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Nonlinear Decoupling Sliding Mode Control of Permanent Magnet Linear Synchronous Motor Based on α -th Order Inverse System Method

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Abstract

In this paper, a nonlinear dynamic decoupling controller is proposed for the permanent magnet linear synchronous motor (PMLSM) servo system to improve dynamic operating performance. Firstly, the reversibility of the PMLSM mathematical model is analyzed, and it is proved that the system is reversible. Then an inverse system method is applied to the PMLSM servo system, and it is decoupled into a linear velocity subsystem and a linear current subsystem based on the α -th order inverse system method. Considering the both ideal linear subsystems are sensitive to parameter disturbances and various disturbances, a variable rate reaching law approach based subsystem sliding mode controller for higher system stability and robustness is proposed. Finally, simulation results are provided to demonstrate the effectiveness of the proposed control method.

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Keywords: Permanent magnet linear synchronous motor; Inverse system; Decoupling control; Sliding mode; Reaching law

1. Introduction

Due to its high force density, low losses and high direct-drive dynamic performance with fast speed and better accuracy, the permanent magnet linear synchronous motor (PMLSM) is increasing applied to the direct-drive mechanical system as actuator in recent years[1-3]. Reported applications include

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semiconductor manufacturing equipment, X-Y driving devices, industrial robots, etc.

As is well-known, the PMLSM is a complicated multi-variable nonlinear strong-coupling system. There are nonlinear couplings among the electromagnetic thrust force, flux linkage and current. Besides, the precision of PMLSM mathematical model is easier to be affected by load disturbance, parameter variations and thrust force ripple, since the direct-drive type mechanical configuration. In order to achieve high servo control precision, it is necessary to realize nonlinear decoupling and robust control of PMLSM.

The field-oriented control (FOC) and the direct thrust control (DTC) are two basic decoupling control approaches of PMLSM[4-8]. However, the FOC approach just can realize a kind of approximate decoupling performance in practice. And only when the flux linkage reaches steady state and remains a constant, the decoupling relationship between the velocity and the flux linkage should be met. Moreover, the FOC approach is very sensitive to variation of the model parameters, which may badly influence the PMLSM decoupling and control performance. The conventional DTC approach realizes a kind of part decoupling performance, it uses look-up table method and Bang-Bang control scheme to achieve the decoupling relationship between the electromagnetic thrust force and the flux linkage. Consequently, it leads to some main drawbacks including high current and thrust force ripple as well as high noise level at low velocity[9-10].

The α -th order inverse system method is a study means of control system design. There involves three major utilizing steps[11]: (1) constructing an α -th order inverse system, which can be realized by feedback linearization technique using the inverse model of system plant; (2) establishing a decoupling linear system via the α -th order inverse system, which it is composed of several linear subsystems; and (3) designing control law for the decoupling linear system to synthesize the system. The α -th order inverse system method takes into account of time varying characteristic of the model parameters, therefore it is good for PMLSM to realize dynamic decoupling control.

In this paper, a nonlinear dynamic decoupling controller is proposed for PMLSM servo system to yield high dynamic operating performance. Using the α -th order inverse system method, PMLSM system is decoupled into a linear velocity subsystem and a linear current subsystem, which can be controlled independently. Consider the both ideal linear subsystems are sensitive to parameter uncertainties and various disturbances, a sliding mode control method based on variable rate reaching law approach[12] is adopted to design the decoupled linear subsystem controllers. Numerical simulations are carried out to demonstrate the effectiveness of the proposed dynamic decoupling control scheme.

2. Analysis of the System Model

2.1. Model of PMLSM

When neglecting the longitudinal end effect, the mathematical model of a PMLSM can be described, in the synchronously rotating d-q frame, by the following differential equations

$$\begin{cases} \frac{di_d}{dt} = -\frac{R_s}{L_d}i_d + \frac{\pi}{\tau_n} \frac{L_q}{L_d} v_m i_q + \frac{1}{L_d} u_d \\ \frac{di_q}{dt} = -\frac{\pi}{\tau_n} \frac{\psi_f}{L_q} v_m - \frac{\pi}{\tau_n} \frac{L_d}{L_q} v_m i_d - \frac{R_s}{L_q} i_q + \frac{1}{L_q} u_q \\ \frac{dv_m}{dt} = \frac{3\pi n_p}{2\tau_n M} [(L_d - L_q) i_d + \psi_f] i_q - \frac{B}{M} v_m - \frac{F_\Sigma}{M} \end{cases} \quad (1)$$

where R_s is the mover resistance; u_d , u_q , i_d , i_q , L_d and L_q are the d-axis and q-axis mover voltages, currents and inductances respectively; ψ_f is the permanent magnet flux linkage; v_m is the mover velocity; F_Σ

stands for the load torque; M is the total mass of the mover and mechanical load; B is the friction coefficient; τ_n is the pole pitch; n_p is the number of pole pairs.

