Abstract

The present study is to investigate the influence of the variable properties and double dispersion on mixed convection flow over a vertical plate embedded in a power-law fluid saturated porous medium. The variable viscosity is assumed to vary as an inverse linear function of temperature and thermal conductivity as a linear function of temperature. Similarity transformations of the governing partial differential equations are obtained using Lie scaling group transformations. These similarity equations are solved numerically by using the Shooting technique. The numerical results for the non-dimensional velocity, temperature and concentration are displayed graphically for different values of variable viscosity, thermal conductivity, thermal dispersion and solutal dispersion. Local heat and mass transfer are shown in a tabular form. The present results are compared with previously published work and are found to be in good agreement.

Keywords: Mixed convection; Thermal dispersion; Solutal dispersion; Variable viscosity; Thermal conductivity; Power-law fluid.

1. Introduction

In recent years, the study of laminar boundary layer flow, heat and mass transfer in non-Newtonian power-law fluid model in porous media has received a lot of attention. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Many materials such as pastes, slurries, gels, clay coating, intrauterine, drilling mud, polymer metals, etc., are examples of non-Newtonian fluids. The power-law fluid is in engineering, industrial and technological applications such as metal extrusion, lubrication, continuous casting, etc. Many engineering applications involve the study of non-Newtonian fluids. Several investigators have extended the convection of heat and mass transfer problems to fluids exhibiting non-Newtonian rheology. An illustrative example of non-Newtonian power-law fluid flow in porous medium can be found in geothermal and oil reservoir engineering in connection with the production of heavy crude oils. Chamkha and Al-Humoud [1] presented the mixed convection for a non-Newtonian fluid.
power-law fluid from a permeable vertical plate embedded in a fluid saturated porous medium with heat generation or absorption effects. Mukhopadhyay and Mandal [2] studied the effects of velocity slip and thermal slip on magnetohydrodynamic boundary layer mixed convection flow heat transfer over a vertical porous plate in the presence of suction/blowing. The double dispersion has engineering applications such as sensible heat storage beds, ceramic processing, oil reservoir and petroleum recovery, etc. The thermal and solutal dispersion effects become more important when the inertial effects are prevalent. Kairi [3] studied the effects of thermal and solutal dispersion on mixed convection heat and mass transfer in vertical surface embedded in a porous medium. Narayana and Sibanda [4] studied the effect of Soret and double dispersion on MHD mixed convection along a vertical flat plate in a fluid saturated in non-Darcy porous medium. Srinivasacharya et al. [5] investigated the magnetic and double dispersion effects on natural convection heat and mass transfer from a vertical plate in non-Darcy porous media saturated with power-law fluid. Kameswaran and Sibanda [6] presented the thermal dispersion on a non-Newtonian power-law nanofluid over an impermeable vertical plate in porous medium. Afify and Elgazery [7] analyzed similarity solution for the mixed convective heat and mass transfer flow over a vertical surface embedded in a porous medium with double dispersion, radiation and melting effects.

The analysis of convection through porous media with variable viscosity and thermal conductivity are important in several engineering applications such as glass fiber, drawing of plastic films, study of spilling pollutant crude oil over the surface of the seawater, cooling of nuclear reactors, food processing, petroleum reservoir operations, casting and welding in manufacturing processes, etc. Several researchers have considered the effect of variable properties in mixed convection flows. Elaiw et al. [8] studied the vortex instability of non-Darcy mixed convection boundary layer flow on a non-isothermal horizontal plate surface in a saturated porous medium with the effect of variable viscosity. Umavathi and Shekar [9] studied the combined effects of viscous dissipation, temperature dependent viscosity and thermal conductivity on the free convection flow of a viscous fluid in a vertical channel.

In this paper, we investigated the effects of variable viscosity, variable thermal conductivity, thermal dispersion and solutal dispersion on the two dimensional steady mixed convection heat and mass transfer flow in a power-law fluid saturated porous medium. Similarity equations are derived by using the Lie scaling group transformations.

