



Reducing Computation Time with a Rolling Horizon Approach Applied to a MILP Formulation of Multiple Urban Energy Hub System

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Abstract

Energy hub model is a powerful concept allowing the interactions of many energy conversion and storage systems to be optimized. Solving the optimal configuration and operating strategy of an energy hub combining multiple energy sources for a whole year can become computationally demanding. Indeed the effort to solve a mixed-integer linear programming (MILP) problem grows dramatically with the number of integer variables. This paper presents a rolling horizon approach applied to the optimisation of the operating strategy of an energy hub. The focus is on the computational time saving realized by applying a rolling horizon methodology to solve problems over many time-periods. The choice of rolling horizon parameters is addressed, and the approach is applied to a model consisting of a multiple energy hubs. This work highlights the potential to reduce the computational burden for the simulation of detailed optimal operating strategies without using typical-periods representations. Results demonstrate the possibility to improve by 15 to 100 times the computational time required to solve energy optimisation problems without affecting the quality of the results.

Keywords: Rolling Horizon, Mixed Integer Linear Programming (MILP), Energy Hub, Computational Time

1 Introduction

The European Commission Directorate-General for Climate Action aims to increase the share of renewable energy to at least 27% by 2030 [General Secretariat of the Council, 2014]. The installed capacity of multi-energy carrier distributed-energy systems is growing, encouraged by government support for more competitive, sustainable and reliable cogeneration systems [Intergovernmental Panel on Climate Change, 2007]. According to this trend, energy networks

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will become more complex, offering a large potential for energy savings, but which need to be better quantified. Simulation of networks allows comparison of the energy-savings between different network configurations combining hybrid centralized or decentralized energy systems.

[Geidl et al., 2007] introduced the energy hub model, a powerful concept allowing the interactions of many energy conversion and storage system to be optimized. New formulations of the energy hub model [Evins et al., 2014] address operational constraints such as operating limits and storage losses. The energy hub has been employed in a bi-level optimisation process to select and size the technologies of an energy centre and determine their optimal operation [Evins and Orehounig, 2014]. An advantage of the energy hub is that it is based on a fast and reliable optimisation approach: Mixed integer linear programming (MILP). The energy hub model uses linear equations to describe the relation between input energy sources and output energy streams with a conversion matrix representing technology efficiencies. Adding integer variables increases the complexity of the problem but allows better representation of part load constraints and on-off constraints.

Multi-objective optimisation of investment and operating costs and environmental impact of a network is computationally challenging, especially for models including many constraints, for instance with hourly energy demand profiles to be met. As the problem complexity increases with the number of variables, and exponentially with the number of integer variables [Domínguez-Muñoz et al., 2011], solving a MILP problem at hourly resolution over a whole year is difficult in reasonable time. “Typical days” are commonly employed to overcome this. [Domínguez-Muñoz et al., 2011], for instance, select N_k user-chosen typical days using the k -medoids clustering method to optimise the configuration and operation of a combined heat and power (CHP) system for a whole year. Based on a k -means clustering algorithm assisted by an ϵ constraint optimisation technique, [Fazlollahi et al., 2014] use typical periods selected from multiple time dependent demand profiles, minimizing the number of typical periods, while accounting for extreme demand peaks. To compute the optimal design of distributed energy systems for a neighbourhood of 10 buildings, [Mehleri et al., 2012] divide a yearly calendar into 18 different time periods (6 periods per day, for summer, mid-season and winter). [Morvaj et al., 2014] when selecting the optimal distributed energy resources for an urban district used 12 typical days. [Lozano et al., 2009] used 24 representative days in a MILP optimisation of a trigeneration system.

Instead of using a typical-period representation, the Rolling Horizon (RH) approach, a common decomposition method (along with Bender decomposition, Lagrangean decomposition, and bi-level decomposition [Grossmann, 2012]) can be used to reduce the computational time needed to solve problems with many variables. Rather than solving a complex problem considering its whole time horizon, the problem is solved for successive planning intervals representing a small part of the horizon, reducing the size of the problem per interval, breaking down one large problem into easily solved sub-problems. [Nease and Adams, 2014] applied a rolling horizon optimization strategy to an integrated solid-oxide fuel cell and compressed air energy storage plant model as a MINLP.