$$y = [y_1, y_2]^T = [h_1(x), h_2(x)]^T = [i_d^2 + i_q^2, v_m]^T \tag{2}$$

State variables are chosen as

$$x = [x_1, x_2, x_3]^T = [i_d, i_q, v_m]^T$$

Input variables are chosen as

$$u = [u_1, u_2]^T = [u_d, u_q]^T$$

Consider that PMLSM is a surface-mounted PMLSM. As a result, $L_d = L_q = L$. Then, Eq.(1) can be simplified as

$$\begin{cases} \dot{x}_1 = k_1 x_1 + k_2 x_2 x_3 + k_3 u_1 \\ \dot{x}_2 = k_4 x_3 - k_2 x_1 x_3 + k_1 x_2 + k_3 u_2 \\ \dot{x}_3 = k_5 x_2 + k_6 x_3 + k_7 F_\Sigma \end{cases} \tag{3}$$

where $k_1 = -R_s / L$, $k_2 = \pi / \tau_n$, $k_3 = 1 / L$, $k_4 = -\pi \psi_f / (\tau_n L)$, $k_5 = 3\pi n_p \psi_f / (2\tau_n M)$, $k_6 = -B / M$, $k_7 = -1 / M$.

2.2. Analysis of the System Reversibility

Applying to the Interactor algorithm, we take the derivatives of $y = [y_1, y_2]^T$. Then, the following equations can be obtained

$$\begin{cases} y_1^{(1)} = L_f h_1(x) = 2k_1 x_1^2 + 2k_4 x_2 x_3 + 2k_3 x_1 u_1 + 2k_3 x_2 u_2 \\ y_2^{(1)} = L_f h_2(x) = k_5 x_2 + k_6 x_3 + k_7 F_\Sigma \\ y_2^{(2)} = L_f^2 h_2(x) = -k_2 k_5 x_1 x_3 + k_1 k_5 x_2 + k_4 k_5 x_3 + k_6 \dot{x}_3 + k_3 k_5 u_2 \end{cases} \tag{4}$$

The Jacobi with respect to the input variables $u = [u_1, u_2]^T$ as follows

$$A(x, u) = \begin{bmatrix} \frac{\partial}{\partial u_1} L_f h_1(x) & \frac{\partial}{\partial u_2} L_f h_1(x) \\ \frac{\partial}{\partial u_1} L_f^2 h_2(x) & \frac{\partial}{\partial u_2} L_f^2 h_2(x) \end{bmatrix} = \begin{bmatrix} 2k_3 x_1 & 2k_3 x_2 \\ 0 & k_3 k_5 \end{bmatrix} \tag{5}$$

From Eq.(5), it is easy to obtain that the $\det[A(x, u)] = 2k_3^2 k_5 x_1$. Since $x_1 \neq 0$, the $\text{rank}[A(x, u)] = 2$, which is equal to the number of the system output variables. So the matrix $A(x, u)$ is nonsingular. The relative orders of the system are $\alpha = [\alpha_1, \alpha_2]^T = [1, 2]^T$. As $\alpha_1 + \alpha_2 = 3$, the system is invertible. So the state feedback linearization method can be adopted to realize system decoupling.

2.3. Linearization Decoupling Model

According to the implicit function existence theorem, we suppose

$$\begin{cases} y_1^{(1)} = L_f h_1(x) = v_1 \\ y_2^{(2)} = L_f^2 h_2(x) = v_2 \end{cases} \tag{6}$$

Then, from Eq.(4), we can obtain the analytic expressions of state feedback formulas are as follows

$$\begin{cases} u_1 = \frac{k_5 v_1 - 2x_2 v_2 + 2k_6 x_2 \dot{x}_3}{2k_3 k_5 x_1} - \frac{k_1 x_1 + k_2 x_2 x_3}{k_3} \\ u_2 = \frac{v_2 - k_6 \dot{x}_3}{k_3 k_5} + \frac{k_2 x_1 x_3 - k_4 x_3 - k_1 x_2}{k_3} \end{cases} \quad (7)$$

All the state variables in the analytic expressions Eq.(7) are measurable, and we can describe the α -th order inverse system equation as

$$u = [u_1, u_2]^T = \phi(x, y_1^{(1)}, y_2^{(2)}) = \phi(x, v_1, v_2) \quad (8)$$

After adopting the α -th order inverse system Eq.(8), the system will be turned into a linear decoupling system which the input and output variables are turned into $\{v_1, v_2\}$ and $\{y_1, y_2\}$ respectively. Fig.(1) shows the schematic diagram of the linear system.

