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The s–d transition in heavy mesons

Da-Xin Zhang [a,b]

[a] *Institute of Theoretical Physics, School of Physics, Peking University, Beijing 100871, PR China*
[b] *Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, PR China*

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Abstract

A class of $|\Delta s| = 1$ transition is analyzed in the D_s^+–D^+ and $\overline{B}_s^0$–$\overline{B}^0$ systems. Short distance Wilson coefficients are calculated within HQET. Novel features of the transitions are discussed. We find that these transitions are unobservable in the standard model.

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1. Introduction

The $\Delta I = 1/2$ rule [1] in the $|\Delta s| = 1$ decays of the kaons and the hyperons is a great puzzle for the physicists to explain. Theoretically, using the techniques of operator product expansion and renormalization group equations, one gets the effective Hamiltonian of a set of local four-quark operators whose Wilson coefficients contain the information at high energy. There are big uncertainties in the hadronic matrix elements of these four-quark operators because the calculations are highly model dependent [2]. Thus a comparison between the data and the prediction is not conclusive in testing the standard model. The $\Delta I = 1/2$ rule is attributed either to the uncertainties in the hadronic matrix elements [2] or to the possible existence of new physics beyond the standard model [3].

We note that there are more transitions induced by the $|\Delta s| = 1$ effective Hamiltonian other than the kaon and the hyperon decays. In the standard model, the effective Hamiltonian for the $|\Delta s| = 1$ transitions including QCD-penguins is [4]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \left[\sum_{i=1}^{2} C_i(\mu)\left(Q_i^u - Q_i^c\right) + \sum_{i=3}^{6} C_i(\mu) \sum_{q=uds}^{cb} Q_i^q \right] \tag{1}$$

between the m_W and the m_b scale, where

$$\begin{aligned}
Q_1^u &= (\bar{d}s)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}},\\
Q_2^u &= (\bar{d}_i s_j)_{\mathrm{V-A}}(\bar{u}_j u_i)_{\mathrm{V-A}},\\
Q_3^q &= (\bar{d}s)_{\mathrm{V-A}}(\bar{q}q)_{\mathrm{V-A}},\\
Q_4^q &= (\bar{d}_i s_j)_{\mathrm{V-A}}(\bar{q}_j q_i)_{\mathrm{V-A}},\\
Q_5^q &= (\bar{d}s)_{\mathrm{V-A}}(\bar{q}q)_{\mathrm{V+A}},\\
Q_6^q &= (\bar{d}_i s_j)_{\mathrm{V-A}}(\bar{q}_j q_i)_{\mathrm{V+A}}.
\end{aligned} \tag{2}$$

E-mail address: dxzhang@phy.pku.edu.cn (D.-X. Zhang).

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As the decay processes are concerned, the operators

$$Q_1^c = (\bar{d}s)_{\mathrm{V-A}}(\bar{c}c)_{\mathrm{V-A}},$$
$$Q_2^c = (\bar{d}_i s_j)_{\mathrm{V-A}}(\bar{c}_j c_i)_{\mathrm{V-A}} \tag{3}$$

and

$$Q_{3\cdots 6}^{c,b} \tag{4}$$

in (1) are integrated out at the mass scale $m_c(m_b)$.

As the standard model is assumed, the Wilson coefficients for the operators with $q = c, b$ and $q = u, d, s$ in (1) are related. We will not consider the electroweak penguins whose inclusion is straightforward and less important. Although the operators in (3) and in (4) do not contribute to the decay processes of the kaon and the hyperons, they could be relevant in other interesting processes in the heavy hadron sectors. $Q_{i=1\cdots 6}^c$ could contribute to the D_s^+–D^+ mixing, while $Q_{i=3\cdots 6}^b$ could contribute to the $\overline{B}_s^0$–$\overline{B}^0$ mixing, if suitable modification by going into the heavy quark effective theory (HQET) [5] where only the heavy degrees of freedom are integrated out.

In Section 2 we present the novel features of the transition. The Wilson coefficients and the hadronic matrix elements of the relevant operators in the standard model are calculated in Sections 3 and 4, respectively. We discuss the result in Section 5.

2. Novel features of the transition

To make a naive estimation for the amplitude of the $\overline{B}_s^0$–$\overline{B}^0$ mixing induced by the penguin operators $Q_{i=3\cdots 6}^b$, we compare these three groups of diagrams:

1. box diagrams $(b\bar{d})$–$(d\bar{b})$, $(b\bar{s})$–$(d\bar{b})$ and $(b\bar{s})$–$(s\bar{b})$;
2. box diagrams $(b\bar{d})$–$(s\bar{b})$ and $(b\bar{d})$–$(b\bar{s})$;
3. box diagram $(b\bar{d})$–$(b\bar{s})$ and penguin diagram $(b\bar{d})$–$(b\bar{s})$.

