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Experimental study of thin wall milling chatter stability nonlinear criterion

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Abstract

The nonlinear dynamic behavior of milling process has been accompanied by the entire cutting process. In order to accurately determine and predict chatter stability of machining process, this article studied at both ends of the fixed thin part nonlinear criterion of milling chatter stability with experimental method. The experiment takes the vibration signal of thin part as the study object. And it analyses the vibration signal of different processing parameters based on the phase plane method, Poincare method and spectral analysis. Then, the relationship between the maximum Lyapunov exponent and the spindle speed and milling depth changes is discussed. Finally, taking the largest Lyapunov exponent as the criterion, the study determines the chatter stability domain of milling by using contour method. The comparative analysis is based on the milling chatter stability domain which obtained from the full discrete method. The experiments obtained the nonlinear stability criterion of aviation aluminium alloy 7075-T6 thin part.

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Keywords: *Thin part; Milling chatter; Nonlinear criterion; Lyapunov exponent*

1. Introduction

The requirements of weight and intensity for the parts which used in modern aerospace are gradually increasing. Thin-walled structures are widely used in plane girder, wall plate, etc. The thickness of main load-carrying structure wall is always 1 mm only. Thin wall parts which have many advantages such as high strength and relatively light weight. But it has many problems during thin-walled parts processing. The vibration buckling (such as flutter) in the process of machining is severely restricts the thin-walled workpiece machining quality. A convenient and effective method for the flutter stability of the machining process for accuracy prediction and judgment which in order to ensure the stability of the milling process by milling parameter optimization is necessary. Spindle-cutting tool-workpiece, and the fixture system [1] is a complex nonlinear system during the machining process. In the traditional linear theory, the requirements of low speed processing can be met by using theory of linear approximation method in nonlinear system when cutting thickness is small.

But with the development of high speed milling, the traditional linear theory has not guarantee the accuracy of the model. It is difficult to predict the flutter critical value and the surface of the workpiece position errors and roughness, etc. So Gradisek et al. [2-3] analyze the influence of different cutting depth of the influence of nonlinear vibration. They put forward the chatter occurs in vibration signal with low dimensional chaotic vibration phenomenon which is based on the bifurcation model. David et al. [4] established type regenerative chatter model with second order delay which is based on the nonlinear dynamic cutting force. They analyzed the system vibration from the Hopf bifurcation to chaos vibration state and got Hopf bifurcation stability boundary conditions, the value of characterization. Szalai et al. [5] simplified the machining for the collision and established the nonlinear dynamic model of high speed milling. They think that chaos vibration exist during high speed milling process. On the other hand, they analyzed the subcritical bifurcation stability boundary. Stefanski et al. [6] pointed out that the

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Lyapunov index of dynamic system (Lyapunov Exponent, LE) is an effective approach for the analysis of chaotic motion. This method is mainly aimed at the nonlinear milling vibration. This is due to the nonsmooth features of dynamic cutting force make the phase space of the dynamic model multidimensional. While calculating the distance between the two trajectories and divergence between adjacent tracks is more difficult, the maximum Lyapunov index can be used to analyze the vibration signals of the milling system. Li Zhongqun et al. [8] solved nonlinear milling dynamics equation through numerical method and got the time-domain flutter stability domain according to the stability criterion. Kong fansen et al. [9] analyzed characteristics of the nonlinear behavior for cutting system from no flutter state to flutter. And this is based on Lyapunov index and Kolmogorov entropy. Wang Xibin et al. [10] found that nonlinear characteristic of the vibration signal processing is very clear in the study. It is hard to get the same processing effect under the condition of the same process parameters. At the same time, the study founded that the nonlinear characteristic of the vibration signal for thin-walled workpiece model is more obvious and complex than the vibration signal which is based on the theory [11] in the experiment of milling process. At present, the study about bifurcation and chaotic motions of several coupling vibration for the thin-walled workpiece has made some achievements. YEH and Wang ping et al.[12-14] studied the thermoelastic coupling vibration of rectangular plates of bifurcation and chaos.