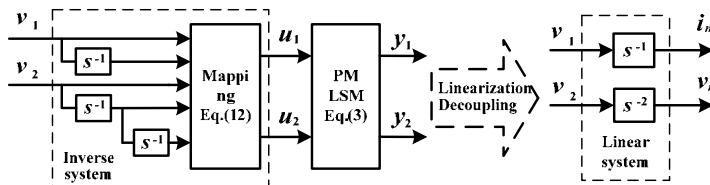


Fig. (1).-Linearization-decoupling-schematic-diagram-based-on- α -th-inverse-system-method

As Fig.(1) shown, the linear system is composed of a first-order current subsystem and a second-order velocity subsystem, and the two subsystems are linear and decoupling.

3. Sliding mode controller design

The two subsystems can be synthesized using the sliding mode control theory. The first step in designing SMC is defining the sliding surface. Let the two subsystem sliding surfaces be

$$\begin{cases} s_1 = e_1 \\ s_2 = c e_2 + \dot{e}_2 \end{cases} \quad (9)$$

where $s_1(t)$ and $s_2(t)$ are the sliding surfaces of the current subsystem and the velocity subsystem respectively; c is the optional sliding surface coefficient, which meets $c > 0$; $e_1(t)$ and $e_2(t)$ are the current tracking error and the velocity tracking error, which are expressed as $e_1 = r_1 - y_1$, $e_2 = r_2 - y_2$, where r_1 and r_2 are the current reference and the velocity reference.

Substituting $e_1(t)$ and $e_2(t)$ into Eq.(9) leads to

$$\begin{cases} s_1 = r_1 - y_1 \\ s_2 = c(r_2 - y_2) + \dot{r}_2 - \dot{y}_2 \end{cases} \quad (10)$$

The second step in designing SMC is to determine a sliding mode control law such that the sliding surface approaches zero and is sustained thereafter.

To ensure the occurrence of the sliding motion, and to avoid chattering, two variable rate reaching laws for the two subsystems are proposed as

$$\begin{cases} \dot{s}_1 = -\varepsilon_1 \|z_1\| \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{s}_2 = -\varepsilon_2 \|z_2\| \operatorname{sgn}(s_2) - k_2 s_2 \end{cases} \quad (11)$$

where k_1 , k_2 , ε_1 and ε_2 are the positive constant gains; $\|z_1\|$ and $\|z_2\|$ are the current subsystem and the velocity subsystem state norms, $\|z_1\| = |e_1|$, $\|z_2\| = |e_2| + |\dot{e}_2|$.

Thus, taking the derivative of Eq.(10) with respect to time, then substituting Eq.(11) and employing Eq(6), we can obtain the sliding mode control laws of the current subsystem and the velocity subsystem as

$$\begin{cases} v_1 = \dot{i}_1 + \varepsilon_1 \|z_1\| \operatorname{sgn}(s_1) + k_1 s_1 \\ v_2 = c(\dot{i}_2 - \dot{y}_2) + \varepsilon_2 \|z_2\| \operatorname{sgn}(s_2) + k_2 s_2 + \dot{i}_2 \end{cases} \quad (12)$$

Furthermore, from Eq.(10) and Eq.(11), there can easily find that

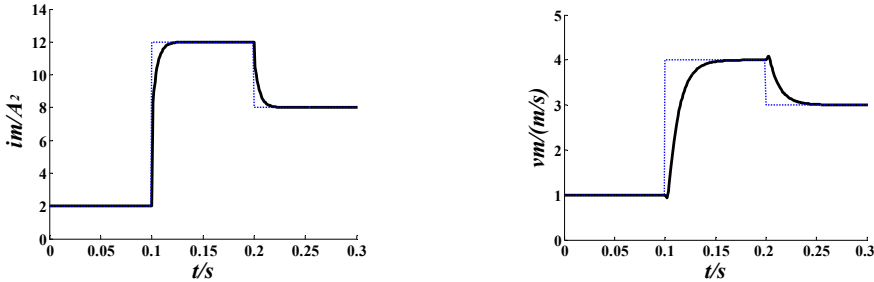
$$\begin{cases} s_1 \dot{s}_1 = -\varepsilon_1 \|z_1\| \operatorname{sgn}(s_1) s_1 - k_1 s_1^2 = -\varepsilon_1 \|z_1\| |s_1| - k_1 s_1^2 < 0 \\ s_2 \dot{s}_2 = -\varepsilon_2 \|z_2\| \operatorname{sgn}(s_2) s_2 - k_2 s_2^2 = -\varepsilon_2 \|z_2\| |s_2| - k_2 s_2^2 < 0 \end{cases} \quad (13)$$