Approaches similar to RH have been considered to deal with uncertainties or unknown data. Model predictive control (MPC) applies the RH approach by solving a control problem, implementing the first stages, then moving forward one timestep and resolving. [Široký et al., 2010] present a comparison of MPC and weather compensated heating control for the control of a building heating system. RH can be employed (as in this paper) as a method for allowing faster optimisation of a detailed model; [Dimitriadis et al., 1997] compared RH to direct simulation over the full time horizon, and found within a 5% optimality margin results lower by 2% and almost 12 times faster.

This work presents a model study of a rolling horizon approach applied to an energy hub used to compute the optimal operating strategy of an office building over a full year. A parameter study is conducted to identify the best way of applying the RH approach. The findings are applied to a more detailed problem at network level.

2 Model implementation

The optimisation problem, formulated as a mixed-integer linear programming (MILP) model, is implemented using AIMMS and solved using IBM CPLEX. The model solved over the full planning horizon is used as a reference case. Then a rolling horizon method is applied to improve the time taken to solve the problem. As for the reference case, the model is implemented in AIMMS and solved using IBM CPLEX solver. The difficulty resides in the choice of the best set of parameters for the rolling horizon approach.

2.1 MILP model description

The energy hub MILP model tries to find the operating sequence for all converters and storages that minimize operating costs (Equation 1), subject to the demands to be met and supplies available (Equation 2a and 2b). The basic principle of an energy hub is that each technology is represented with a constant efficiency to retain linear formulation. Therefore, the output of the technology is equal to an efficiency factor time input. The output can again be an input for the next technology in case of cascade systems (e.g. heat output of CHP can be input for absorption chiller which produces cool energy).

$$\min f = F \times I(t) \tag{1}$$

$$O(t) = \Theta \times P(t) + A_- Q_-(t) - A_+ Q_+(t) \tag{2a}$$

$$E(t + 1) = E(t) + Q_+(t) - Q_-(t) \tag{2b}$$

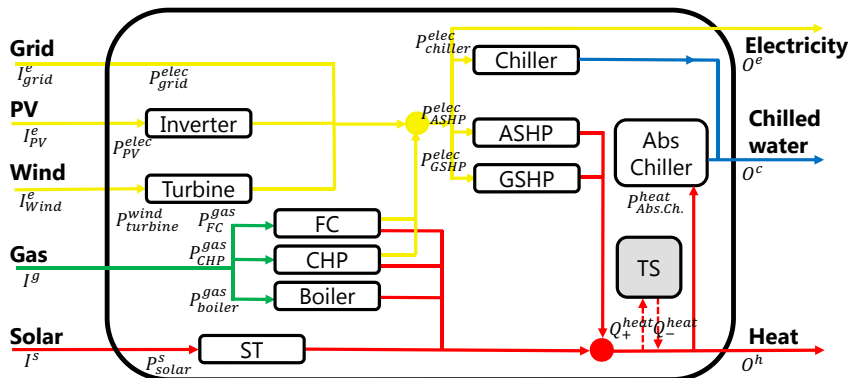


Figure 1: Energy hub: schematic showing the inputs, converters, storages and outputs

Equation 1 represents the objective function to be minimized with F cost factors of input energy streams $I(t)$ at each timestep. The Equation 2a represents the output energy streams

(energy demand) $O(t)$ function of the decision variables $P(t)$ (constrained by the availability of the energy streams $I(t)$) times the conversion matrix (systems efficiency factors) Θ and the storage vectors $Q_-(t)$ and $Q_+(t)$. Equation 2b represents the energy stored $E(t)$ at each timestep, equal to the content at the previous timestep plus any charging, minus any discharging. A schematic of the energy hub¹ used is presented in Figure 1 and the conversion matrix Θ with efficiency factors for the technologies is given in Table 1.