Although the nonlinear dynamic model holds an enormous advantage in flutter prediction, how to judge the stability of the thin-walled cutting based on the nonlinear theory, to predict the flutter stability of the domain on the basis of experiment, to establish flutter stability criterion for specific artifacts milling process are still challenges.

The study takes 7075-T6 thin part which is fixed on both ends as the research object. And it takes the maximum Lyapunov index of workpiece vibration signal as the threshold. Then, the cutting chatter stability domain is determined through the milling experiment. On the other hand, the nonlinear criterion of flutter stability domain for thin part which is fixed both ends is determined. The study lays the nonlinear stability foundation for cutting vibration of thin part under thermal and mechanical coupling effect.

2. Flutter analysis method

The cutting vibration has obvious nonlinear characteristics due to the nonlinear characteristics of the milling system (dynamic milling force of smoothness, systems of nonlinear damping and stiffness, etc.). And Maximum Lyapunov Exponent index, Poincare ichnography, Poincare mapping and phase analysis methods are effective methods for the analysis of nonlinear dynamic behavior of milling chatter.

2.1Maximum Lyapunov exponent

According to the motion relationship between the motion axis and the motion joint of the five axes vertical machining center Mikron UCP 710, the model of the kinematic chain

Maximum Lyapunov exponent describes that two points closed infinitely separate with time evolution at the initial time. As the characteristic parameter of chaotic motion, maximum Lyapunov exponent means the maximum divergence degree of phase trajectory or the maximum sensitivity degree for the initial value [15].

Set the milling vibration signal of variable cutting depth as x_1, x_2, $\cdots$ x_N (univariate time series). In formula, N is the total number of time series. According to packed and others [16], the idea of time delay is proposed to reconstruct the phase space of the dynamics system observed. Based on this thought, time series is made the phase space reconstruction and get reconstructed trajectory X, and it can be expressed as

$$X = [X_1, X_2, \cdots, X_M]^T \quad (1)$$

In formula, M is the number of track points after reconstruction of the phase space, X_i is the state of milling vibration system in the discontinuous time point i, and it can be expressed as,

$$X_i = \left[x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\cdot\tau}\right] \quad (2)$$

In formula, τ is the time delay. And m is embedded dimension. $M = N-(m-1)\cdot\tau$. Reconstructing phase space is divided into N sections: $[X_1, X_2, \cdots, X_T]$, $[X_{T+1}, X_{T+2}, \cdots X_{2T}]$, $\cdots$, $\left[X_{(n-1)T+1}, X_{(n-1)T+2}, \cdots, X_{nT}\right]$. Each section length $T = M/n$ is called evolutionary time.

Set initial point X_1, looking for the nearest neighbor points $X_{1'}$, the distance is $L_1 = \|X_1 - X_{1'}\|$. In formula, $\| \ \|$ stands for Euclidean norm. After the evolution time T, the distance changes into $L_1' = \|X_{1+T} - X_{1'+T}\|$. The nearest neighbor points of X_{1+T} is $X_{(1+T)'}$, getting the distance L_2. After the evolution time T, the distance changes into L_2' . By that analogy, maximum Yeli spectrum is

$$\lambda = \frac{1}{M\tau}\sum_{i=1}^{n}\log\frac{L_i'}{L_i} \quad (3)$$

In formula, Δt is a sample interval .

Under the different phase space dimension, vibration signal in milling thin part is analyzed and calculated in order to research milling chatter stability under different processing parameters.

2.2 Poincare mapping and phase plane portrait

Analytical method of Poincare mapping is that abscissa is displacement value of system response and ordinate is speed value of the system response. The calculation methods are conducting data extraction for each time interval. If the milling system is periodic vibration, it corresponds to an isolated point on the Poincare mapping. If the cycle of vibration signal is N, there are n independent points on the Poincare mapping, and the number of cycle is the same as the number of isolated points. If vibration signal changes drastically and flutter occurs, it is shown discrete points accumulated figure [16].

