DEVELOPMENT OF THE THEORY AND METHODOLOGY OF THE ANALYTIC HIERARCHY PROCESS AND ITS APPLICATIONS IN CHINA†

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Abstract—It is less than 4 years since the Analytic Hierarchy Process (AHP) was introduced in China. During this time there have been many applications in planning, forecasting and decision making. In this paper, we review the development of the theory and applications of the AHP.

1. INTRODUCTION

People in the area of socio-economic systems engineering often face decision making with qualitative and intangible factors. How does one make decisions under such conditions? Obviously, conventional operations research and decision analysis, of which data analysis combined with computer-based numerical algorithms are characteristic methodological tools, are incapable of accomplishing such a task.

H. Nezhad introduced the Analytic Hierarchy Process (AHP) to China when he came here for the 1st Sino-U.S. Conference on Energy, Resources and Environment in Beijing in November 1982. Since then many Chinese scholars have studied the AHP with great interest. The first paper that introduced the AHP in Chinese was written by Xu et al. [1]. Since then thousands of Chinese people have used the method to solve problems encountered in their work. Research on the theory and methodology of the AHP has been one of the major areas of attention in several institutes. In the summer of 1985, a seminar on the AHP at Tianjin University attracted more than 50 scholars. In addition, a book giving a comprehensive introduction to the AHP has been published by Scientific Press in China [2].

Why does the AHP have such a strong appeal to many Chinese scholars? A quote from Professor Diao, from Tiayuan Manufacturing College, may provide an answer:

"The vitality of the AHP in the area of decision making is that it is able to solve problems quickly."

In this paper, we review the development of the theory of the AHP and its applications in China. Section 2 briefly states some theoretical results. A survey on applications of the AHP is given in Section 3. The references consist of 38 papers and reports related to the AHP in Chinese [1–38].

2. SOME THEORETICAL AND METHODOLOGICAL DEVELOPMENTS

The Chinese people have paid great attention to the theoretical foundations of the AHP. Among the topics that have been studied are the mathematical foundations of the AHP, the test of judgment consistency, fuzzy extensions of the AHP, dynamic priorities, absolute and impact priorities and so on.

Xu [3] discussed the mathematical principles of the AHP. In his paper, he gives a complete description of the mathematical foundations of the eigenvalue method, the properties of reciprocal matrices and their sensitivity analysis. Xu and Gong [4] computed again the average consistency random index (R.I.). When a reciprocal matrix is randomly generated, the consistency index \( \lambda_{\text{max}} - n/(n - 1) \) is called the R.I. The computed average R.I. for matrices of order 1–15 using a sample size of 1000 on an IBM-PC at Tianjin University is given below:

<table>
<thead>
<tr>
<th>Matrix size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>Average R.I.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
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<tr>
<th>Matrix size</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
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<tbody>
<tr>
<td>Average R.I.</td>
<td>1.45</td>
<td>1.49</td>
<td>1.51</td>
<td>1.53</td>
<td>1.55</td>
<td>1.56</td>
<td>1.59</td>
</tr>
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</table>

†To avoid further delay, this paper has been published without the authors' corrections.
This result is regarded as an improvement on that of Oak Ridge National Laboratory.

In the AHP the judgments or pairwise comparisons are assigned numerical values from the 1–9 scale. The degrees of preference: equal, weak, essential, very strong and absolute importance, could in fact be mathematically expressed by membership functions or fuzzy numbers. Xu [3] showed how to use fuzzy information in the structure of the matrix of pairwise comparisons. He substitutes fuzzy sets for the values assigned from the 1–9 scale and then calculates priority weights by means of fuzzy operations. In his paper, Xu described some concepts based on the theory of fuzzy sets, and showed that the use of fuzzy numbers is an extension of the AHP. King and Xu [5] use fuzzy numbers to obtain a mathematical model for diagnosis of acute myocardial infarction.

When judgments change over time, the eigenvalue problem that must be solved is given by $A(t)w(t) = \lambda_{\text{max}}(t)w(t)$. The solution should be a time-dependent function. In this case we are concerned with dynamic priorities. Xu [6] presented a new dynamic model whose pairwise comparison matrix has the following form:

$$A(t) = M(t)A_0 M(t) - 1,$$

where

$$M(t) = \begin{bmatrix} m_1(t) \\ m_2(t) \\ \vdots \\ m_n(t) \end{bmatrix}$$

and $m_i(t_0) = 1, (i = 1, 2, \ldots, n),$ and $A_0 = A(t_0)$ is the pairwise comparison matrix at the initial time $t_0$. The function $m_i(t)$ represents how the $i$th factor's importance changes over time. Several theorems related to dynamic priorities are proved in this paper. In addition, using the dynamic priority model, Xu discussed the development of a strategy to deal with major energy resources in China until the year 2000.

When the number of alternatives is large and decisions are based on well-established criteria, one should use the method of scoring. In scoring, the absolute priorities of the criteria are used to score the alternatives. He et al. [7] presented a mathematical model to evaluate achievement in scientific research using this method.

3. APPLICATIONS OF THE AHP IN CHINA

The AHP has been recognized in China and elsewhere as a powerful tool in planning, decision making, forecasting and systems analysis. It has been applied in China in a variety of areas, such as economic analysis and planning, energy and resource policy analysis, scientific research management, decision making in industry, manpower prediction and planning, tourist trade development, livestock planning and medical treatment. The reports and papers issued on this topic number about 22. We briefly review some of them.