In the first group of diagrams, the first and the third box diagrams are responsible for the B_d–$\overline{B}_d$ and B_s–$\overline{B}_s$ mixings, respectively. The second diagram in the first group is drawn by replacing the s-quark by d-quark in the third diagram. It induces a $|\Delta B| = 2$ but $|\Delta s| = 1$ mixing $\overline{B}_s$–B_d, which is smaller than the B_s–$\overline{B}_s$ mixing in amplitude by a factor V_{td}/V_{ts} but is bigger than the B_d–$\overline{B}_d$ mixing by a factor V_{ts}/V_{td}. In the second group, the second diagram is drawn by exchanging the external lines $s \to b$ and $\bar{b} \to \bar{s}$ in the first diagram. Note that in the box diagrams the internal quark lines run over u, c, t. If the top quark gives the dominant contribution to the second diagram, then these two diagrams differ in amplitude by a factor $\frac{V_{ts}^* V_{tb}}{V_{ts} V_{tb}^*}$ which is purely a phase factor. We know that in the case of $K\overline{K}$ mixing, the power suppressed contribution from the charm quark is larger than highly CKM-suppressed contribution from the top quark, thus the second box diagram is analogously larger than the first diagram in the second group. In the third group, the QCD-penguin diagram is approximately larger than the box diagram by a factor α_s/α_2 in amplitude. We arrive at that the penguin induced $\overline{B}_s^0$–$\overline{B}^0$ mixing is comparable with B_s–$\overline{B}_s$ mixing in amplitude. The amplitude for D_s^+–D^+ mixing is even larger since the tree-diagrams are not GIM suppressed.

Differing from the well known mechanism for the neutral meson mixings of K^0–$\overline{K}^0$, D^0–$\overline{D}^0$ and $B_{d,s}^0$–$\overline{B}_{d,s}^0$, the cases of D_s^+–D^+ and $\overline{B}_s^0$–$\overline{B}^0$ mixings have some novel features. The system under consideration is non-degenerate states. Both the mass difference $\delta^D \equiv m_{D_s} - m_{D_d}$ and $\delta^B \equiv m_{B_s} - m_{B_d}$ are of the order of 100 MeV [6], comparing to the very small upper limit in the K^0–$\overline{K}^0$ due to the CPT invariance. Defining

$$\Delta^D \equiv \frac{1}{2m_D}\langle D_s^+ | H_{\mathrm{eff}} | D^+ \rangle,$$
$$\Delta^B \equiv \frac{1}{2m_B}\langle \overline{B}_s^0 | H_{\mathrm{eff}} | \overline{B}^0 \rangle, \tag{5}$$

the mass matrices of the $D_s - D_d$ and $B_s - B_d$ are

$$\widehat{M}_D = \begin{pmatrix} m_{D_s} & \Delta^D \\ \Delta^{D\dagger} & m_D \end{pmatrix},$$
$$\widehat{M}_B = \begin{pmatrix} m_{B_s} & \Delta^B \\ \Delta^{B\dagger} & m_B \end{pmatrix}. \tag{6}$$

The mass shifts are $\pm|\Delta^{D,B}|^2/\delta^{D,B}$, due to the see-saw mechanism. The additional factors $\Delta^{D,B}/\delta^{D,B}$ suppress the mass shifts strongly to be unobservable, although the transition amplitudes $\Delta^{D,B}$ can be much larger than Δm_K, etc.

In the presence of s–d transition in the heavy meson sector, we need to view the observed states $D^+(1869)$ and $\overline{B}^0(5279)$ as mass eigenstates which contain small amounts of valence $\bar{s}$-quark. Similar observation applies to $D_s^+(1968)$ and $\overline{B}_s^0(5369)$. Consequently, at the B-factories a fraction $|\Delta^{D,B}/\delta^{D,B}|^2$ of the neutral $\overline{B}^0(5279)$ mesons decay as $\overline{B}_s$, which have typical channel like

$$\overline{B}^0(5279) \to (b\bar{s}) \to \big(D_s^+(1968), D_s^{+*}(2112)\big) + l\bar{\nu}. \tag{7}$$

Note that the semileptonic branching ratio of $\overline{B}_s$ into $D_s^{+(*)}$ is about 10% [6], these channels can be observed if we have good mass-reconstruction of the final states. Observation of the D_s^+–D^+ mixing effects need to assume ideal mixing of ρ–ω–ϕ so that ϕ is a pure $s\bar{s}$ state, then

$$D^+(1869) \to (c\bar{s}) \to \phi\bar{l}\nu \tag{8}$$

is a characteristic channel.

