i = 1,...,n_1 = \deg_{x_1}\{r(x_1)\}\} \). Each of the roots \( x_{1,i} \), are then substituted back into \( p_5(x_{1,i}, x_2), p_7(x_{1,i}, x_2) \) and the polynomials \( p_5(x_{1,i}, x_2) = 0 \) and \( p_7(x_{1,i}, x_2) = 0 \) are each solved for \( x_2 \). Finally, their common roots \( \{x_{2,j}, j = 1,...,n_2\} \) are used to get the pairs \( (x_{1,i}, x_{2,j}) \) that satisfy the original system.

This algorithm was used to find the switching angles for each phase in the case of a seven-level USAMI. The results for phase \( a \) are plotted in Fig. 4 versus \( m \), where \( m \in [0.50, 3.00] \) with a step of 0.01. One can notice that for \( m \) in the interval \([1.15, 2.52]\) there is at least one solution. Furthermore, there are two different sets of solutions in the interval \([1.49, 1.85]\). On the other side, for \( m \in [0, 0.8] \), \( m \in [0.83, 1.14] \), and \( m \in [2.53, 2.77] \) there are no solutions. Interestingly, for \( m \approx 0.8, m \approx 0.81 \) and \( m \approx 2.76 \) there are isolated solutions.

In the case of two possible solutions of an angle \( \theta_i \), one clear way to choose a particular solution is simply to pick the one that results in the lowest THD given by

\[
\text{THD} = \sqrt{\sum_{n=5,7,...}^{\infty} \left( \frac{1}{n} \sum_{i=1}^{n-3} \cos(n\theta_i) \right)^2 / \sum_{i=1}^{n=3} \cos(\theta_i)}
\]  

(14)

The THD corresponding to the solutions given in Fig. 4 is represented by Fig. 5. Selecting the switching angles that lead to the lowest THD results in the solutions given by Fig. 6. The corresponding THD is shown on Fig. 7.

Fig. 4. All switching angles versus \( m \) for a seven-level USAMI
Fig. 5. THD versus $m$ for all switching angles

Fig. 6. Switching angles versus $m$ which give the lowest THD

Fig. 7. THD versus $m$ for the switching angles that give the lowest THD
4. Test Results

In this Section, simulations were performed to validate the theoretical approach presented in Section 3. The simulations have been achieved using Matlab-Simulink. The seven-level USAMI composed of \( k = 2 \) partial inverters per phase supplied by the dc-voltage sources \( U_{d1} = 100 \text{V} \) and \( U_{d2} = 200 \text{V} \) was attached to a three-phase induction motor with the following data: rated power \( P_n = 1.5 \text{KW} \), rated speed \( 1420 \text{rpm} \), rated voltage \( 220/380 \text{V} \), stator resistance \( R_s = 4.850 \Omega \), rotor resistance \( R_r = 3.805 \Omega \), stator inductance \( L_s = 0.274 \text{H} \), rotor inductance \( L_r = 0.274 \text{H} \), mutual inductance \( L_m = 0.258 \text{H} \), number of pole pairs \( P = 2 \), rotor inertia \( J = 0.031 \text{kg.m}^2 \), viscous friction coefficient \( K_f = 0.00136 \text{Nm.s.rad}^{-1} \).

In the test, the value of \( m \) was chosen to 1.75 in order to produce a fundamental voltage of \( U_1 = m(4U_{d1}/\pi) = 1.75(4 \times 100/\pi) = 222.8 \text{V} \) along with \( f = 50 \text{Hz} \). As can be seen in Fig. 4, there are two different solution sets for \( m = 1.75 \). The solution set that gave the lowest THD, i.e., 7.9% as indicated by Fig. 7, was used. The output voltages \( U_{p1} \) and \( U_{p2} \) of each partial inverter and \( V_a \), the phase \( a \) voltage, are represented by Fig. 8. The harmonic content of \( V_a \) is calculated using the Fast Fourier Transform (FFT) and it is represented by Fig. 9. This figure shows that the fifth and seventh harmonics are absent from the waveform as predicted. The triple harmonics (3th, 6th, 9th, etc.) in each phase do not need be canceled as they are automatically cancelled in the line-line voltages. The THD of the line-line voltage was computed and was found to be 7.93% which compares favourably with the value of 7.9% predicted with Fig. 7.

![Fig. 8. Output voltages of each partial inverter and the resulting \( V_a \), i.e., the phase \( a \) voltage using the solution set with the lowest THD (with \( m = 1.75 \)).](image-url)
Fig. 10 shows the phase $a$ current $i_a$ corresponding to $V_a$. The harmonic content of this current is given by Fig. 11. One can notice that the harmonic content of $i_a$ is lower than the one of $V_a$. This is due to the filtering effect introduced by the motor’s inductance. The THD of this current amounts to 2.23%.
5. Conclusion

A procedure to eliminate harmonics in a seven-level USAMI has been presented. This approach transforms the sets of transcendental harmonic elimination equations into a single set of polynomial equations. The resultant theory is then used to find the complete set of solutions to these polynomial equations. In the presence of several solutions, the one which leads to the lowest value of the THD is chosen. The modulation index $m$ is obtained and the switching angles for the partial cells composing the multilevel inverter are deduced. Simulation results have been presented, they corroborate the theoretical approach.

References