3.1. Economic analysis and planning

The judgments of decision makers play an important role in economic analyses and planning. The key problem is how to elicit systematic judgments from unstructured information. The AHP is a powerful tool for this purpose. He and Xu [8] investigated a method to adjust the industry structure of Tianjin city using the AHP along with macroeconomic development goals. The output is used as a guide by the city government to issue industry development policies. Similarly, Fu [9] studied the relative importance of various industry sectors in the economic development of the Zhejiang Province; Xie and Chen [10] assigned priorities to alternative designs of industry structure for the Tianjin Economic–Technological Developing Area. Wang [11] examined the distribution of nonproductive capital investment for medium-range economic development planning of the Yibin region in Sichuan—the biggest province in China. The common denominator in all the
applications of the AHP in economic areas has been its supporting function to help decision makers in governmental economic departments.

3.2. Energy policy analysis

Energy policy analysis and resource distribution is the first area in which the AHP was applied in China. Xu and He [12] examined the energy supply problem of Tianjin. Le and Wang [13] investigated the development policy of the coal base in Shanxi Province. They set an important guide for Shanxi social and economic development. Wu [14] used the AHP to study the distribution of energy in Zhejiang, and the relationship between development and energy conservation, and engineering economic policies for developing mini-hydroelectric stations.

Using the AHP in his Master’s Thesis, Jiang [15] forecast the commercial energy demand of Tianjin by 1990, and examined the priorities of alternative ways of supplying energy. The result was extremely helpful in establishing the integrated environmental pollution control program of Tianjin.

Rural energy problems have been the focus of attention in China recently. Zhu [16] and Ge [17] built AHP models for setting the rural energy policy of Henan Province. Zhu’s model consists of five levels:

- **U-level** Focus: developing rural energy in order to promote the economy of Henan Province
- **S-level** Energy strategy: \( S_1 = \) increasing commercial energy; \( S_2 = \) developing renewable energy; \( S_3 = \) energy conservation; \( S_4 = \) developing livestock.
- **C-level** Constraints: \( C_1 = \) monitoring shortage; \( C_2 = \) science and technology; \( C_3 = \) population; \( C_4 = \) environment; \( C_5 = \) energy supply; \( C_6 = \) demand of energy; \( C_7 = \) demand of food; \( C_8 = \) low level of manufacturing.
- **P-level** Policy: \( P_1-P_{17}. \) The priorities of the 17 policies are given in Zhu’s paper [16].

3.3. Management of science and technology

In this area, Zhao et al. [18] investigated the problem of selecting research projects in science institutes. As the report abstract of the research program of SSTC, He et al. [7] gave a mathematical method based on the AHP to integrate and evaluate scientific research achievements. The method has been used by Chinese authorities to award science prizes.

In recent years, many agencies of local governments have used the AHP in planning and technology development. For example, Hu [19] studied science and technology development planning for the Yibin region in the province of Shichuan. The study uses the forward and backward processes in the medium-term social and economic development planning of the region.

3.4. Other areas of application

In China the first application of the AHP in industry management can be seen in the work of Zeng [20], an engineer in the largest Chinese oil refinery. After learning the AHP, he developed a model to set management goals which has led to remarkable improvements in plant management.

L. Lu, a graduate student in the Department of Management at Tianjin University, in her Master’s Thesis [21], gave a quantitative method for selection and assessment of technology imports. The AHP is introduced as an effective methodology for dealing with the economic, social and technological realities confronting technological change in enterprises. A result of the analysis is the interpretation and possible explanation of the interrelations and interactions between technology and the enterprise’s economic development strategy. Furthermore, Lu also studied problems such as the consistency and sensitivity analysis of judgment matrices and the adjustment of initial judgments.

Recently, considerable attention has been paid to manpower planning. A research team at Nanjing Technology College forecast the special manpower demand of the nation using the AHP. The Institute of Systems Engineering at Xian Jiaotong University is among those involved in research and application of the AHP in China. Guo [22] examined the problem of the development of the tourist trade in Shanxi Province. In his paper he gave a systematic analysis of how to develop policies to address the issue of tourism and how to manage and invest the revenues.

Applications of the AHP can also be seen in the medical arena. King and Xu [5] used the AHP
in the diagnosis of heart disease. Using a large number of case studies, they built a mathematical model using fuzzy numbers for the diagnosis of acute myocardial infarction.

Finally, a decision support system for the Chinese language based on the AHP is under development.

The AHP has also found its way into the following areas: production testing, weapon performance evaluation, financial management, education planning, transport systems engineering and so on. More and more engineers and project managers are realizing that the AHP is a simple and convincing tool for decision making. It can be expected that applications of the AHP will continue to increase in China helping people achieve their goals in a more systematic and effective way.

4. CONCLUSION

Many researchers in the fields of systems engineering and operations research in China are interested in developing the theory and methodology of the AHP. Some institutes have included the AHP as part of their research program. Progress has been made in its mathematical foundations, fuzzy AHP, sensitivity analysis of reciprocal matrices, the consistency test of the judgment matrix, dynamic priorities and the scoring method.

Applications of the AHP have been reported in the areas of economics, energy, science and technology management, industry management, manpower planning, planning of tourist trade and medical diagnosis. Many areas such as production testing, weapon performance evaluation, financial management, education planning and transport systems analysis benefit from the AHP methodology.

In the coming years, the AHP will become a factor in the thinking of the Chinese people, and further rapid development of the AHP both in its research and applications will become apparent.

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