3. The short distance analysis

In the HQET, the effective Hamiltonian below the m_Q scale is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{4} C_i^Q(\mu) O_i^Q, \tag{9}$$

where

$$\begin{aligned} O_1^Q &= (\bar{d}s)_{\text{V}-\text{A}}\big(\bar{h}^{(Q)} h^{(Q)}\big)_{\text{V}-\text{A}}, \\ O_2^Q &= (\bar{d}_i s_j)_{\text{V}-\text{A}}\big(\bar{h}_j^{(Q)} h_i^{(Q)}\big)_{\text{V}-\text{A}}, \\ O_3^Q &= (\bar{d}s)_{\text{V}-\text{A}}\big(\bar{h}^{(Q)} h^{(Q)}\big)_{\text{V}+\text{A}}, \\ O_4^Q &= (\bar{d}_i s_j)_{\text{V}-\text{A}}\big(\bar{h}_j^{(Q)} h_i^{(Q)}\big)_{\text{V}+\text{A}}. \end{aligned} \tag{10}$$

$h^{(Q)}(Q = c, b)$ is the heavy quark field in the HQET.

In the case of D_s^+–D^+ mixing all these Q_1 to Q_6 contribute to the matching at $\mu = m_c$

$$\begin{aligned} C_1^c(m_c) &= -C_1(m_c) + C_3(m_c), \\ C_2^c(m_c) &= -C_2(m_c) + C_4(m_c), \\ C_3^c(m_c) &= C_5(m_c), \qquad C_4^c(m_c) = C_6(m_c), \end{aligned} \tag{11}$$

and for $\overline{B}_s^0$–$\overline{B}^0$ mixing only penguin operators contribute

$$\begin{aligned} C_1^b(m_b) &= C_3(m_b), \qquad C_2^b(m_b) = C_4(m_b), \\ C_3^b(m_b) &= C_5(m_b), \qquad C_4^b(m_b) = C_6(m_b). \end{aligned} \tag{12}$$

We now calculate the anomalous dimension matrix in the HQET for the operators O_i^Q and find

$$\hat{\gamma} = \begin{pmatrix} \hat{\gamma}_2 & 0 \\ 0 & \hat{\gamma}_2 \end{pmatrix}, \quad \hat{\gamma}_2 = \frac{\alpha_s}{4\pi}\begin{pmatrix} 0 & 0 \\ 3 & -9 \end{pmatrix}. \tag{13}$$

Using the renormalization group equations

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \widehat{C}^Q = \hat{\gamma}^T C^Q \tag{14}$$

the Wilson coefficients $C_i^Q(\mu)$ are then running to the hadronic scale where the hadronic matrix elements of O_i^Q are to be calculated. We have

$$\begin{aligned} C_{1(3)}^c(\mu) &= C_{1(3)}^c(m_c) - \frac{1}{3}\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{27/58} - 1\right] C_{2(4)}^c(m_c), \\ C_{2(4)}^c(\mu) &= \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{27/58} C_{2(4)}^c(m_c) \end{aligned} \tag{15}$$

for the D_s^+–D^+ mixing, and

$$\begin{aligned} C_{1(3)}^b(\mu) &= C_{1(3)}^b(m_b) - \frac{1}{3}\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{27/58} \times \left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{9/20} - 1\right] C_{2(4)}^b(m_b), \\ C_{2(4)}^b(\mu) &= \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{27/58} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{9/20} \times C_{2(4)}^b(m_b) \end{aligned} \tag{16}$$

for the $\overline{B}_s^0$–$\overline{B}^0$ mixing. The scale independence of $3C_1 + C_2$ and $3C_3 + C_4$ follows simply the form of the anomalous dimension (13).