The trajectory of solution of vibration system in the phase space forms the motion curve. Phase plane portrait is the motion curve in the phase space. For the certain motor system, several forms are as follows: When the cycle of vibration signal is 1 or N, corresponding phase plane portrait appears 1 or N closed curve. When vibration signals have strong flutter properties, closed curve will gradually expand to clutter group of curves.

3. Milling machining experiment of thin part

Experiment is conducted on the type of VDL-1000E machine tool. Workpiece model is thin part of both ends fixed. The length of the workpiece is 120mm. The width of it is 80mm and wall thickness is 5mm. Experimental facility and sensors are shown as Fig. 1. The type of dynamometer is KISTLER 9257B. Vibration acceleration sensors are set at the back of the machining surface, whose type is PCB356A25 and sensitivity is 10.42mv/g. Cutter is Sandvik R216.64-08030-AO09G 1610, whose diameter is 8mm, and helical angle is 30° . The cutter is four edges cemented carbide ball nose milling cutter. Data collection system is Donghua DH5922 and sample frequency is 5k Hz. Processing method is climb cutting. Processing parameter is set as follows. Radial milling width is $a_e = 0.2mm$. Feed speed is $f_z = 0.1$ mm/tooth. The relative angle of the workpiece and cutting tool is 30° in milling experiment. Tool-path is feed back and forth along the horizontal direction.

Tab.1 Modal parameters of the workpiece

modality	Natural quency f_0 (Hz)	Rigidity K (N/m)	Damping ratio ξ (%)
1	4840	3.8×106	25
2	5041	3.5×106	'5

Assuming that tool is the rigid and workpiece is the flexibility, the former second order modal parameters of workpiece are shown as Tab1. First of all, based on double degree-of-freedom system of tool - the workpiece model, milling chatter stability domain of thin part of two ends fixed is obtained as shown in Fig. 3 according to full discrete method [17]. Milling experiment is done according to milling parameter points marked in figure. Each parameter points correspond to certain cutting depth and the spindle speed. In this way, it will conduct 121 cutting vibration experiments in different processing parameters. As shown in Fig. 2, the number of thin part sample of both ends fixed is two. For milling experiment of the first sample, when each speed serves as a set of experiments, there are 13 experiments and the spindle speed is from 3000 rpm up to 6000 rpm. In the same way, for the second sample, 13 sets of experiments are done and its spindle speed is from6000 rpm down to 3000 rpm. Processing area on the workpiece is shown in Fig. 2.

Each group consists of 10 or 9 different cutting depths. Because of the workpiece material removal, the influence of former experiment is ignored in the process of same group experiment. At the same time, before the next set of experiments, the workpiece is processed and repaired to make the thickness and height of workpiece thin parts ensure the same size in each group of experiment. So we can try to make sure relative consistency of modal characteristics of workpiece in each group experiment. When two experimental samples are in the same cutting parameters, vibration condition of thin part is basically the same and the overall error is less than 9%. Then focused on each condition, based on the phase plane method, Poincare method and frequency spectrum, the measured vibration signals of the workpiece are analysed to decide if the flutter occurs under the processing parameters. And the maximum Lyapunov exponent of the vibration signal is calculated.

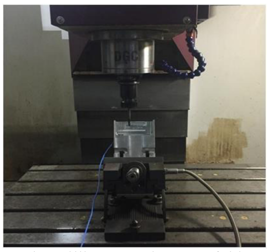


Fig. 1. The milling experiments of thin wall.

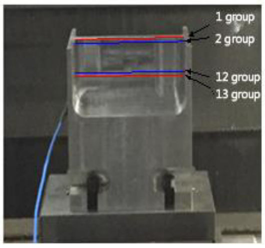


Fig. 2. The 13 groups at both ends of the fixed thin wall processing regional distribution.

Meanwhile, we found in the experiment that the chaos characteristic of instantaneous milling force signal is less obvious than that of vibration signal. So the vibration signals of the thin part are taken as the research object in the thin part milling experiments.