2. Mathematical Formulation

Consider a steady, laminar, viscous incompressible, mixed convection heat and mass transfer boundary layer flow over the vertical plate in a non-Newtonian power-law fluid saturated Darcy porous medium. Choose the two dimensional coordinate system such that the \( \bar{x} \) -axis is along the vertical plate and \( \bar{y} \) -axis normal to the plate. The wall is maintained at constant temperature and concentration \( T_w \) and \( C_w \) respectively, and these values are assumed to be greater than the ambient temperature and concentration \( T_\infty \) and \( C_\infty \) respectively.

By Boussinesq approximation and boundary layer approximation, the governing equations, namely, the equations of continuity, momentum, energy and concentration for the non-Newtonian power-law fluid are given by

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]

\[
\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left[ \frac{gK \rho_\infty}{\mu} \left( \beta_T (T - T_\infty) + \beta_C (C - C_\infty) \right) \right]
\]

\[
\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left[ (D + \bar{C} \alpha) \frac{\partial C}{\partial \bar{y}} \right]
\]

where \( \bar{x} \) and \( \bar{y} \) are the cartesian coordinates, \( \bar{u} \) and \( \bar{v} \) are the velocity components in \( \bar{x} \) and \( \bar{y} \) directions respectively, \( T \) is the temperature, \( C \) is the concentration, \( \beta_T \) and \( \beta_C \) are the thermal and concentration expansion coefficients respectively, \( \nu \) is the kinematic viscosity of the fluid, \( K \) is the permeability, \( K_T \) is the thermal diffusion ratio, \( \alpha \) is the thermal conductivity, \( D \) is the molecular diffusivities, \( \gamma \) is the mechanical thermal dispersion, \( \zeta \) is the mechanical solutal dispersion and \( d \) is the pore diameter, \( n \) is the index in the power-law variation of viscosity, \( n < 1 \) for the
pseudo-plastic fluids (shear-thinning fluid), $n > 1$ for the dilatant fluids (shear-thickening fluids) and $n = 1$ for the Newtonian fluids.

The boundary conditions are

$$\tilde{v} = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \quad \text{and} \quad \tilde{u} = u_\infty, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty \quad (5)$$

The fluid properties are assumed to be isotropic and constant except fluid viscosity and thermal conductivity. The viscosity $\mu$ of the fluid is assumed to be an inverse linear function of temperature and it can be expressed as (see Lai [10])

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T - T_\infty)] \quad \text{or} \quad \frac{1}{\mu} = a(T - T_\infty) \quad (6)$$

where $a = \frac{\dot{\rho}}{\mu}, T_e - T_\infty = -\frac{1}{\beta}, \mu_\infty$ is the coefficient of the viscosity far away from the plate and both $a, T_\infty$ are constants and their values depend on the reference state and the thermal property of the fluid, i.e $\delta$ in general $a > 0$ for liquids and $a < 0$ for gases.

The variable thermal conductivity can be written in the non-dimensional form (see Slattery [11]) as

$$\alpha = \alpha_0(1 + \beta\theta) \quad (7)$$

where $\beta = E(T_w - T_\infty)$ is the thermal conductivity parameter. $E$ is a constant depending on the nature of the fluid.

Introducing the stream function $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ the following dimensionless variables

$$x = \frac{\tilde{x}}{L}, \quad y = \frac{\tilde{y}}{L} Pe^\frac{1}{2}, \quad u = \frac{\tilde{u}L}{\alpha_0 Pe}, \quad v = \frac{\tilde{v}L}{\alpha_0 Pe^\frac{1}{2}} \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (8)$$

into Eq.(2)-(4), we get the following transformed momentum, energy and concentration equations:

$$\Delta_1 = n\left(\frac{\partial \psi}{\partial y}\right)^{n-1} \frac{\partial^2 \psi}{\partial y^2} + \frac{\alpha_0}{\alpha_0} \left[\theta + B\phi\right] \frac{\partial \theta}{\partial y} - \lambda^2 \left[\frac{\partial \theta}{\partial y} + B\frac{\partial \phi}{\partial y}\right] \left[1 - \frac{\theta}{\theta_e}\right] = 0 \quad (9)$$