Obviously, the negative definiteness of $s_1 \dot{s}_1$ and $s_2 \dot{s}_2$ infer that the asymptotical convergence of s_1 and s_2 . Therefore, the control law v_1 and v_2 in Eq.(12) actually achieve a stable sliding mode control system.

4. Simulation Results

The PMLSM parameters are $R_s=2.4\Omega$, $L_d=L_q=27.8\text{mH}$, $M=6.8\text{kg}$, $B=2\text{Ns/m}$, $\psi_f=0.45\text{Wb}$, $\tau_n=30\text{mm}$, $n_p=1$. The controller parameters are $c=100$, $\varepsilon_1=10$, $\varepsilon_2=20$, $k_1=200$, $k_2=1000$.

Fig.(2) shows the tracking response curves of the real current i_m and the real velocity v_m with the PMLSM initial conditions of $F_\Sigma=60\text{N}$, $i_{m0}=2\text{A}$, $v_{m0}=1\text{m/s}$.



(a) real current i_m tracking response (b) real velocity v_m tracking response

Fig.(2). Decoupling control with constant load

From Fig.(2), we can see that both the real current i_m and the real velocity v_m can track their own reference input signal. The coupling characteristic between the current subsystem and the velocity subsystem is weak.

decoupling control method based on the α -th order inverse system theory and the S- a good control performance.

Fig.(3) shows the transient response curves of the velocity v_m , the d-axis current i_d and the q-axis current i_q , with the initial conditions of $F_\Sigma=60\text{N}$, $i_{m0}=2\text{A}$, $v_{m0}=1\text{m/s}$, and there has a step load disturbance of $\Delta F_\Sigma=+240\text{N}$ at $t=0.1\text{s}$ and $\Delta F_\Sigma=-180\text{N}$ at $t=0.3\text{s}$.

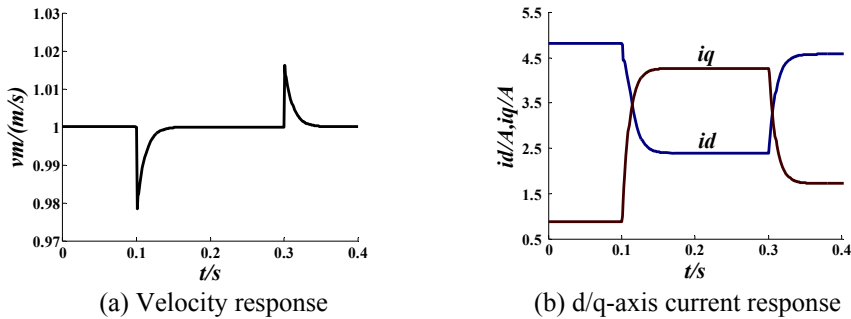


Fig.(3). Decoupling control with load disturbance

From Fig.(3), we can see that when the system subjects a step load disturbance, the q-axis current i_q has ability to increase or decrease in very fast speed to resist the influence of external mechanical force disturbance. As the same time, the d-axis current i_d can adjust relatively to guarantee the real current i_m to maintain at an invariable value. During the entire period of load disturbance, the adjust response of the current i_q and i_d are rapid and without over-shoot, the velocity fluctuation is small and has no steady-state error. Fig.(3) shows that the proposed α -th order inverse dynamic decoupling control method can ensure PMLSM to meet the need of high accuracy servo application with load disturbance condition.

5. Conclusions

In this paper, we have developed an inverse dynamic decoupling controller for PMLSM servo system application. Using α -th order inverse system method, the PMLSM system is decoupled into a linear current subsystem and a linear velocity subsystem, each subsystem has no coupling and can be controlled independently. A variable rate reaching law sliding mode control scheme is adopted to design the two subsystems controllers. The controllers can alleviate the chattering and reduce steady state tracking error. The simulation results have shown that the proposed α -th order inverse dynamic decoupling controller is effectiveness and has good dynamic and static performance.

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