Satellite Constellation Design with Adaptively Continuous Ant System Algorithm

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Abstract

The ant system algorithm (ASA) has proved to be a novel meta-heuristic algorithm to solve many multivariable problems. In this paper, the earth coverage of satellite constellation is analyzed and a $n+1$-fold coverage rate is put forward to evaluate the coverage performance of a satellite constellation. An optimization model of constellation parameters is established on the basis of the coverage performance. As a newly developed method, ASA can be applied to optimize the constellation parameters. In order to improve the ASA, a rule for adaptive number of ants is proposed, by which the search range is obviously enlarged and the convergence speed increased. Simulation results have shown that the ASA is more quick and efficient than other methods.

Keywords: ant system algorithm; satellite constellation; optimization design; coverage performance; adaptive adjusting

1 Introduction

Satellite constellation optimization remains a hot topic among constellation designers. Most constellations are used for communication, navigation, or similar functions by taking advantage of the Earth coverage provided by satellites[1]. So far, some integrated and applied methods have been employed for symmetrical constellation optimization design. For example, Walker’s method has been used to optimize global or zonal coverage. However, no absolute rules exist for regional coverage[1], and it is difficult to achieve satisfied results just with the aid of the method of celestial geometry[2].

With the rapid development of computer technology and calculation methods, many algorithms for satellite constellation design, such as annealing and genetic algorithms[3], have been introduced to optimize the constellation parameters in the purpose of minimizing the number of satellites or achieving the best coverage performance.

This paper presents a method for regional coverage through the ant system algorithm (ASA), a heuristic technique based on the artificial ant colony searching strategy[4]. It has been used to solve many multivariable problems. Moreover, a rule for adaptive number of ants is proposed to improve the algorithm. It is shown that the improved ASA used for satellite constellation design can efficiently determine the optimum parameters of constellation.

2 Earth Coverage

2.1 Earth coverage of a single satellite

As a fundamental index to the performance of most spacecrafts, the earth coverage rate serves as a key parameter in orbit and constellation design. It could be used to determine the number of the required satellites and some other useful data. Most often, continuous coverage refers to a certain region, or, perhaps, the whole world which could be cov-
ered at any time by at least one satellite in the constellation\textsuperscript{[1]}. Thus, from the angle of a ground station, if at least one satellite can be seen at a certain time, the constellation is meant to be in sight that time.

For a single satellite to be observed, the spacecraft elevation from the ground station, $\theta$, must be greater than the minimum $\theta_{\text{min}}$. For ground station communication, the satellite typically must be more than $5^\circ$ above the horizon, hence $\theta_{\text{min}} = 5^\circ$\textsuperscript{[1]}.

Fig.1 shows the angular relationship of the spacecraft elevation $\theta$ between satellite, ground station and earth’s center. $O$ is the center of the earth, and $S$ is the position of the $i$-th satellite in the constellation. $F$ is the $j$-th ground station fixed on the Earth to be communicated with. The spacecraft elevation angle $\theta$ is measured at the station between the satellite and the local horizontal.

As shown in Fig.1, $r_S$ is the geocentric vector from $O$ to $S$, $r_F$ is the geocentric vector from $O$ to $F$, and $r_{FS}$ is the vector from $F$ to $S$. If $r_S$ and $r_F$ are given, then

$$r_{FS} = r_S - r_F$$

Meanwhile, the elevation angle between the $i$-th satellite and the $j$-th station is

$$\theta_{ij} = \arcsin \left( \frac{r_{FS,z}}{r_{FS}} \right)$$

where $r_{FS,z}$ is the $z$-axis value of $r_{FS}$ in the local horizontal coordinate system of the ground station.

Thus, the coverage function from the $i$-th satellite to the $j$-th station could be defined by

$$f_{ij}(t) = \begin{cases} 1 & \theta(t) \geq \theta_{\text{min}} \\ 0 & \theta(t) < \theta_{\text{min}} \end{cases}$$

2.2 Coverage performance of constellation

Computer simulation inclusive of sampling and data collection is usually used to analyze the coverage rate of constellation. The process of sampling could be divided into two steps: geographic sampling and time sampling\textsuperscript{[3]}. The coverage function $f_j(t)$ from the constellation to the $j$-th station could be obtained by combining the coverage functions of every satellite.