$$\Delta_2 = \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} - \beta \left(\frac{\partial \theta}{\partial y}\right)^2 - (1 + \beta\theta) \frac{\partial^2 \theta}{\partial y^2} - Pe_y \left[\frac{\partial \psi}{\partial y} \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \psi}{\partial y^2} \frac{\partial \theta}{\partial y}\right] = 0 \quad (10)$$

$$\Delta_3 = \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} - \frac{1}{Le} \frac{\partial^2 \phi}{\partial y^2} - Pe_\psi \left[\frac{\partial \psi}{\partial y} \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \psi}{\partial y^2} \frac{\partial \phi}{\partial y}\right] = 0 \quad (11)$$

The corresponding boundary conditions Eq. (5) now becomes

$$\frac{\partial \psi}{\partial x} = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0 \quad \text{and} \quad \frac{\partial \psi}{\partial y} = 1, \quad \theta = 0, \quad \phi = 0 \quad \text{at} \quad y \to \infty \quad (12)$$

Where $Pe = \frac{U_\infty L}{\alpha_0}$ is the Peclet number. $B = \frac{\beta_e(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ is the Buoyancy ratio, $Le = \frac{\alpha_0}{D}$ is the Lewis number, $Ra = \frac{L}{\alpha_0} \left[\frac{Kg\beta_T(T_w - T_\infty)}{\gamma}\right]^{1/n}$ is the generalized Rayleigh number, $\theta_e = \frac{T_e - T_\infty}{T_w - T_\infty}$ is the variable viscosity. $Pe_\psi = \frac{\gamma U_\infty d}{\alpha_0}$ is the thermal dispersion parameter. $Pe_\psi = \frac{\gamma U_\infty d}{\alpha_0}$ is the solutal parameter. $\lambda^2 = \left(\frac{Ra}{Pe}\right)$ is the mixed convection parameter.

2.1. Application of scaling group of transformation

Finding the similarity solutions of Eqs. (9)-(11) is equivalent to determining the invariant solutions of these equations under a particular continuous one parameter group (see Seddeek [12]) one of the methods is to search for a transformation group from the elementary set of one parameter scaling transformation, defined by the following group \((\Gamma)\)

$$\Gamma : x^* = xe^{\alpha_1}, \quad y^* = ye^{\alpha_2}, \quad \psi^* = \psi e^{\alpha_3}, \quad \theta^* = \theta e^{\alpha_4}, \quad \phi^* = \phi e^{\alpha_5} \quad (13)$$
Here \(\epsilon \neq 0\) is the parameter of the group and \(\alpha\)'s are arbitrary real numbers whose interrelationship will be determined by our analysis. Transformation in Eq.(13) may be treated as a point transformation, transforming the coordinates \((x, y, \psi, \theta, \phi) = (x', y', \psi', \theta', \phi')\).

Using the transformations Eq.(13) in Eqs. (9) - (11) and the property that group transformations Eq. (13) keeps the system invariant, we get the values of the parameters as \(\alpha_1 = 2\alpha_3,\ \ \ \alpha_2 = \alpha_3,\ \ \ \alpha_4 = \alpha_5 = 0\).

The set of transformation \(\Gamma\) reduces to

\[
x^* = xe^{2\alpha_3}, \quad y^* = ye^{2\alpha_3}, \quad \psi^* = \psi e^{2\alpha_3}, \quad \theta^* = \theta, \quad \phi^* = \phi.
\] (14)

Expanding by the Taylor series in power of \(\epsilon\), keeping the term up to the first degree (neglecting higher power of \(\epsilon\)), we get

\[
x^* - x = 2\epsilon\alpha_3 x, \quad y^* - y = \epsilon\alpha_3 y, \quad \psi^* - \psi = \epsilon\alpha_3 \psi, \quad \theta^* = \theta, \quad \phi^* = \phi.
\] (15)

The characteristic equations are

\[
\frac{dx}{2\alpha_3 y} = \frac{dy}{\alpha_3 y} = \frac{d\psi}{\alpha_3 \psi} = \frac{d\theta}{0} = \frac{d\phi}{0}
\] (16)