4. The hadronic matrix elements

For the heavy meson mixings, the matrix elements $\langle D_s^+|O_i^c|D^+\rangle$ and $\langle \overline{B}_s^0|O_i^b|\overline{B}^0\rangle$ are much easier to calculate without the notoriously difficulties in the hadronic matrix elements of Q_i for the s-quark decay processes. Lacking lattice calculations of these matrix elements at present, we use $SU(3)$ symmetry to relate them to those matrix elements of four-quark operators

$\langle \overline{B}^0_{(s)} | (\bar{q}q)(\bar{h}^Q h^Q) | \overline{B}^0_{(s)} \rangle$

and

$\langle \overline{B}^0_{(s)} | (\bar{q}_i q_j)(\bar{h}^Q_j h^Q_i) | \overline{B}^0_{(s)} \rangle$

and get

$$\langle \overline{B}_s^0 | O_1^b | \overline{B}^0 \rangle = \langle \overline{B}_s^0 | O_3^b | \overline{B}^0 \rangle \simeq \frac{1}{3} B_1 f_B^2 M_B^2,$$
$$\langle \overline{B}_s^0 | O_2^b | \overline{B}^0 \rangle = \langle \overline{B}_s^0 | O_4^b | \overline{B}^0 \rangle \simeq B_1 f_B^2 M_B^2 \quad (17)$$

and similarly for $\langle D_s^+ | O_i^c | D^+ \rangle$, with $B_1 \simeq 1$ calculated by QCD sum rules [7].

5. Results and discussions

Within the HQET, the Wilson coefficients are calculated at a scale μ_0 where $\alpha_s(\mu_0) \simeq 1$ to match the matrix elements [8]. We also take $f_B = f_D = 200$ MeV in estimations. We get

$$\Delta^D = 1.4 \times 10^{-7},$$
$$\Delta^B = 2.8 \times 10^{-8}. \quad (18)$$

Correspondingly, the mixing effects is 2×10^{-12} for the D_s^+–D^+ system and is 5×10^{-14} for the $\overline{B}_s^0$–$\overline{B}^0$ system in probabilities. We find that these small effects are unobservable in the standard model.

Could any new physics exist beyond the standard model, the Wilson coefficients for the heavy quark operators in (3) and (4) are not necessary to be related to those which are relevant to the kaon and the hyperon decays. Furthermore, new operators with other Lorentz structures are possibly relevant to the processes discussed here. To extract a bound of order $\mathcal{O}(0.5)$ on these Wilson coefficients at the hadronic scale, $\overline{B}^0$ decay events of a number of 10^{13} will be the minimal if the new physics which induces the $\overline{B}_s^0$–$\overline{B}_d^0$ transition is at the scale of 1 TeV, provided these events are good reconstructed and the semileptonic decay fraction is 10%. It will be very interesting to construct such kind of new physics models which will be relevant to the processes discussed here.

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References

[1] M.K. Gaillard, B.W. Lee, Phys. Rev. Lett. 33 (1974) 108; M.B. Wise, E. Witten, Phys. Rev. D 20 (1979) 1216; J.F. Donoghue, E. Golowich, B.R. Holstein, Dynamics of the Standard Model, Cambridge Univ. Press, Cambridge, England, 1992.
[2] S. Bertolini, J.O. Eeg, M. Fabbrichesi, E.I. Lashin, Nucl. Phys. B 514 (1998) 63, hep-ph/9705244; M. Harada, Y. Keum, Y. Kiyo, T. Morozumi, T. Onogi, N. Yamada, Phys. Rev. D 62 (2000) 014002, hep-ph/9910201; Y.L. Wu, Phys. Rev. D 64 (2001) 016001, hep-ph/0012371.
[3] A.L. Kagan, Phys. Rev. D 51 (1995) 6196, hep-ph/9409215; C.S. Huang, W.J. Huo, Y.L. Wu, Phys. Rev. D 64 (2001) 016009, hep-ph/0005227.
[4] G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125, hep-ph/9512380.
[5] N. Isgur, M.B. Wise, Phys. Lett. B 232 (1989) 113; N. Isgur, M.B. Wise, Phys. Lett. B 237 (1990) 527; For reviews, see: M. Neubert, Phys. Rep. 245 (1994) 259, hep-ph/9306320.
[6] D.E. Groom et al., Particle Data Group Collaboration, Eur. Phys. J. C 15 (2000) 1.
[7] M.S. Baek, J. Lee, C. Liu, H.S. Song, Phys. Rev. D 57 (1998) 4091, hep-ph/9709386; H.Y. Cheng, K.C. Yang, Phys. Rev. D 59 (1999) 014011, hep-ph/9805222; D. Pirjol, N. Uraltsev, Phys. Rev. D 59 (1999) 034012, hep-ph/9805488.
[8] B. Grinstein, W. Kilian, T. Mannel, M.B. Wise, Nucl. Phys. B 363 (1991) 19.