Let the satellite number of the constellation be $N$. $f_j(t)$ could be found from

$$f_j(t) = \sum_{i=1}^{N} f_{ij}(t) \quad (3)$$

If the $n$-fold ($n > 0$) continuous coverage to the $j$-th station is required, the following inequality would be found

$$n \leq \min f_j(t) \quad (4)$$

And, the $n$-fold coverage rate to the $j$-th station, $C_{r_j}(n)$, could be expressed by

$$C_{r_j}(n) = \frac{\int_{t_{\text{begin}}}^{t_{\text{end}}} F_j(t,n)\,dt}{t_{\text{end}} - t_{\text{begin}}} \quad (5)$$

where, $F_j(t,n)$ is the $n$-fold coverage index, $t_{\text{begin}}$ the start time of simulation and $t_{\text{end}}$ its end time.

Now, for the convenience of computation in evaluating constellation coverage performances, $C_{r_j}(n+1)$ could be defined as a new coverage index called $n+1$-fold coverage rate, which is equal to the value of $C_{r_j}(n+1)$ while the value of $C_{r_j}(n)$ equals to 1. For example, if $C_{r_j}(1) = 1$, the $1+1$-fold coverage rate, $C_{r_j}(1+1)$, is equal to $C_{r_j}(2)$. The expression of $n+1$-fold coverage rate to the $j$-th station can be defined by

$$C_{r_j}(n+1) = \begin{cases} C_{r_j}(n+1) & C_{r_j}(n) = 1 \\ 0 & C_{r_j}(n) \neq 1 \end{cases} \quad (6)$$

Let $m$ be the number of the sampling stations in the region under current discussion, the average $n+1$-fold coverage rate of the region could be found from

$$A_{CR}(n+1) = \frac{1}{m} \sum_{j=1}^{m} C_{r_j}(n+1) \quad (7)$$
Given the model of average \( (n+1) \)-fold coverage rate, the work is to optimize the constellation parameters for a minimum satellite number of constellation, \( N \), and the maximum value of \( A_{CR}(n+1) \). Obviously, \( N \) is a natural number and \( A_{CR}(n+1) \) ranges [0, 1]. Thus the optimization model could be found from

\[
\min(N - A_{CR}(n+1))
\]  

It means that a constellation with \( N \) satellites is better than the one with \( M \) (\( M > N \)) satellites, though \( A_{CR}(n+1) \) of \( N \) satellites is less than that of \( M \) satellites.

3 Ant Systems for Constellation

3.1 Basic ant system theory

Inspired by the behavior of real ants and firstly derived by M. Dorigo[6], the ASA was originally used to solve travel salesman problem (TSP). In this algorithm, a set of artificial ants cooperate to optimize a problem by exchanging information via pheromone deposited on graph edges[4]. The core of ASA is composed of two parts: a state transition rule used by an ant to determine the next destination of a complete tour, and a pheromone updating rule for the use of allocating a greater amount of pheromone to shorten the tour.

3.2 The algorithm realization for constellation

The optimization of the constellation parameters is an issue concerning the continuous multi-dimension function. Let the vector \( X \) (\( X = [x_1, \cdots, x_j, \cdots, x_s] \)) be the parameters to be optimized in the constellation, first, it is needed to estimate the definition field of \( X \), i.e., \( x_j \in D(x_j) = [x_{j,\text{low}}, x_{j,\text{up}}] \) (\( i = 1, \cdots, s \)), where \( s \) is the total number of the parameters in the constellation to be optimized. The definition field \( D(x_j) \) could be divided into \( N_g \) subspaces[7], and the middle of each sub-space defines a nod. A single artificial ant \( k \) (\( k = 1, \cdots, N \)) would choose to move from one nod to the other. Therefore, there would be \( N_g \) nodes in each \( D(x_j) \). Meanwhile, the length of each sub-space \( h_i \) can be expressed by

\[
h_j = \frac{x_{j, \text{up}} - x_{j, \text{low}}}{N_g}
\]  

Thus the multi-dimension function optimization could be regarded as a problem of multilevel decision[8]. As each level has \( N_g \) nodes on it, there are \( N_g \times s \) nodes in total. As shown in Fig.2, the travel index \([4, 2, \cdots, 1]\) is the state vector of the ant \( k \) which has completed its tour.

Fig.2 State space graph for continuous ASA.

Then the value of the ant \( k, X \), could be found from the following expression

\[
\begin{bmatrix} x_1, x_2, \cdots, x_j \end{bmatrix} = \begin{bmatrix} x_{1,\text{low}} + (4 - 0.5)h_1, x_{2,\text{low}} + (2 - 0.5)h_2, \cdots, x_{j,\text{low}} + (1 - 0.5)h_j \end{bmatrix}
\]

The state transition rule of the ant \( k \) could be expressed by the following equation[8]

\[
P_{ij} = \frac{\tau_{ij}}{\sum_{i=1}^{s} \tau_{ij}}
\]  

where \( P_{ij} \) is the probability of the ant \( k \)’s moving to the \( i \)-th node on the \( j \)-th level, and \( \tau_{ij} \) is the amount of the pheromone at the node.