When thin part of both ends fixed is under different processing parameters A（3500 rpm，0.3 mm）、B（3500 rpm，0.5 mm）、C（4500 rpm，0.3 mm）and D（5000 rpm，0.7 mm）, Figure. 4 is respective result of vibration signals of workpiece acceleration, phase plane portrait, Poincare sectional view and spectral analysis.

As known in Fig. 4, when the processing parameter is A （3500rpm，0.3mm） in the milling experiments, time-domain signal amplitude of vibration acceleration of the workpiece is about 5m/s2. In research, we take 4096 sampling points of vibration signals and find that the energy is near the place where frequency of the blade cutting is 3500 × 4/60=233Hz and harmonic frequency is 467Hz after filtering and the discrete Fourier transform. What's more, energy distribution is even as shown in Fig. 4(d). From the phase plane portrait 4 (b) of the vibration signal, the vibration response of workpieces gradually converges into the closed curve around the center point. But the points in Poincare sectional view are also relatively concentrated and just have a few points. Although the maximum Lyapunov exponent of vibration signal (The embedding dimension is 2) is 0.2773 in this processing parameter, thin part of both ends fixed in this milling processing parameters is in a state of cutting stability without the flutter after testing the surface roughness, the phase plane portrait, Poincare sectional view and spectrum analysis.

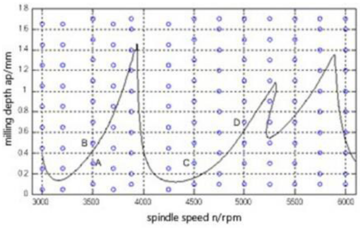
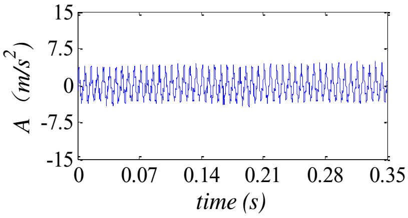
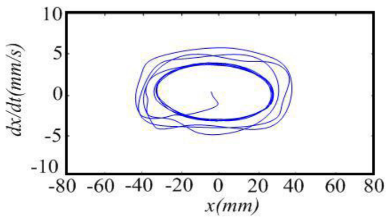


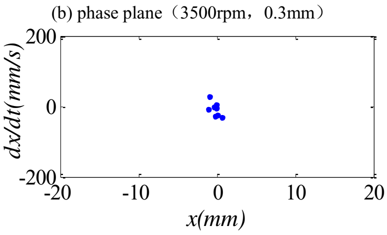
Fig. 3. Milling chatter stability region of fixed at both ends of the thin wall(○ is the parameter point of milling).



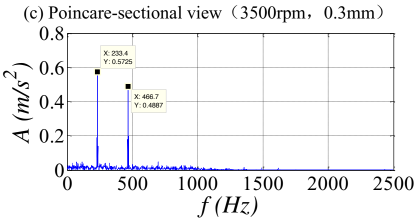
(a) vibration signal（3500rpm，0.3mm）



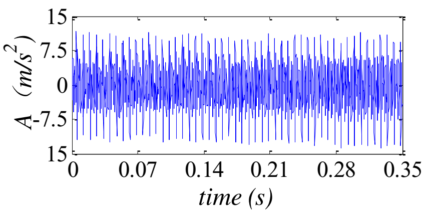
(b) phase plane（3500rpm，0.3mm）



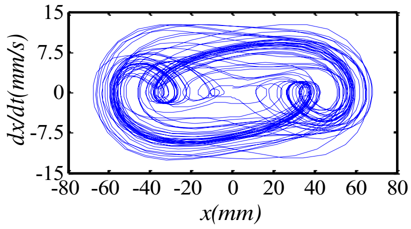
(c) Poincare-sectional view（3500rpm，0.3mm）