The energy hub model (Figure 1) used for this study has been chosen for its large size (289,082 constraints and 201,482 variables, with 26,280 integers²). Electrical, heating and cooling demand is considered for a whole year (hourly resolution) and can be satisfied by combining the operations of different energy converters and storages. Electricity can be imported from the grid or generated via photovoltaics (PV), combined heat and power (CHP), fuel cell (FC) or wind. The cooling demand can be satisfied by a chiller or an absorption chiller and the heating demand is satisfied by the ground source heat pump (GSHP), air source heat pump (ASHP), boiler, CHP, FC or solar thermal collectors (ST). The storage device is represented by a Thermal Storage (TS) in Figure 1. Demand profiles were obtained from an example building simulation, and are presented for three days in Figure 2, with energy available from wind and solar radiation shown in Figure 3. The weekly optimal operating strategy results for the whole year are presented in the Figure 4, and the hourly operations for the same three days (corresponding to the first three days of the week 39 highlighted in the red colored column Figure 4) are shown in Figure 5.

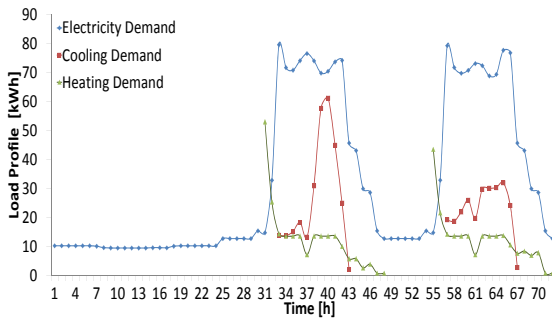


Figure 2: Energy demand for 72h

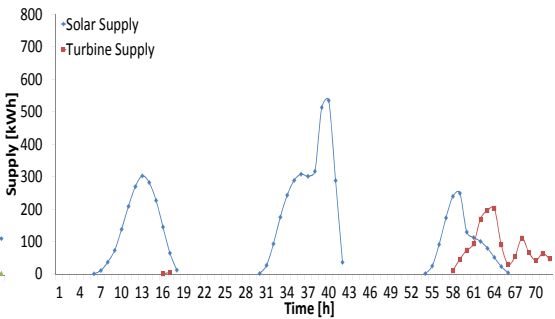


Figure 3: Renewable energy availability for 72h

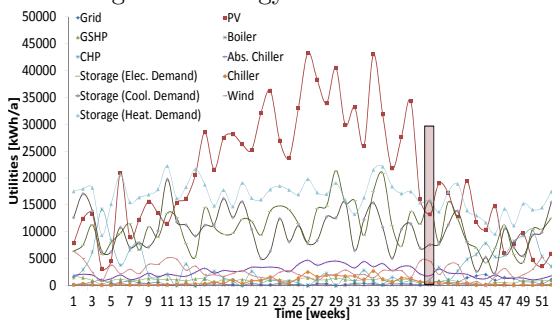


Figure 4: Optimal operation over a year

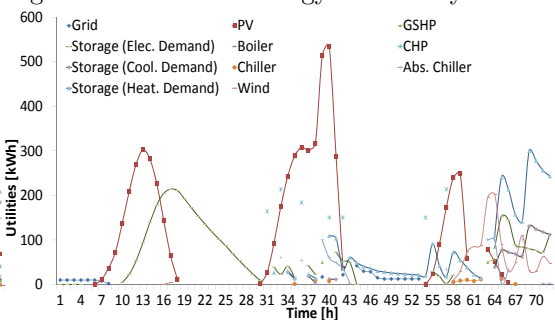


Figure 5: Optimal operation week 39

¹A detailed formulation of an energy hub problem can be found in [Evins et al., 2014].