When all the ants have finished their tours, the pheromone updating rule could be found from the following equation

\[
\tau_{ij} = (1 - \rho)\tau_{ij} + QL_{ij}
\]  

where \( 0 < \rho < 1 \) is a pheromone decay parameter, \( Q \) the quantity of pheromone laid by an ant per iteration cycle, and \( \tau_0 = \text{const} \) the initial value of \( \tau_{ij} \). In ant colony system, only the best ant of each searching period is allowed to deposit pheromone. \( L_{ij} \) can thus be given as follows.
\[ L_{ij} = \begin{cases} A_{C_R}(n+1), & \text{if } (i,j) \text{ is best tour} \\ 0, & \text{otherwise} \end{cases} \quad (12) \]

### 3.3 The modification of ASA

Like other bionics algorithms, the ASA also has some disadvantages such as the ease of converging at the locally optimal solution and its slow speed in convergence etc [9].

In order to raise the efficiency and avoid the local optimal solution, the following rule for pheromone decay parameter, \( \rho \), is used to improve the ASA,

\[
\rho(N_c + 1) = \begin{cases} 0.9\rho(N_c), & 0.9\rho(N_c) > \rho_{\min} \\ \rho_{\min}, & 0.9\rho(N_c) \leq \rho_{\min} \end{cases} \quad (13)
\]

where \( N_c \) is an iteration step shown in Fig.3, and \( \rho_0 \) the initial value of \( \rho \).

#### 4 Experiments and the Simulation Study

### 4.1 Parameters set for experiments

**Experiment I**

In this section, an experiment is chosen from Ref. [3] to verify the validity and efficiency of the ASA. Supposing that a constellation contains three satellites with the same altitude and inclination, the constellation has 12 parameters in total to be optimized in order to ensure a single continuous coverage of most areas of China.

Considering the perturbation, let \( J_2 \) be the main perturbation of the constellation. The simulation time starts at 12 o’clock on 16 April, 2006 and lasts five days. The time interval used in sampling is 30 s. The constraint conditions of orbital elements are:

\[
a \leq 15,000 \text{ km}, \quad e \leq 0.2, \quad \Omega, \omega, \arg \leq 360^\circ, \quad M \leq 360^\circ
\]

where \( a \) is the semi-major axis, \( e \) the eccentricity, \( i \) the inclination, \( \Omega \) the right ascension of ascending node, \( \omega \) the argument of perigee, and \( M \) the mean anomaly.

The coverage areas are given by the following...
inequalities, 
\[ 75^\circ \leq \lambda \leq 135^\circ, \quad 0^\circ \leq \varphi \leq 55^\circ \]
where \( \lambda \) is the longitude of ground station, and \( \varphi \) the latitude.

**Experiment II**

In this section, an experiment is illustrated from Ref.[5], where a retrograde orbit is used to design regional coverage satellite constellations. The optimized result in Ref.[5] is a retrograde Walker \( \delta \) Constellation 24/6/1 with an orbit altitude being 1201.88 km and an orbital inclination 41°.

In order to enhance the comparability between the experiment II and that in Ref.[5], a Walker \( 24/p/F \) is chosen as an example. Thus four parameters need to be optimized. They are \( p, F \), the orbit altitude \( H \) and the inclination \( i \). The constraint conditions of \( H \) and \( i \) are as follows,

\[ 1000 \text{ km} \leq H \leq 1500 \text{ km}, \quad 0^\circ \leq i \leq 90^\circ \]

As shown in experiment I, \( J_2 \) is the main perturbation of the constellation. The simulation time starts at 12 o’clock on 16 April 2006 and lasts 5 d. The time interval used in sampling is 30 s.

### 4.2 Simulation results

From the simulation have been obtained some useful data sufficient to validate the improved ASA.

**Results from the experiment I**

Determined by the experiments, the initial parameters of the ASA are

\[ \varepsilon = 0.01, \quad \tau_0 = 0.5, \quad Q = 10, \quad \rho_0 = 1 \]
\[ \rho_{\text{min}} = 0.5, \quad N_{e \text{max}} = 20, \quad N_g = 10, \quad N_{a \text{min}} = 50 \]

Table 1 shows the correlated data of each item. From it, clearly, a satisfied solution could hardly be found when the number of ants equals to 100 with the basic ASA. However, as shown in the table, the efficiency and effects of optimization have been improved with the improved ASA. Compared with the Item 4, the Item 3 is more efficient, though the average \( 1+1 \)-fold coverage rate of the Item 3 is less than that of the Item 4.