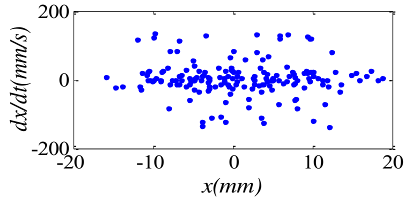
(d) spectrum analysis（3500rpm，0.3mm）



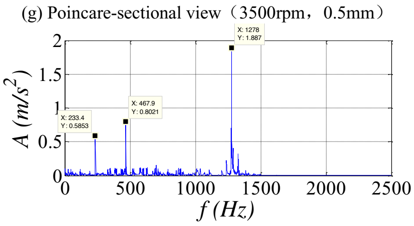
(e) vibration signal（3500rpm，0.5mm）



(f) phase plane（3500rpm，0.5mm）



(g) Poincare-sectional view（3500rpm，0.5mm）



(h) spectrum analysis（3500rpm，0.5mm）

Fig. 4. Workpiece vibration signal and its phase plane, Poincare-sectional view, spectrum analysis under different processing parameters.

The workpiece vibration acceleration time-domain signal under the parameter B（3500rpm，0.5mm） of milling experiments is shown in Fig. 4(e). And the vibration amplitude is 11.2 m/s2. A plan view of the vibration signal 4(f) shows that the vibration response of the workpiece gradually diverges into multiple curves without a rule. On the other hand, the point distribution of Poincare section 4(g) becomes uneven. The maximum Lyapunov exponent (embedding dimension equal to 2) of the vibration signal under the processing parameters is 0.6131. The spectrum after filtering and the Fourier transform is shown in figure 4(h). And the peak is found that not only exist around cutting frequency (3500 × 4/60 = 233Hz) and harmonic frequency (467Hz). The energy has a tendency to aggregate to the tool structure frequency (1278 Hz).The milling state of thin part which is fixed both two ends will be comprehensively judged under the processing parameters after testing the surface roughness, calculating the phase plane, plotting the Poincare sectional view and taking the spectrum analysis.

4. Prediction of nonlinear stability domain

The chatter will happen or not under these process parameters is determined by using the phase plane method, Poincaré method and spectrum analysis method. The calculation results of the maximum Lyapunov exponent (embedding dimension is 2) calculation for workpiece vibration acceleration signals are shown in Fig. 5. Fig. 5 shows that the maximum Lyapunov exponent value are subsequently changed with the change of each speed milling depth varies. And it will mutate when the maximum

Lyapunov exponent (embedding dimension is 2) is about 0.3-0.6. However, there is a sudden drop of the line when milling speed reaches 5250rpm and the milling depth is 0.9mm. Its phase plane, Poincare sectional view, spectral analysis also shows that this processing parameters in a stable cutting state. The point is in the hopf bifurcation lobe and it is near the boundary of chatter stability region.

The rotational speed of milling tool and milling depth are two major factors which influence the milling stability. So maximum Lyapunov exponent (embedding dimension is 2) can be obtained by using interpolation method. The relationship of spindle speed changes and milling depth changes are shown in Fig. 6. So that we can more intuitive see influence of the nonlinear characteristics of vibration signals caused by the changes of milling spindle speed and milling depth.

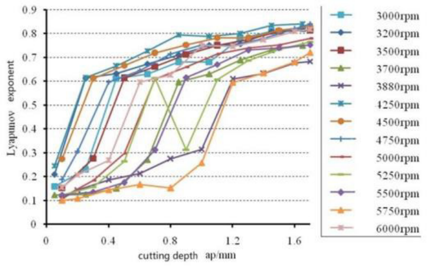


Fig. 5. The maximum Lyapunov exponent of vibration acceleration signals of different milling depth.

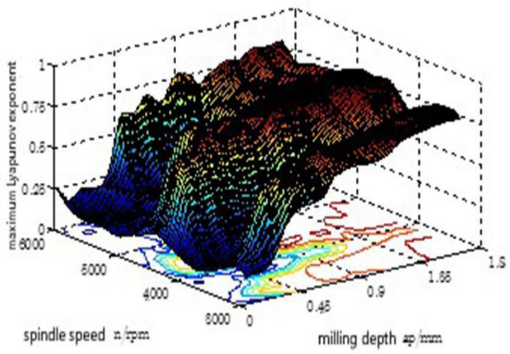


Fig. 6. Changes in the relationship between the maximum Lyapunov exponent of vibration acceleration signals with the speed of spindle and milling depth.