²Three integer variables per hourly time step: two binary variables for CHP and GSHP (on/off for system with part load constraint) and one for storage (in/out). In total 23 variables and 33 constraints per time step plus 2 variables and 2 constraints at the first time step.

Table 1: Conversion matrix Θ with efficiency factors

	Grid	PV	GSHP	ASHP	Boiler	CHP	FC	ST	Chiller	Abs. Chiller	Wind
Electricity	1	0.2	-1	-1		0.3	0.35		-1		0.95
Cooling									2.5	0.7	
Heating			4.5	3.5	0.8	0.49	0.3	0.6		-1	

2.2 Rolling horizon implementation

The principle purpose of the rolling horizon approach (RH) is illustrated by a simple example. Figure 6 presents an hourly optimisation problem to be solved on a $20h$ time horizon (8760h in our study case). Considering a certain number n_v of variables to be determined per time step and a time horizon H , the total number of variables is $N = n_v * H$ (201,482 variables for our study case). When increasing the number of variables, particularly integer variables, the computation time can increase dramatically. In order to reduce the solving time, a difficult problem can be reduced to smaller and easier to solve sub-problems with the number of variables in each reduced to $N_{RH} = n_v * L_{int}$ per planning interval. The overall problem is solved by optimizing successive planning intervals, each shifted L_{step} steps forward. Figure 6 shows the key parameters:

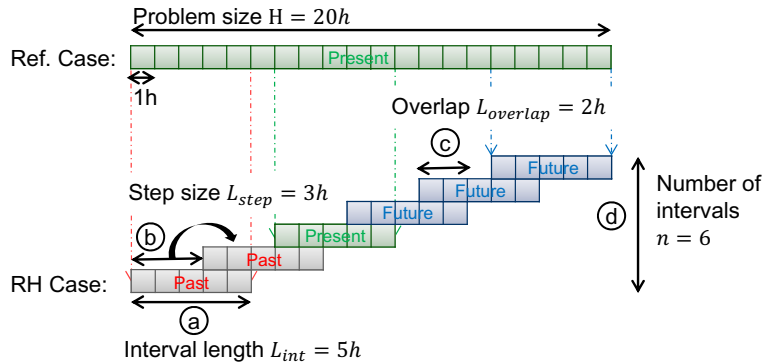


Figure 6: Rolling horizon approach

(a) **Interval length** L_{int} characterizes the length of one sub-problem. (b) **Step size** L_{step} is the number of periods before rolling to another planning interval and (c) **overlap** $L_{overlap}$ is the number of periods from the past interval reconsidered in the present interval. Finally (d) the **number of planning intervals** n describes the number of sub-problems to be solved to cover the whole period³. The overlap and the number of intervals are defined by the interval length (L_{int}), the step size (L_{step}) and the total problem length (H). The parameters need to be carefully chosen in order to reduce the total solving time without affecting the quality of the results. If L_{int} is too small the optimal operating strategy found will be less efficient than the full horizon problem, where all periods are known from the outset.

³If n is not an integer, a different interval length must be used for the final planning interval.

2.3 Simulation environment

A parameter study has been conducted to identify the best combinations of rolling horizon parameters (results are shown in Table 2). All cases have been computed with the same machine which has an Intel Xeon 3.1 GHz CPU with 8 cores and 64 Gb of RAM. The CPLEX solver was set to use a deterministic approach restricted to 1 core.

CPU time is preferred to ‘ticks’ as the unit of measure to compare solving time between runs. Although tick values, set by CPLEX to quantify the amount of work done deterministically, are slightly more reliable as they do not vary between machines, CPU time is more useful for our study since it allows the pre-solving steps performed prior to each optimisation to be included. Pre-solving steps refer to the steps done in AIMMS to transform the mathematical program written in AIMMS to a CPLEX readable code, i.e. steps done before passing the problem to CPLEX for solving.