Table 2 lists a set of optimized orbital elements of each satellite in the constellation which meets the single continuous coverage need in China with the aid of the algorithm of Item 3 in Table 1. It took one Pentium IV 1 GHz computer using Matlab Language 0.88 h to obtain the results. Table 3 lists the results in Ref.[3] with the help of the distributed genetic algorithm. It took four Pentium III 1 GHz computers 1.93 h.

### Table 1 Comparison between basic ASA and improved ASA

<table>
<thead>
<tr>
<th>Item</th>
<th>Comparative item</th>
<th>( A_{\text{cr}}(% \text{ cr}) )</th>
<th>Time/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Basic ASA, ( N_e = 100 )</td>
<td>error</td>
<td>error</td>
</tr>
<tr>
<td>2</td>
<td>Basic ASA, ( N_e = 500 )</td>
<td>30.202</td>
<td>3.76</td>
</tr>
<tr>
<td>3</td>
<td>Im-ASA, ( N_e = 500, \beta = 0.6 )</td>
<td>37.972</td>
<td>0.88</td>
</tr>
<tr>
<td>4</td>
<td>Im-ASA, ( N_e = 500, \beta = 0.8 )</td>
<td>38.017</td>
<td>1.97</td>
</tr>
</tbody>
</table>

### Table 2 Optimum orbital elements with ASA

<table>
<thead>
<tr>
<th>Sat.</th>
<th>( a/\text{km} )</th>
<th>( e )</th>
<th>( i(\text{°}) )</th>
<th>( \Omega(\text{°}) )</th>
<th>( \omega(\text{°}) )</th>
<th>( \mu(\text{°}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26 550</td>
<td>0.60</td>
<td>66.00</td>
<td>187.6</td>
<td>302.9</td>
<td>72.4</td>
</tr>
<tr>
<td>2</td>
<td>26 508</td>
<td>0.52</td>
<td>18.97</td>
<td>173.3</td>
<td>302.9</td>
<td>32.6</td>
</tr>
<tr>
<td>3</td>
<td>26 508</td>
<td>0.52</td>
<td>18.97</td>
<td>173.3</td>
<td>302.9</td>
<td>32.6</td>
</tr>
</tbody>
</table>

### Table 3 Optimum orbital elements with GA in Ref.[6]

<table>
<thead>
<tr>
<th>Sat.</th>
<th>( a/\text{km} )</th>
<th>( e )</th>
<th>( i(\text{°}) )</th>
<th>( \Omega(\text{°}) )</th>
<th>( \omega(\text{°}) )</th>
<th>( \mu(\text{°}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26 508</td>
<td>0.52</td>
<td>18.97</td>
<td>173.3</td>
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<td>32.6</td>
</tr>
</tbody>
</table>

### Results from the experiment II

In comparison with experiment I, there were less parameters to be optimized in experiment II. Therefore, the adaptive ants’ number rule would not be used to improve ASA in this experiment. The initial parameters of the ASA were

\[ \varepsilon = 0.01, \quad \tau_0 = 0.5, \quad Q = 10, \quad \rho_0 = 1 \]
\[ \rho_{\text{min}} = 0.5, \quad N_{e \text{max}} = 20, \quad N_g = 10, \quad N_{a \text{min}} = 50 \]
The results of experiment II show that a Walker 24/6/1 constellation can meet the single coverage need in China. The orbital elements of the first satellite in the first orbital plane are

\[ H = 1200 \text{ km}, \quad e = 0, \quad i = 45^\circ \]
\[ \Omega = 288^\circ, \quad \omega = 0^\circ, \quad M = 36^\circ \]

It is possible to derive the orbital elements of other satellites out of the characteristics of Walker constellation.

Table 4 compares the coverage performances calculated in experiment II with those listed in Ref.[5]. On the base of Ref.[5], the data of the coverage rate in sixteen densely populated cities of China are also compared in Table 4.

As shown in Table 4, thirteen cities with 100% single-fold coverage rate can be continuously covered by the constellation in the experiment. In contrast, only five cities meet the same requirements with the method mentioned in Ref.[5]. And the two-fold coverage rate of the former constellation is much better than the latter one.

Moreover, only 777.56 s (less than 13 min) in total were needed to achieve the optimum LEO constellation using the same computer as in the experiment I.

4 Conclusion

In this paper, a new method for satellite constellation optimization (ASA) is proposed to optimize the constellation parameters. Having analyzed the Earth coverage of constellation, a \( n+1 \)-fold coverage rate is presented to assess the coverage performances. Then, by associating the constellation optimization with the ASA, an adaptive ants’ number rule is put forward to improve the algorithm. Simulation results listed in Tables.1-4 show that the improved ASA is feasible and efficient for constellation optimization design.

References


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