Solving the above characteristic equation, we have following similarity transformations:

\[
\eta = yx^{-1/2}, \quad \psi = x^{1/2} f(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta)
\] (17)

On the use of Eq. (17) in Eqs.(9)-(11), we get the following similarity equations:

\[
n(f')^{-1} f'' = -\lambda^e (\theta + B\phi) \frac{\theta'}{\theta_e} + \lambda^e (\theta' + B\phi') \left(1 - \frac{\theta}{\theta_e}\right)
\] (18)

\[
\beta(\theta')^2 + (1 + \beta\theta) \theta'' + Pe_\gamma (f''\theta' + f'\theta'') + \frac{1}{2}f\theta' = 0
\] (19)

\[
\frac{1}{Le} \phi'' + Pe_\xi (f'\phi' + f''\phi') + \frac{1}{2}f\phi' = 0
\] (20)

The transformed boundary conditions Eq.(12) becomes

\[
f(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0.
\] (21)

2.2. Heat and Mass transfer coefficients

The non dimensional heat and mass transfer coefficients in terms of local Nusselt number \(Nu\) and the local Sherwood number \(Sh\) are respectively given by:

\[
q_w = -k_e \left[\frac{\partial T}{\partial y}\right]_{y=0} = -(k + k_d) \left[\frac{\partial T}{\partial y}\right]_{y=0} \quad \text{and} \quad q_m = -D_e \left[\frac{\partial C}{\partial y}\right]_{y=0} = -(D + D_d) \left[\frac{\partial C}{\partial y}\right]_{y=0}
\] (22)

The local Nusselt number is defined as: \(Nu_x = \frac{q_w x}{k(T_w - T_\infty)}\) and local Sherwood number \(Sh_x = \frac{q_m x}{D(C_w - C_\infty)}\) are given by

\[
\frac{Nu_x}{Pe^{1/2}} = -\left[1 + Pe_\gamma f'(0)\right] \theta'(0) \quad \text{and} \quad \frac{Sh_x}{Pe^{1/2}} = -\left[1 + Pe_\xi f'(0)\right] \phi'(0)
\] (23)
3. Results and Discussions

The set of ordinary differential Eqs.(18)-(20) subject to the boundary conditions (21) have been solved numerically using the Runge-Kutta fourth order with shooting technique by giving proper guess values for \( f'(0), \theta(0) \) and \( \phi(0) \) respectively. In the present study, the boundary condition for \( \eta \) at \( \infty \) are replaced by sufficiently large value of \( \eta \), where the velocity \( f'(0) \), temperature \( \theta(0) \) and concentration \( \phi(0) \) approach zero. We have discussed the effect of variable viscosity \( \theta \), thermal conductivity \( \beta \), thermal dispersion \( Pe \), and solutal dispersion \( Pe_{\gamma} \) parameters.

Table 1 shows the comparison of the results of the values of \( \lambda \) of the present problem for different values of \( n \) and fixed values of \( Pe_{\gamma} = 0, Pe_{\xi} = 0, B = 0, Le = 1, \theta_e \rightarrow \infty, \beta = 0 \) with the results obtained by Chaoyang et al. [13]. It is shown that these results are in excellent agreement.

<table>
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Figure 1 represents the non dimensional velocity \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) profiles for different values of the variable viscosity \( \theta_e \) and power-law index \( n \) for \( n=0.6 \) (pseudo-plastic fluid), \( n=1.0 \) (Newtonian fluid) and \( n=1.5 \) (dilatant fluid). Figure (1a) shows that increase in the value of variable viscosity tends to increase the momentum boundary layer thickness near the plate and decrepation far away from the plate. While Figure (1b) depicts that the non dimensional temperature \( \theta(\eta) \) of the fluid in the medium is decreases with increase in the value of the variable viscosity. The concentration \( \phi(\eta) \) profile for different values of variable viscosity are given by Figure (1c), which shows that the concentration slightly decreases with increase in the variable viscosity.