Optimality gap is defined as the difference between the best linear solution and the best integer solution found so far. It is commonly used as a metric of solution quality for MILP problems. A target optimality gap of 2% was used for all cases unless otherwise stated ([Dimitriadis et al., 1997] compared RH to direct simulation over the full time horizon within a 5% optimality margin). As this target has to be reached for every planning interval for the rolling horizon approach, the overall optimality gap reached is recalculated based on the overall solution and the linear solution of the reference case problem.

3 Results

3.1 Parameter study

The parameter study results are presented in Table 2: (a) shows the CPU time taken for each set of rolling horizon parameters and (b) the optimality gap reached. In Table (a) dark red shows cases performing worst or with similar performance than the reference case in terms of computation time. This is due to the small step-size (12 – 144) and large interval (> 120), thus giving a relatively large number of variables. In dark green are the fastest cases, lying on the diagonal (representing overlapping parameter⁴ ≤ 24). However the optimality gap reached is affected in comparison with other cases as shown in (b). To summarise, RH does not perform well in terms of solving time when the interval length is large and the step size is small. It does not perform well in term of quality when the overlap of planning intervals is smaller than 1 day. The best balance of computing time and quality is given by the solutions labelled A-F. The influence of the rolling horizon approach on the operating strategies obtained is studied for those labelled solutions in Section 3.2.

All cases presented in Table 2 used the same optimality gap limit of 2%. When using smaller limit for the optimality gap criteria the difference in solving-time between runs further increases, highlighting the benefit of using a rolling horizon approach. Figure 9 in Section 4 shows results for an application case computed with an optimality gap limit of 1%.

⁴For the cases on the diagonal of Table 2, in order to consider storage at least 1h of overlap is required between two planning intervals, so L_{step} was reduced by 1.

Table 2: Parameter study results: (a) CPU time [s] - (b) Optimality gap reached [%]

(a)	Interval Step	12	24	36	48	72	96	120	144	168	240	336	CPU time [s]
	12	449	410	549	981	1'341	2'148	2'587	3'376	3'999	7'232	14'321	100
	24		225	287	414	733	1'149	1'772	2'376	1'977	3'428	12'488	400
	36			186	266	470	759	1'082	1'506	1'975	4'062	4'737	700
	48				199	350(A)	518	883	1'160	1'502	2'956	6'337	1000
	72					250	386(B)	557	733	1'032	1'206	4'014	1300
	96						291	382(C)	575	758	907	3'516	1600
	120							341	455(D)	571	1'165	1'344	1900
	144								410	505(E)	690	2'308	2100
	168									418	442(F)	1'784	2400
	240										601	1'226	2700
	336											882	> 3000 ← Ref. Case = 3233 s

(b)	Interval Step	12	24	36	48	72	96	120	144	168	240	336	Optimality gap [%]
	12	23.19	4.76	2.43	2.10	2.06	2.01	2.03	2.16	2.16	2.18	2.20	2 ← Ref. Case = 2.00 %
	24		14.70	3.59	2.31	2.07	2.04	2.03	2.08	2.01	2.12	2.16	3
	36			10.71	3.37	2.01	1.93	2.03	1.97	2.01	2.12	2.14	4
	48				9.71	2.22(A)	2.02	1.99	2.03	2.01	2.10	2.08	5
	72					6.98	2.10(B)	2.04	1.99	2.10	2.06	2.10	6
	96						5.62	2.00(C)	2.05	2.03	2.09	2.03	7
	120							4.52	2.11(D)	2.09	2.12	2.10	8
	144								4.02	2.08(E)	2.05	2.11	9
	168									3.70	1.96(F)	2.12	10
	240										3.52	2.14	11
	336											2.33	> 12