Fig. 6 shows that there is a smaller Lyapunov exponent area around the spindle speed 4000rpm and 5500rpm. It is easy to obtain a better processing quality by selecting these process parameters (the surface roughness is less than other working conditions). The maximum Lyapunov indices were selected as 0.6, 0.59 and 0.61 as a threshold for drawing contour lines in Fig. 6. And to analyze the relationship between the steady-state critical milling depth and milling spindle speed by the stability boundary. Fig. 7 is comparison of milling stable prediction chart in different maximum Lyapunov exponent value and the whole region and the fully Discrete Method for forecasting milling chatter stability.

In Fig. 7, which marked "×" represents the cutting parameters experiment will lead to chatter occurs. And which marked "○" indicates that the cutting parameters cutting experiment in a stable cutting state. The critical curve 1 is obtained by using full discrete method. Curve 4 is the vibration signal stability critical curve when the maximum Lyapunov exponent value reaches 0.61.The coincidence of these two curves is good and the overall error less than 9%.This also proves the validity of this method. Curve 2 and 3 are milling stable prediction charts under the different maximum Lyapunov exponent threshold. The limit cutting depth on critical curve of stability which obtained in this study is less than it on the curve of full discrete method. This is due to the processing system is very complicated and there are quite a lot of non-linear factors.

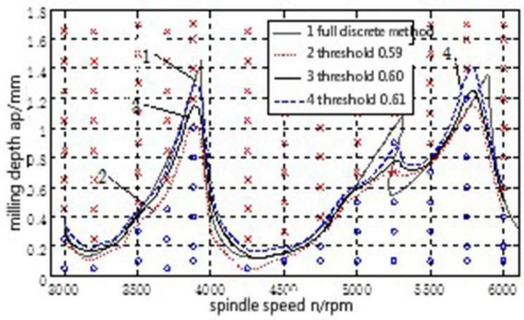


Fig. 7. Milling chatter stability region critical curve and experimental results (× The unstable cutting parameters; ○ The stable cutting parameters).

The stability curves obtained in experiments have some error when compare with the results of experiments. The reason for the error may be due to the following factors:

1. Milling stability boundary will change in a certain range with the change of dynamic parameters.
2. The relationship diagram of maximum Lyapunov exponent and milling spindle speed and depth which is obtained by interpolation technique itself has a certain error.
3. A large number of experimental data are needed to further refine this critical flutter stability curve.
4. The boundary of the milling stability in practical processing domain has some uncertainty.

5. Conclusion

The study takes aerospace aluminium alloy 7075-T6 thin part which fixed at both ends as the research object and studies on the stability of chatter in milling which takes the maximum Lyapunov exponent as a threshold value. The conclusions are as follows:

1. The vibration signals are analyzed by using the phase plane method and Poincaré method. The chatter will occur or not is determined.

2. Lyapunov index is an important criterion to measure the characteristics of the system dynamics uncertainty. Irregular vibration contains chaos and chaos is caused by the nonlinear of the milling system. It will lead irregular vibration. Irregular

varying degrees of vibration are different between steady milling and the chatter milling.

3. The study extracts maximum Lyapunov exponent of the vibration signal under different processing parameters. And it draws and discusses the relation between maximum Lyapunov exponent and the change of milling spindle speed or milling depth. The maximum Lyapunov exponent increases with the milling depth increasing. It was found that the exponent will mutate at the some milling depths. So the maximum Lyapunov exponent of vibration signals can be used as a nonlinear stability criterion.

4. The maximum Lyapunov exponent value 0.61 of vibration signals is taken as a milling chatter nonlinear criterion. The flutter stability critical curve is obtained by milling contour lines method. Although there some errors between critical curve of stability which obtained in the study and full discrete method. Those errors are acceptable. In addition, the maximum Lyapunov exponent of the workpiece vibration signal processing can be a means of optimizing the processing parameters.

Acknowledgements

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