The behavior of the non dimensional velocity \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) profiles according to the variation of the thermal conductivity \( \beta \) and power-law index \( n \) is plotted in Figure 2. It is interesting to note that the increase in the thermal conductivity \( \beta \) substantially decreases the velocity \( f'(\eta) \) near the plate and increases far away from the plate as show in Figure (2a). It is observed from Figure (2b) that increasing the thermal conductivity tends to enhance the thermal boundary layer. This is attributed to the fact that the non-linear drag is more pronounced when the velocity is larger. Moreover, Figure (2c) shows that concentration boundary layer strongly decreases with increasing the thermal conductivity.

Figure 3 displays the effect of thermal dispersion and the power-law index \( n \) for fixed values of the parameters on the non dimensional velocity \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) profiles respectively. Figure (3a) shows that enhancing the thermal dispersion increases the non dimensional velocity \( f'(\eta) \). Figure (3b) shows that increasing values of thermal dispersion, increases the non dimensional temperature \( \theta(\eta) \). i.e thermal dispersion enhances the transport of heat along the normal direction to the wall as compared with the case where dispersion is neglected (i.e., \( Pe_{\gamma} = 0 \)). Figure (3c) displays that concentration boundary layer \( \phi(\eta) \) reduces with increasing the value of thermal dispersion.

Figure 4 presents the non dimensional velocity \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) profiles for different values of power-law fluid index \( n \) and solutal dispersion. It is observed from the Figure (4a) that non dimensional velocity \( f'(\eta) \) profile increases with an increase in the solutal dispersion. Also an increase in the value of the solutal dispersion slightly decreases the thermal boundary layer thickness as shown in Figure (4b). Figure (4c) depicts that the concentration \( \phi(\eta) \) profile continuously increases with increasing the value of solutal dispersion. Hence the concentration boundary layer thickness enhances with an enhance in the solutal dispersion parameter.
Table 2: Variation of Heat and Mass Transfer Coefficients for Varying Values of \( n, \theta_e, \beta, Pe_\gamma, Pe_\zeta \) and \( \lambda \).

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Table 2 describes the rate of heat and mass transfer for different values of power law index \( n \), variable viscosity \( \theta_e \), thermal conductivity \( \beta \), thermal dispersion \( Pe_\gamma \) and solutal dispersion \( Pe_\zeta \) parameters for fixed values of \( B = 1.0 \) and \( Le = 1.0 \). It is observed that increasing the value of power-law index \( n \) decreases the heat and mass transfer. It is noticed that enhancing the variable viscosity decreases both the heat and mass transfer coefficient. It can be seen that the increase in the value of thermal conductivity and thermal dispersion parameters decreases the heat transfer, but a reverse trend is observed in the case of mass transfer. An enhancement in the value of solutal dispersion parameter increases in case of heat transfer, but decreases the mass transfer. It can be observed that increase in the value of mixed convection parameter increases the both heat and mass transfer.

4. Conclusion

Lie scaling group transformations are applied to find the similarity solutions of mixed convection flow over a permeable vertical plate in a Darcy power-law fluid saturated porous medium to study the effect of variable viscosity, thermal conductivity, double dispersion. The results obtained from our analysis are as follows:

- Enhancement in the value of variable viscosity increases the velocity near the plate and slightly decreases far away from the plate and decreases in temperature and concentration profiles.
- An increase in the value of thermal conductivity, decreases the concentration and increases the velocity and temperature profiles.
- The higher values of the thermal dispersion parameter, result in higher velocity and temperature distributions but lower concentration distribution.
- Velocity and concentration distributions increases with increase in solutal dispersion parameter while we noticed that opposite results are reported for temperature profile.
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References


Fig. 1: (a) velocity profile (b) temperature profile and (c) the concentration profile for various value of variable viscosity ($\theta_1$).
Fig. 2: (a) velocity profile (b) temperature profile and (c) the concentration profile for various value of thermal conductivity ($\beta$).

Fig. 3: (a) velocity profile (b) temperature profile and (c) the concentration profile for various value of thermal dispersion($Pe_\gamma$).

Fig. 4: (a) velocity profile (b) temperature profile and (c) the concentration profile for various value of solutal dispersion($Pe_\zeta$).