The total computing time to solve the problem is composed of AIMMS computing time (i.e. pre-solving determination of the MILP for each sub-problem) and CPLEX solving time. AIMMS computing time is the time necessary to translate a mathematical program from AIMMS to a CPLEX readable code in general, additionally in the RH context, the time necessary to aggregate data of each sub-problems solved. CPLEX solving time is the time required by CPLEX solver to build the search tree of the MILP problem and solve it with the branch and bound method. Figure 7 shows the breakdown of AIMMS (red) and CPLEX (blue) time for selected cases⁵ with different interval lengths but the same overlap. A general trend (true for all cases) can be seen, with the minimum total time dependant on the balance between the number of sub-problems to solve and the number of variables of each sub-problem. In order to keep CPLEX time low, the period length should not exceed one month in our problem. Minimizing the number of sub-problems, i.e. increasing the step size will reduce AIMMS computing time and also CPLEX time. This can be observed in Figure 7 for step size higher than 48h. In order to maintain the quality of the results, the overlapping parameter should be higher than 24 hours as observed in Table 2 (b).

⁵Optimality gap set at 8% for this study (Figure 7), chosen for the reasonable amount of time required to compute RH with $L_{int} > 336$

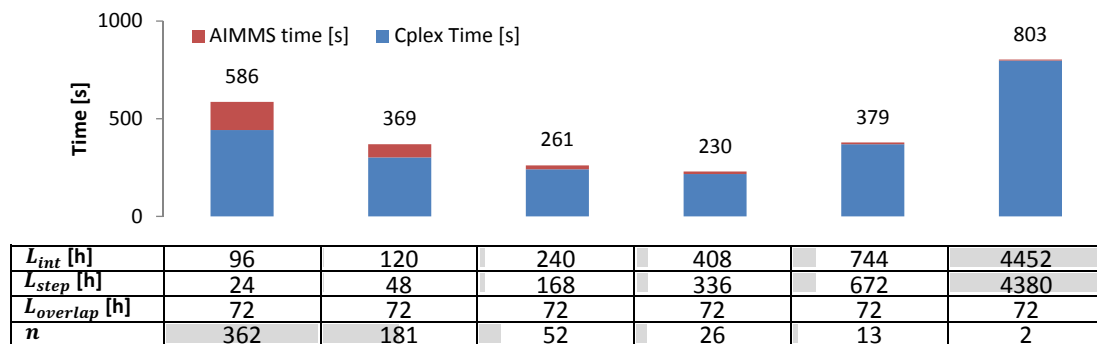


Figure 7: Solving time breakdown for selected cases

3.2 Influence on operation schedules obtained

The influence of the rolling horizon approach on the operating strategies obtained is studied for the optimal cases chosen from Table 2. The computing time to solve these cases is reduced by 6 to 10 times compared to the reference case (average of 7 minutes compared to 54 minutes for the reference case) and the optimality gap is among the best results (average of 2.07% compared to 2.00% for the reference case). Figure 8 represents the difference (in percent) in the annual energy production of each energy source compared to the reference case. Considering the optimality gap target of 2%, the difference between cases is small: less than 5% for all except GSHP and Abs. Chiller. The cross-correlation has been studied and shows a strong correlation between the operating strategy per hour and per utility for all cases.

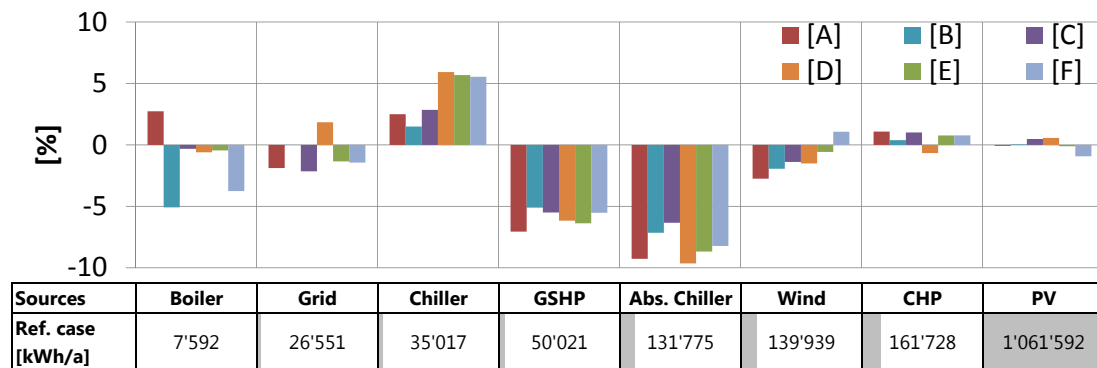


Figure 8: Differences in supply compared to reference case

4 Application case

4.1 Application to a multiple hub model

The RH approach has been applied to a more detailed model used to compute the optimal operating strategy of a network of 12 buildings at neighbourhood scale. The multiple energy hub model has been implemented by Morvaj, and consists of an extended version of the model

used in [Morvaj et al., 2014]. The optimisation problem considers a fixed network and fixed capacities of the utilities present in each building. The objective function is to minimize the total operating costs of the 12 buildings.

The technologies installed in the 12 buildings are thermal storage, ASHP, CHP, Boiler and PV. This multiple energy hub problem is larger than the previous case with 2,848,241 constraints, 2,418,928 variables (210,240 integers). In order to solve this problem in reasonable time, the rolling horizon approach is needed. The best combinations of parameters determined for our reference problem are used for this application. Results for the best set of parameters are compared with an opportunistic approach (average-value on 20 runs) with CPLEX parallel mode enabled on 8 cores. The optimality gap is set to 1%⁶ and the results are presented in Figure 9, highlighting the time savings possible: on average 130 times faster than solving the full horizon problem.

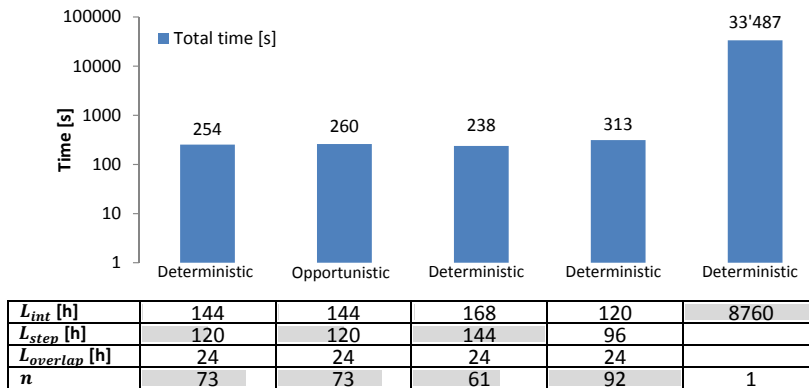


Figure 9: Comparison cases at 1% optimality gap

5 Conclusions and future development

A rolling horizon approach has been implemented and the best parameters studied in order to reduce the computing time necessary to find the optimal operating strategy of an energy hub, thus avoiding the use of typical periods. Results have shown that the computation time of large size problems can be drastically reduced (from 10 to 100 times⁷) without severely sacrificing the quality of the results. The memory necessary to solve the “branch and bound” MILP search tree is also reduced by using a rolling horizon approach. From the results of the parameter study, the planning interval size should be between 4 and 14 days (the choice within this range should be based on the number of variables). The overlap should be between 1 and 2 days in order to solve the overall problem quickly without severely sacrificing quality of results. There are many possible avenues for further research in this area. A bi-level approach using a genetic algorithm can be combined with the rolling horizon approach in order to consider the design level optimization of plant capacities. The optimal designed variables of the energy system are defined by the genetic algorithm and the optimal operating strategy is solved for each design

⁶Limit chosen in function of the reasonable amount of time needed to reach 1% optimality gap in the full horizon ($L_{int} = 8760h$) deterministic case.

⁷For the computational performance comparison, a time complexity analysis will be included in future works, in order to clarify the benefits of the RH implementation with respect to the dataset size.

using the MILP formulation combined with the rolling horizon approach. An aggregated version of the rolling horizon approach could be employed to consider long-term energy storage while solving a detailed model for the short-term operation strategy. With the aggregated model, design variables could also be considered and fixed in the first planning interval.

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